Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.1-Sine/72-4.1.1.3-g-tan-^p-a+b-sin-^m

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September 27, 2022 Compiled on September 27, 2022 at 4:52am

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [208]. This is test number [72].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.04 (206)	0.96 (2)
Mathematica	97.60 (203)	2.40 (5)
Fricas	85.58 (178)	14.42 (30)
Maple	85.58 (178)	14.42 (30)
Mupad	74.04 (154)	25.96 (54)
Giac	74.04 (154)	25.96 (54)
Maxima	68.27 (142)	31.73 (66)
Sympy	2.40 (5)	97.60 (203)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

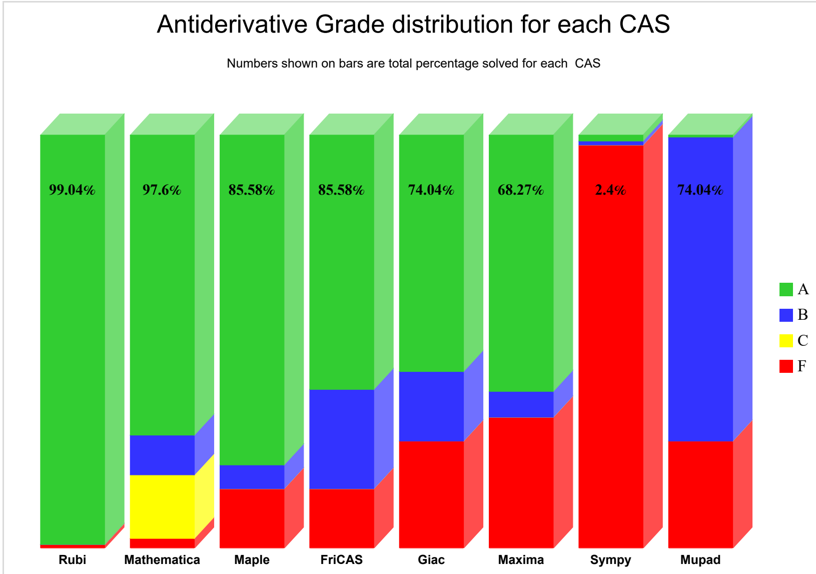
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

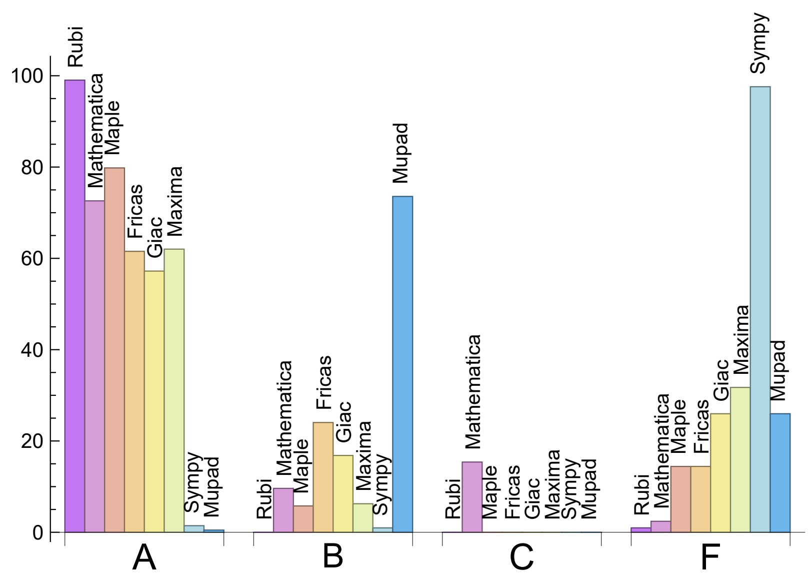
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.04	0.00	0.00	0.96
Maple	79.81	5.77	0.00	14.42
Mathematica	72.60	9.62	15.38	2.40
Maxima	62.02	6.25	0.00	31.73
Fricas	61.54	24.04	0.00	14.42
Giac	57.21	16.83	0.00	25.96
Sympy	1.44	0.96	0.00	97.60
Mupad	N/A	73.56	0.00	25.96

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	2	100.00 %	0.00 %	0.00 %
Mathematica	5	100.00 %	0.00 %	0.00 %
Maple	30	100.00 %	0.00 %	0.00 %
Fricas	30	86.67 %	13.33 %	0.00 %
Giac	54	55.56 %	40.74 %	3.70 %
Maxima	66	69.70 %	7.58 %	22.73 %
Sympy	203	95.07 %	1.97 %	2.96 %
Mupad	54	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

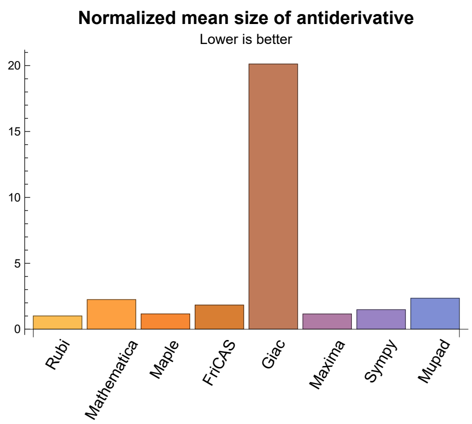
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

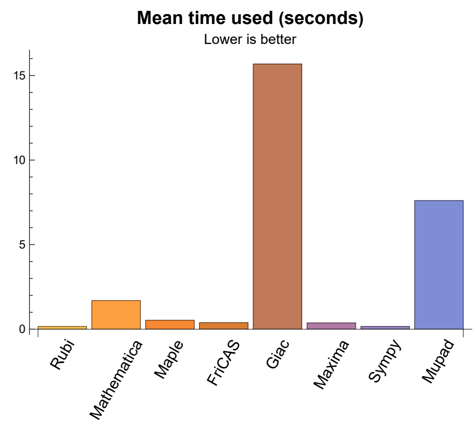
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	133.35	1.01	113.00	1.00
Mathematica	1.69	298.96	2.25	113.00	1.00
Maple	0.53	141.37	1.16	119.00	1.06
Maxima	0.37	125.56	1.16	95.00	1.00
Fricas	0.39	266.66	1.83	157.50	1.48
Sympy	0.16	90.00	1.48	78.00	1.73
Giac	15.68	1719.32	20.12	156.50	1.44
Mupad	7.60	306.68	2.35	231.50	2.31

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
5
10
15
20
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
5
10
15
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{208}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{115, 119\}$

Mathematica $\{114, 115, 117, 118, 123, 124, 126, 127, 128, 129, 136, 138, 203, 204, 205, 206, 207\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

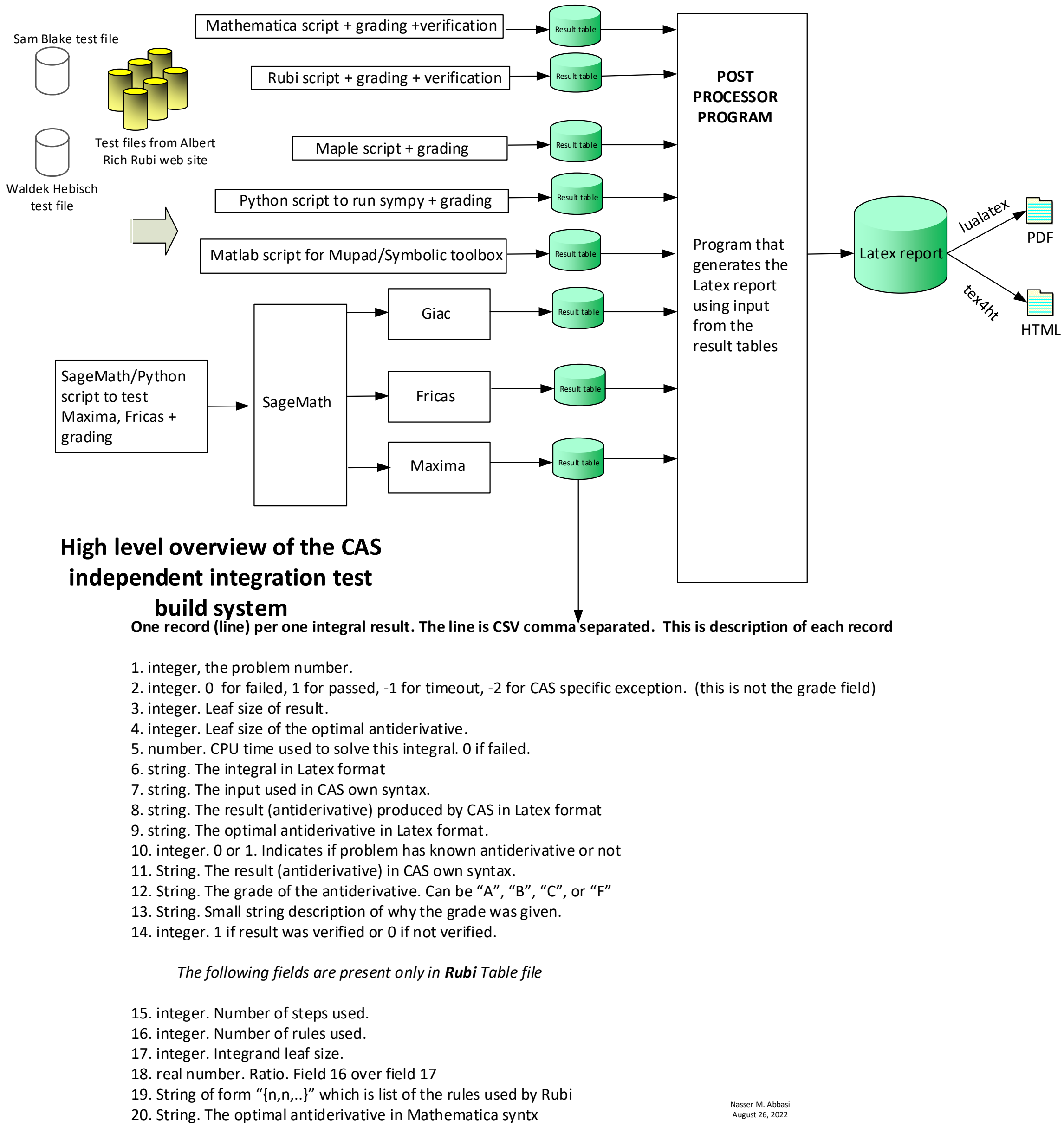
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 208 }

B grade: { }

C grade: { }

F grade: { 206, 207 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 94, 96, 97, 98, 99, 100, 101, 102, 120, 130, 131, 132, 133, 134, 137, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 207, 208 }

B grade: { 21, 42, 55, 56, 57, 58, 59, 60, 88, 89, 90, 93, 105, 106, 110, 126, 129, 158, 179, 206 }

C grade: { 11, 12, 13, 91, 92, 95, 103, 104, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 127, 128, 136, 138, 147, 148, 149, 203, 204, 205 }

F grade: { 121, 122, 125, 135, 139 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 27, 28, 31, 32, 33, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 208 }

B grade: { 17, 25, 26, 29, 30, 34, 35, 38, 39, 53, 54, 60 }

C grade: { }

F grade: { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 96, 100, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 182, 183, 184, 185, 186, 193, 194, 195, 196, 197, 208 }

B grade: { 53, 54, 57, 58, 59, 60, 87, 88, 89, 90, 99, 181, 192 }

C grade: { }

F grade: { 91, 92, 93, 94, 95, 97, 98, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 176, 177, 178, 179, 180, 187, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 14, 15, 16, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 91, 92, 95, 96, 99, 100, 103, 107, 111, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 183, 184, 187, 188, 198, 199, 208 }

B grade: { 10, 11, 12, 13, 17, 24, 30, 57, 58, 59, 60, 74, 84, 88, 89, 90, 93, 94, 97, 98, 101, 102, 104, 105, 106, 108, 109, 110, 112, 113, 114, 147, 148, 149, 181, 182, 185, 186, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202 }

C grade: { }

F grade: { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

2.1.6 Sympy

A grade: { 22, 56, 208 }

B grade: { 32, 40 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207 }

2.1.7 Giac

A grade: { 4, 5, 6, 7, 12, 13, 17, 18, 22, 28, 32, 33, 37, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 97, 98, 101, 102, 103, 104, 107, 108, 110, 111, 112, 114, 142, 143, 144, 148, 149, 152, 153, 154, 157, 158, 159, 162, 163, 164, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 208 }

B grade: { 2, 3, 10, 11, 16, 21, 23, 24, 27, 36, 42, 53, 57, 58, 59, 60, 92, 95, 96, 99, 100, 105, 109, 113, 140, 141, 146, 147, 151, 156, 161, 167, 168, 181, 193 }

C grade: { }

F grade: { 1, 8, 9, 14, 15, 19, 20, 25, 26, 29, 30, 31, 34, 35, 38, 39, 91, 106, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 145, 150, 155, 160, 165, 166, 203, 204, 205, 206, 207 }

2.1.8 Mupad

A grade: { 208 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202 }

C grade: { }

F grade: { 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	121	95	159	0	0	235
N.S.	1	1.00	1.07	1.05	0.83	1.38	0.00	0.00	2.04
time (sec)	N/A	0.053	0.315	0.202	0.275	0.372	0.000	0.000	6.628

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	77	81	51	87	0	26228	154
N.S.	1	1.00	1.08	1.14	0.72	1.23	0.00	369.41	2.17
time (sec)	N/A	0.035	0.083	0.141	0.276	0.367	0.000	171.227	6.469

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	38	23	25	27	0	1456	43
N.S.	1	1.00	1.27	0.77	0.83	0.90	0.00	48.53	1.43
time (sec)	N/A	0.015	0.018	0.132	0.280	0.353	0.000	7.546	6.623

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	22	24	0	23	38
N.S.	1	1.00	1.08	0.83	0.92	1.00	0.00	0.96	1.58
time (sec)	N/A	0.014	0.026	0.092	0.275	0.363	0.000	6.036	6.578

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	60	67	45	69	0	60	146
N.S.	1	1.00	1.11	1.24	0.83	1.28	0.00	1.11	2.70
time (sec)	N/A	0.027	0.086	0.176	0.267	0.361	0.000	4.887	6.540

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	87	101	69	110	0	82	207
N.S.	1	1.00	1.07	1.25	0.85	1.36	0.00	1.01	2.56
time (sec)	N/A	0.032	0.141	0.214	0.280	0.358	0.000	4.607	6.697

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	111	143	91	158	0	104	267
N.S.	1	1.00	0.97	1.24	0.79	1.37	0.00	0.90	2.32
time (sec)	N/A	0.041	0.267	0.238	0.273	0.379	0.000	3.460	7.368

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	110	135	87	116	0	0	351
N.S.	1	1.00	1.09	1.34	0.86	1.15	0.00	0.00	3.48
time (sec)	N/A	0.065	0.045	0.189	0.498	0.349	0.000	0.000	11.223

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	65	88	0	0	231
N.S.	1	1.00	1.12	1.36	0.90	1.22	0.00	0.00	3.21
time (sec)	N/A	0.055	0.038	0.185	0.493	0.350	0.000	0.000	9.418

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	47	59	39	80	0	1008	111
N.S.	1	1.00	1.21	1.51	1.00	2.05	0.00	25.85	2.85
time (sec)	N/A	0.078	0.033	0.132	0.514	0.336	0.000	7.403	6.768

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	75	49	54	84	0	108	108
N.S.	1	1.00	1.83	1.20	1.32	2.05	0.00	2.63	2.63
time (sec)	N/A	0.039	0.034	0.116	0.509	0.369	0.000	4.234	6.906

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	125	86	92	160	0	141	228
N.S.	1	1.00	1.52	1.05	1.12	1.95	0.00	1.72	2.78
time (sec)	N/A	0.059	0.043	0.124	0.497	0.395	0.000	3.722	6.602

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	164	129	125	222	0	199	291
N.S.	1	1.00	1.34	1.06	1.02	1.82	0.00	1.63	2.39
time (sec)	N/A	0.074	0.047	0.164	0.487	0.362	0.000	4.800	6.673

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	75	206	96	168	0	0	283
N.S.	1	1.00	0.63	1.73	0.81	1.41	0.00	0.00	2.38
time (sec)	N/A	0.059	0.164	0.184	0.279	0.369	0.000	0.000	6.567

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	54	136	58	90	0	0	204
N.S.	1	1.00	0.75	1.89	0.81	1.25	0.00	0.00	2.83
time (sec)	N/A	0.044	0.072	0.168	0.292	0.348	0.000	0.000	7.111

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	40	70	43	45	0	6695	178
N.S.	1	1.00	0.77	1.35	0.83	0.87	0.00	128.75	3.42
time (sec)	N/A	0.028	0.026	0.116	0.272	0.360	0.000	5.259	6.669

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	94	53	76	0	47	56
N.S.	1	1.00	0.93	3.13	1.77	2.53	0.00	1.57	1.87
time (sec)	N/A	0.028	0.029	0.208	0.274	0.347	0.000	3.622	6.669

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	86	228	107	206	0	121	392
N.S.	1	1.00	0.65	1.73	0.81	1.56	0.00	0.92	2.97
time (sec)	N/A	0.053	0.150	0.232	0.281	0.397	0.000	4.022	11.182

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	174	251	152	152	0	0	392
N.S.	1	1.00	1.17	1.68	1.02	1.02	0.00	0.00	2.63
time (sec)	N/A	0.124	0.574	0.237	0.512	0.366	0.000	0.000	10.917

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	159	186	120	196	0	0	287
N.S.	1	1.00	1.32	1.55	1.00	1.63	0.00	0.00	2.39
time (sec)	N/A	0.141	0.851	0.243	0.506	0.369	0.000	0.000	10.072

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	145	117	84	125	0	5370	213
N.S.	1	1.00	2.04	1.65	1.18	1.76	0.00	75.63	3.00
time (sec)	N/A	0.068	0.291	0.204	0.494	0.358	0.000	12.030	8.693

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	34	52	47	41	78	38	123
N.S.	1	1.00	0.76	1.16	1.04	0.91	1.73	0.84	2.73
time (sec)	N/A	0.010	0.130	0.085	0.287	0.357	0.079	5.655	6.583

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	94	80	79	105	0	143	201
N.S.	1	1.00	1.27	1.08	1.07	1.42	0.00	1.93	2.72
time (sec)	N/A	0.077	0.396	0.129	0.497	0.357	0.000	7.514	6.520

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	191	146	139	192	0	209	293
N.S.	1	1.00	1.95	1.49	1.42	1.96	0.00	2.13	2.99
time (sec)	N/A	0.119	4.014	0.203	0.512	0.387	0.000	6.022	6.517

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	99	392	133	240	0	0	398
N.S.	1	1.00	0.62	2.45	0.83	1.50	0.00	0.00	2.49
time (sec)	N/A	0.077	0.362	0.230	0.278	0.396	0.000	0.000	6.469

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	204	72	104	0	0	262
N.S.	1	1.00	0.73	2.24	0.79	1.14	0.00	0.00	2.88
time (sec)	N/A	0.051	0.110	0.167	0.280	0.366	0.000	0.000	7.460

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	52	108	57	61	0	28789	281
N.S.	1	1.00	0.74	1.54	0.81	0.87	0.00	411.27	4.01
time (sec)	N/A	0.032	0.032	0.157	0.292	0.359	0.000	32.144	7.265

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	67	116	80	118	0	94	253
N.S.	1	1.00	0.68	1.18	0.82	1.20	0.00	0.96	2.58
time (sec)	N/A	0.046	0.139	0.204	0.292	0.384	0.000	8.263	6.760

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	243	359	209	289	0	0	438
N.S.	1	1.00	1.35	1.99	1.16	1.61	0.00	0.00	2.43
time (sec)	N/A	0.250	3.240	0.273	0.499	0.364	0.000	0.000	11.047

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	177	266	165	220	0	0	371
N.S.	1	1.00	1.49	2.24	1.39	1.85	0.00	0.00	3.12
time (sec)	N/A	0.140	1.358	0.278	0.507	0.371	0.000	0.000	10.500

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	115	167	117	154	0	0	288
N.S.	1	1.00	1.29	1.88	1.31	1.73	0.00	0.00	3.24
time (sec)	N/A	0.092	0.324	0.227	0.537	0.365	0.000	0.000	10.331

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	74	72	54	121	55	156
N.S.	1	1.00	0.70	1.17	1.14	0.86	1.92	0.87	2.48
time (sec)	N/A	0.039	0.214	0.146	0.289	0.358	0.125	6.705	8.919

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	106	94	93	121	0	162	264
N.S.	1	1.00	1.15	1.02	1.01	1.32	0.00	1.76	2.87
time (sec)	N/A	0.099	0.741	0.125	0.500	0.403	0.000	6.295	6.774

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	83	393	109	154	0	0	379
N.S.	1	1.00	0.64	3.05	0.84	1.19	0.00	0.00	2.94
time (sec)	N/A	0.067	0.301	0.210	0.285	0.373	0.000	0.000	7.880

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	76	267	85	116	0	0	320
N.S.	1	1.00	0.71	2.50	0.79	1.08	0.00	0.00	2.99
time (sec)	N/A	0.056	0.105	0.174	0.277	0.354	0.000	0.000	7.564

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	62	143	70	74	0	67058	131
N.S.	1	1.00	0.70	1.62	0.80	0.84	0.00	762.02	1.49
time (sec)	N/A	0.037	0.047	0.178	0.276	0.351	0.000	19.831	6.633

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	78	129	82	131	0	96	298
N.S.	1	1.00	0.76	1.26	0.80	1.28	0.00	0.94	2.92
time (sec)	N/A	0.045	0.093	0.210	0.280	0.376	0.000	15.679	6.398

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	252	360	238	247	0	0	437
N.S.	1	1.00	1.76	2.52	1.66	1.73	0.00	0.00	3.06
time (sec)	N/A	0.147	1.082	0.313	0.510	0.350	0.000	0.000	11.050

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	125	231	181	179	0	0	363
N.S.	1	1.00	1.11	2.04	1.60	1.58	0.00	0.00	3.21
time (sec)	N/A	0.117	0.710	0.267	0.528	0.344	0.000	0.000	10.255

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	57	111	108	70	224	72	237
N.S.	1	1.00	0.66	1.28	1.24	0.80	2.57	0.83	2.72
time (sec)	N/A	0.058	0.269	0.208	0.283	0.354	0.233	3.678	8.591

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	136	136	117	135	0	194	295
N.S.	1	1.00	1.17	1.17	1.01	1.16	0.00	1.67	2.54
time (sec)	N/A	0.116	1.019	0.128	0.514	0.367	0.000	8.123	6.779

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	685	222	218	219	0	274	384
N.S.	1	1.00	4.89	1.59	1.56	1.56	0.00	1.96	2.74
time (sec)	N/A	0.161	6.232	0.226	0.498	0.393	0.000	12.879	6.709

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	283	353	313	291	0	339	454
N.S.	1	1.00	1.43	1.78	1.58	1.47	0.00	1.71	2.29
time (sec)	N/A	0.288	1.074	0.263	0.526	0.394	0.000	14.062	6.853

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	101	115	175	167	0	136	388
N.S.	1	1.00	0.78	0.88	1.35	1.28	0.00	1.05	2.98
time (sec)	N/A	0.117	0.645	0.239	0.279	0.369	0.000	16.364	10.740

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	84	91	130	147	0	116	281
N.S.	1	1.00	0.79	0.86	1.23	1.39	0.00	1.09	2.65
time (sec)	N/A	0.098	0.216	0.214	0.297	0.376	0.000	12.551	10.425

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	67	89	125	0	96	172
N.S.	1	1.00	0.66	0.82	1.09	1.52	0.00	1.17	2.10
time (sec)	N/A	0.080	0.116	0.211	0.290	0.356	0.000	5.718	9.111

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	58	28	43	47	58	0	58	61
N.S.	1	1.57	0.76	1.16	1.27	1.57	0.00	1.57	1.65
time (sec)	N/A	0.047	0.026	0.155	0.313	0.368	0.000	9.666	6.658

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	31	28	0	33	32
N.S.	1	1.00	1.00	0.84	0.97	0.88	0.00	1.03	1.00
time (sec)	N/A	0.027	0.013	0.113	0.275	0.348	0.000	9.183	6.531

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	30	26	30	0	26	23
N.S.	1	1.00	0.75	0.94	0.81	0.94	0.00	0.81	0.72
time (sec)	N/A	0.049	0.023	0.115	0.279	0.353	0.000	6.348	6.576

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	30	49	46	63	0	46	45
N.S.	1	1.00	0.59	0.96	0.90	1.24	0.00	0.90	0.88
time (sec)	N/A	0.065	0.037	0.243	0.286	0.349	0.000	5.302	6.560

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	67	66	96	0	66	63
N.S.	1	1.00	0.90	0.99	0.97	1.41	0.00	0.97	0.93
time (sec)	N/A	0.067	0.094	0.244	0.297	0.368	0.000	4.263	6.795

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	87	86	127	0	86	83
N.S.	1	1.00	0.92	1.04	1.02	1.51	0.00	1.02	0.99
time (sec)	N/A	0.071	0.134	0.324	0.301	0.353	0.000	4.224	6.767

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	146	175	338	95	0	172	99
N.S.	1	1.00	1.74	2.08	4.02	1.13	0.00	2.05	1.18
time (sec)	N/A	0.070	0.213	0.198	0.306	0.345	0.000	8.844	8.486

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	106	130	214	75	0	120	73
N.S.	1	1.00	1.54	1.88	3.10	1.09	0.00	1.74	1.06
time (sec)	N/A	0.067	0.222	0.217	0.298	0.351	0.000	17.136	6.732

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	106	70	90	47	0	68	47
N.S.	1	1.00	2.12	1.40	1.80	0.94	0.00	1.36	0.94
time (sec)	N/A	0.063	0.109	0.187	0.307	0.341	0.000	5.991	6.402

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	48	22	27	42	27	21	21
N.S.	1	1.00	2.09	0.96	1.17	1.83	1.17	0.91	0.91
time (sec)	N/A	0.009	0.032	0.088	0.303	0.344	0.371	8.194	6.428

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	69	44	70	62	0	65	25
N.S.	1	1.00	2.38	1.52	2.41	2.14	0.00	2.24	0.86
time (sec)	N/A	0.038	0.176	0.180	0.283	0.353	0.000	10.356	6.636

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	124	94	155	111	0	127	115
N.S.	1	1.00	2.14	1.62	2.67	1.91	0.00	2.19	1.98
time (sec)	N/A	0.065	0.358	0.213	0.287	0.357	0.000	8.765	6.635

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	189	148	234	155	0	187	183
N.S.	1	1.00	2.30	1.80	2.85	1.89	0.00	2.28	2.23
time (sec)	N/A	0.082	0.518	0.233	0.295	0.355	0.000	8.235	6.662

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	284	200	315	198	0	244	387
N.S.	1	1.00	2.68	1.89	2.97	1.87	0.00	2.30	3.65
time (sec)	N/A	0.100	0.627	0.253	0.308	0.371	0.000	12.649	8.221

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	112	127	202	218	0	146	444
N.S.	1	1.00	0.59	0.67	1.07	1.15	0.00	0.77	2.35
time (sec)	N/A	0.107	1.061	0.269	0.284	0.402	0.000	14.212	10.464

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	91	103	167	198	0	126	361
N.S.	1	1.00	0.62	0.71	1.14	1.36	0.00	0.86	2.47
time (sec)	N/A	0.080	0.299	0.267	0.285	0.365	0.000	8.192	10.504

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	70	79	110	178	0	102	240
N.S.	1	1.00	0.67	0.76	1.06	1.71	0.00	0.98	2.31
time (sec)	N/A	0.062	0.222	0.243	0.273	0.358	0.000	6.293	10.052

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	36	55	70	104	0	90	116
N.S.	1	1.00	0.60	0.92	1.17	1.73	0.00	1.50	1.93
time (sec)	N/A	0.035	0.057	0.218	0.283	0.355	0.000	4.267	7.640

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	37	46	59	0	45	87
N.S.	1	1.00	0.69	0.71	0.88	1.13	0.00	0.87	1.67
time (sec)	N/A	0.035	0.040	0.170	0.285	0.368	0.000	3.604	6.590

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	49	55	76	0	115	103
N.S.	1	1.00	0.75	0.75	0.85	1.17	0.00	1.77	1.58
time (sec)	N/A	0.041	0.048	0.246	0.279	0.373	0.000	4.132	6.528

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	36	57	0	36	36
N.S.	1	1.00	0.69	0.71	0.65	1.04	0.00	0.65	0.65
time (sec)	N/A	0.035	0.050	0.248	0.279	0.349	0.000	5.704	6.335

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	49	46	94	0	46	46
N.S.	1	1.00	1.00	0.67	0.63	1.29	0.00	0.63	0.63
time (sec)	N/A	0.040	0.048	0.322	0.276	0.364	0.000	26.116	6.375

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	78	79	76	127	0	76	76
N.S.	1	1.00	0.61	0.62	0.60	1.00	0.00	0.60	0.60
time (sec)	N/A	0.051	0.098	0.336	0.287	0.348	0.000	11.055	6.542

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	86	162	0	86	85
N.S.	1	1.00	0.61	0.61	0.59	1.12	0.00	0.59	0.59
time (sec)	N/A	0.058	0.143	0.457	0.280	0.360	0.000	14.518	6.633

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	118	119	116	195	0	116	116
N.S.	1	1.00	0.59	0.60	0.58	0.98	0.00	0.58	0.58
time (sec)	N/A	0.070	0.224	0.582	0.278	0.381	0.000	10.218	6.849

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	102	115	188	248	0	136	418
N.S.	1	1.00	0.60	0.67	1.10	1.45	0.00	0.80	2.44
time (sec)	N/A	0.090	0.450	0.315	0.291	0.371	0.000	12.502	10.051

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	82	91	146	226	0	114	302
N.S.	1	1.00	0.65	0.72	1.16	1.79	0.00	0.90	2.40
time (sec)	N/A	0.065	0.245	0.276	0.278	0.377	0.000	9.470	9.925

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	52	67	98	154	0	81	186
N.S.	1	1.00	0.63	0.82	1.20	1.88	0.00	0.99	2.27
time (sec)	N/A	0.041	0.103	0.264	0.302	0.360	0.000	13.213	8.819

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	49	72	104	0	59	148
N.S.	1	1.00	0.70	0.66	0.97	1.41	0.00	0.80	2.00
time (sec)	N/A	0.040	0.126	0.197	0.277	0.354	0.000	9.824	6.648

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	61	61	80	147	0	154	169
N.S.	1	1.00	0.71	0.71	0.93	1.71	0.00	1.79	1.97
time (sec)	N/A	0.050	0.134	0.312	0.282	0.370	0.000	7.562	6.701

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	69	67	75	131	0	174	171
N.S.	1	1.00	0.72	0.70	0.78	1.36	0.00	1.81	1.78
time (sec)	N/A	0.050	0.212	0.302	0.289	0.389	0.000	8.397	6.717

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	46	84	0	46	46
N.S.	1	1.00	0.66	0.67	0.63	1.15	0.00	0.63	0.63
time (sec)	N/A	0.039	0.069	0.298	0.295	0.342	0.000	7.554	6.693

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	66	117	0	66	66
N.S.	1	1.00	0.62	0.63	0.61	1.07	0.00	0.61	0.61
time (sec)	N/A	0.046	0.055	0.371	0.280	0.354	0.000	6.477	6.629

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	86	152	0	86	86
N.S.	1	1.00	0.61	0.61	0.59	1.05	0.00	0.59	0.59
time (sec)	N/A	0.067	0.078	0.453	0.278	0.406	0.000	6.054	6.811

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	86	185	0	86	85
N.S.	1	1.00	0.61	0.61	0.59	1.28	0.00	0.59	0.59
time (sec)	N/A	0.057	0.086	0.567	0.281	0.382	0.000	6.735	6.822

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	112	127	213	290	0	146	476
N.S.	1	1.00	0.57	0.65	1.09	1.49	0.00	0.75	2.44
time (sec)	N/A	0.099	0.953	0.322	0.288	0.384	0.000	24.279	10.661

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	50	81	95	102	0	76	172
N.S.	1	1.00	0.38	0.61	0.72	0.77	0.00	0.58	1.30
time (sec)	N/A	0.065	0.074	0.253	0.290	0.359	0.000	16.615	7.508

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	62	79	121	198	0	91	240
N.S.	1	1.00	0.59	0.75	1.15	1.89	0.00	0.87	2.29
time (sec)	N/A	0.049	0.176	0.323	0.281	0.351	0.000	17.146	10.070

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	73	71	103	196	0	185	228
N.S.	1	1.00	0.69	0.67	0.97	1.85	0.00	1.75	2.15
time (sec)	N/A	0.058	0.547	0.331	0.283	0.401	0.000	7.974	6.610

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	89	89	95	186	0	232	235
N.S.	1	1.00	0.66	0.66	0.70	1.38	0.00	1.72	1.74
time (sec)	N/A	0.060	0.110	0.385	0.284	0.359	0.000	9.920	6.807

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	124	158	356	129	0	146	231
N.S.	1	1.00	0.98	1.24	2.80	1.02	0.00	1.15	1.82
time (sec)	N/A	0.228	0.272	0.271	0.293	0.395	0.000	14.263	7.577

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	315	119	288	369	0	135	203
N.S.	1	1.00	2.92	1.10	2.67	3.42	0.00	1.25	1.88
time (sec)	N/A	0.249	0.278	0.306	0.301	0.371	0.000	7.343	12.052

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	359	143	285	445	0	179	171
N.S.	1	1.00	2.99	1.19	2.38	3.71	0.00	1.49	1.42
time (sec)	N/A	0.185	4.608	0.294	0.292	0.373	0.000	7.788	7.826

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	733	165	279	439	0	204	209
N.S.	1	1.00	5.51	1.24	2.10	3.30	0.00	1.53	1.57
time (sec)	N/A	0.197	6.072	0.333	0.301	0.358	0.000	9.178	7.672

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	195	394	172	0	217	0	0	-1
N.S.	1	1.20	2.43	1.06	0.00	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.635	5.384	1.868	0.000	0.387	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	114	89	0	184	0	455	-1
N.S.	1	1.00	1.13	0.88	0.00	1.82	0.00	4.50	-0.01
time (sec)	N/A	0.121	0.232	2.154	0.000	0.390	0.000	10.711	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	206	125	0	306	0	157	-1
N.S.	1	1.00	2.31	1.40	0.00	3.44	0.00	1.76	-0.01
time (sec)	N/A	0.128	0.666	2.000	0.000	0.388	0.000	3.093	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	309	170	0	417	0	223	-1
N.S.	1	1.00	1.90	1.04	0.00	2.56	0.00	1.37	-0.01
time (sec)	N/A	0.254	1.074	2.366	0.000	0.394	0.000	3.616	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	195	141	139	0	260	0	974	-1
N.S.	1	1.17	0.84	0.83	0.00	1.56	0.00	5.83	-0.01
time (sec)	N/A	0.655	5.381	1.910	0.000	0.400	0.000	154.082	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	46	55	157	52	0	222	-1
N.S.	1	1.00	0.52	0.62	1.78	0.59	0.00	2.52	-0.01
time (sec)	N/A	0.125	1.978	1.050	0.512	0.355	0.000	42.480	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	233	144	0	344	0	193	-1
N.S.	1	1.00	1.93	1.19	0.00	2.84	0.00	1.60	-0.01
time (sec)	N/A	0.210	0.531	2.319	0.000	0.373	0.000	5.066	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	334	196	0	463	0	260	-1
N.S.	1	1.00	1.70	0.99	0.00	2.35	0.00	1.32	-0.01
time (sec)	N/A	0.336	1.039	2.275	0.000	0.371	0.000	9.890	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	208	112	87	301	106	0	1580	-1
N.S.	1	1.38	0.74	0.58	1.99	0.70	0.00	10.46	-0.01
time (sec)	N/A	0.671	5.310	1.565	0.549	0.363	0.000	162.755	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	60	67	207	75	0	1502	-1
N.S.	1	1.00	0.51	0.57	1.75	0.64	0.00	12.73	-0.01
time (sec)	N/A	0.139	5.316	1.356	0.508	0.358	0.000	44.078	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	261	162	0	394	0	235	-1
N.S.	1	1.00	1.73	1.07	0.00	2.61	0.00	1.56	-0.01
time (sec)	N/A	0.287	0.834	2.157	0.000	0.358	0.000	11.803	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	360	222	0	526	0	306	-1
N.S.	1	1.00	1.59	0.98	0.00	2.32	0.00	1.35	-0.00
time (sec)	N/A	0.417	1.162	2.590	0.000	0.375	0.000	5.160	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	241	118	231	0	250	0	229	-1
N.S.	1	1.61	0.79	1.54	0.00	1.67	0.00	1.53	-0.01
time (sec)	N/A	0.615	0.466	2.341	0.000	0.367	0.000	42.653	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	118	130	0	219	0	164	-1
N.S.	1	1.00	1.10	1.21	0.00	2.05	0.00	1.53	-0.01
time (sec)	N/A	0.123	0.177	1.697	0.000	0.360	0.000	20.306	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	138	103	0	288	0	138	-1
N.S.	1	1.00	2.23	1.66	0.00	4.65	0.00	2.23	-0.02
time (sec)	N/A	0.070	0.225	2.480	0.000	0.381	0.000	10.741	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	292	144	0	404	0	0	-1
N.S.	1	1.00	2.16	1.07	0.00	2.99	0.00	0.00	-0.01
time (sec)	N/A	0.400	0.415	2.274	0.000	0.381	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	195	334	289	0	294	0	231	-1
N.S.	1	1.10	1.89	1.63	0.00	1.66	0.00	1.31	-0.01
time (sec)	N/A	0.799	0.254	2.432	0.000	0.412	0.000	46.831	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	128	202	0	259	0	127	-1
N.S.	1	1.00	0.96	1.51	0.00	1.93	0.00	0.95	-0.01
time (sec)	N/A	0.145	0.294	2.053	0.000	0.364	0.000	12.389	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	206	135	0	460	0	215	-1
N.S.	1	1.00	1.82	1.19	0.00	4.07	0.00	1.90	-0.01
time (sec)	N/A	0.149	1.506	2.471	0.000	0.386	0.000	5.309	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	294	144	0	418	0	176	-1
N.S.	1	1.00	2.04	1.00	0.00	2.90	0.00	1.22	-0.01
time (sec)	N/A	0.363	0.524	2.239	0.000	0.375	0.000	9.801	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	260	394	353	0	334	0	186	-1
N.S.	1	1.26	1.90	1.71	0.00	1.61	0.00	0.90	-0.00
time (sec)	N/A	0.974	0.371	2.740	0.000	0.390	0.000	39.976	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	284	266	0	304	0	147	-1
N.S.	1	1.00	1.70	1.59	0.00	1.82	0.00	0.88	-0.01
time (sec)	N/A	0.193	0.267	2.648	0.000	0.372	0.000	23.147	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	451	219	0	589	0	275	-1
N.S.	1	1.00	3.20	1.55	0.00	4.18	0.00	1.95	-0.01
time (sec)	N/A	0.225	0.501	2.231	0.000	0.392	0.000	9.943	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	332	182	0	616	0	252	-1
N.S.	1	1.00	1.74	0.95	0.00	3.23	0.00	1.32	-0.01
time (sec)	N/A	0.636	1.582	2.587	0.000	0.415	0.000	50.435	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	982	982	318	0	0	0	0	0	-1
N.S.	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.894	9.628	0.198	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	290	0	0	0	0	0	-1
N.S.	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	9.366	0.141	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	2692	0	0	0	0	0	-1
N.S.	1	1.00	33.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	19.002	0.105	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	2796	0	0	0	0	0	-1
N.S.	1	1.00	34.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	17.903	0.148	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	128	0	0	0	0	0	-1
N.S.	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	0.530	0.170	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	100	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.337	0.142	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	7.343	0.121	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	5.069	0.162	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	4715	0	0	0	0	0	-1
N.S.	1	1.00	17.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	22.876	1.218	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	1452	0	0	0	0	0	-1
N.S.	1	1.00	7.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	13.241	0.972	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	1.105	0.250	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	232	0	0	0	0	0	-1
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	1.744	0.296	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	710	0	0	0	0	0	-1
N.S.	1	1.00	5.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	3.037	1.347	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	1200	0	0	0	0	0	-1
N.S.	1	1.00	4.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	5.442	0.668	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	367	0	0	0	0	0	-1
N.S.	1	1.00	3.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	1.419	0.181	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	105	0	0	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.167	0.163	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.050	0.121	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.044	0.112	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	68	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.133	0.150	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	83	0	0	0	0	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.183	0.145	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.252	0.765	0.149	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	4043	0	0	0	0	0	-1
N.S.	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	6.257	0.129	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	90	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.115	0.053	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	5048	0	0	0	0	0	-1
N.S.	1	1.00	56.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	16.881	0.120	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.510	0.106	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	77	81	73	90	0	26228	176
N.S.	1	1.00	0.88	0.92	0.83	1.02	0.00	298.05	2.00
time (sec)	N/A	0.053	0.087	0.171	0.280	0.380	0.000	230.657	6.735

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	41	43	45	0	1456	74
N.S.	1	1.00	0.69	0.75	0.78	0.82	0.00	26.47	1.35
time (sec)	N/A	0.028	0.015	0.119	0.290	0.362	0.000	7.517	6.636

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	43	23	22	24	0	23	47
N.S.	1	1.00	1.79	0.96	0.92	1.00	0.00	0.96	1.96
time (sec)	N/A	0.015	0.026	0.100	0.364	0.362	0.000	9.453	6.564

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	60	67	45	69	0	60	146
N.S.	1	1.00	1.11	1.24	0.83	1.28	0.00	1.11	2.70
time (sec)	N/A	0.028	0.138	0.215	0.289	0.366	0.000	23.209	6.631

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	87	101	69	110	0	82	207
N.S.	1	1.00	1.07	1.25	0.85	1.36	0.00	1.01	2.56
time (sec)	N/A	0.036	0.172	0.224	0.435	0.383	0.000	15.242	6.670

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	65	73	0	0	110
N.S.	1	1.00	1.12	1.36	0.90	1.01	0.00	0.00	1.53
time (sec)	N/A	0.055	0.031	0.158	0.576	0.344	0.000	0.000	10.183

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	59	39	47	0	1008	55
N.S.	1	1.00	1.24	1.55	1.03	1.24	0.00	26.53	1.45
time (sec)	N/A	0.042	0.028	0.157	0.542	0.350	0.000	10.582	6.595

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	75	49	54	84	0	108	158
N.S.	1	1.00	1.83	1.20	1.32	2.05	0.00	2.63	3.85
time (sec)	N/A	0.038	0.030	0.099	0.645	0.386	0.000	9.075	6.597

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	125	86	92	160	0	141	225
N.S.	1	1.00	1.52	1.05	1.12	1.95	0.00	1.72	2.74
time (sec)	N/A	0.057	0.033	0.171	0.682	0.379	0.000	9.990	6.294

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	164	129	125	222	0	199	288
N.S.	1	1.00	1.34	1.06	1.02	1.82	0.00	1.63	2.36
time (sec)	N/A	0.071	0.041	0.157	0.668	0.395	0.000	4.370	6.299

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	108	135	105	140	0	0	232
N.S.	1	1.00	0.97	1.22	0.95	1.26	0.00	0.00	2.09
time (sec)	N/A	0.124	0.288	0.193	0.314	0.380	0.000	0.000	6.722

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	69	70	74	0	7855	150
N.S.	1	1.00	0.82	0.88	0.90	0.95	0.00	100.71	1.92
time (sec)	N/A	0.053	0.091	0.191	0.281	0.374	0.000	36.444	6.366

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	40	40	42	0	41	117
N.S.	1	1.00	1.00	0.87	0.87	0.91	0.00	0.89	2.54
time (sec)	N/A	0.027	0.019	0.089	0.322	0.365	0.000	13.323	6.436

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	93	69	115	0	99	221
N.S.	1	1.00	0.83	1.11	0.82	1.37	0.00	1.18	2.63
time (sec)	N/A	0.050	0.158	0.237	0.555	0.390	0.000	9.048	6.385

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	107	157	105	177	0	138	310
N.S.	1	1.00	0.85	1.25	0.83	1.40	0.00	1.10	2.46
time (sec)	N/A	0.072	0.476	0.241	0.282	0.387	0.000	8.734	6.444

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	176	185	119	118	0	0	235
N.S.	1	1.00	1.18	1.24	0.80	0.79	0.00	0.00	1.58
time (sec)	N/A	0.114	0.472	0.220	0.571	0.353	0.000	0.000	10.036

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	77	116	83	81	0	7670	147
N.S.	1	1.00	0.82	1.23	0.88	0.86	0.00	81.60	1.56
time (sec)	N/A	0.087	0.325	0.228	0.715	0.398	0.000	29.434	9.287

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	116	79	79	118	0	148	277
N.S.	1	1.00	1.49	1.01	1.01	1.51	0.00	1.90	3.55
time (sec)	N/A	0.063	0.281	0.153	0.496	0.384	0.000	14.775	7.273

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	293	145	138	218	0	241	584
N.S.	1	1.00	2.20	1.09	1.04	1.64	0.00	1.81	4.39
time (sec)	N/A	0.104	6.176	0.237	0.559	0.376	0.000	18.620	8.997

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	351	216	183	306	0	337	888
N.S.	1	1.00	1.74	1.07	0.91	1.51	0.00	1.67	4.40
time (sec)	N/A	0.126	0.746	0.248	0.504	0.395	0.000	11.451	11.280

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	141	206	162	194	0	0	366
N.S.	1	1.00	0.94	1.37	1.08	1.29	0.00	0.00	2.44
time (sec)	N/A	0.163	0.175	0.194	0.293	0.375	0.000	0.000	6.981

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	90	110	113	116	0	55225	226
N.S.	1	1.00	0.86	1.05	1.08	1.10	0.00	525.95	2.15
time (sec)	N/A	0.078	0.129	0.192	0.280	0.377	0.000	31.933	6.746

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	56	57	66	0	58	118
N.S.	1	1.00	1.00	0.84	0.85	0.99	0.00	0.87	1.76
time (sec)	N/A	0.032	0.019	0.125	0.286	0.394	0.000	6.381	6.678

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	97	118	98	153	0	131	312
N.S.	1	1.00	0.84	1.02	0.84	1.32	0.00	1.13	2.69
time (sec)	N/A	0.068	0.201	0.261	0.337	0.390	0.000	4.234	6.946

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	144	212	142	225	0	185	424
N.S.	1	1.00	0.87	1.28	0.86	1.36	0.00	1.12	2.57
time (sec)	N/A	0.104	0.696	0.246	0.447	0.370	0.000	3.299	6.969

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	226	268	167	157	0	0	297
N.S.	1	1.00	1.03	1.22	0.76	0.71	0.00	0.00	1.35
time (sec)	N/A	0.166	0.462	0.243	0.640	0.344	0.000	0.000	9.197

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	113	169	119	116	0	0	249
N.S.	1	1.00	0.77	1.16	0.82	0.79	0.00	0.00	1.71
time (sec)	N/A	0.130	0.522	0.269	0.591	0.382	0.000	0.000	9.152

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	143	96	95	143	0	199	289
N.S.	1	1.00	1.40	0.94	0.93	1.40	0.00	1.95	2.83
time (sec)	N/A	0.081	0.873	0.154	0.617	0.399	0.000	12.658	6.857

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	355	186	187	293	0	421	405
N.S.	1	1.00	1.83	0.96	0.96	1.51	0.00	2.17	2.09
time (sec)	N/A	0.139	6.184	0.267	0.515	0.375	0.000	6.464	6.809

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	346	289	252	412	0	471	507
N.S.	1	1.00	1.19	0.99	0.87	1.42	0.00	1.62	1.74
time (sec)	N/A	0.172	1.727	0.291	0.657	0.399	0.000	9.466	7.058

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	184	189	288	261	0	343	498
N.S.	1	1.00	0.90	0.93	1.41	1.28	0.00	1.68	2.44
time (sec)	N/A	0.283	0.929	0.430	0.307	0.486	0.000	15.798	7.426

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	117	121	142	157	0	177	217
N.S.	1	1.00	0.93	0.96	1.13	1.25	0.00	1.40	1.72
time (sec)	N/A	0.143	0.337	0.317	0.291	0.392	0.000	6.650	7.035

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	87	71	65	63	0	71	91
N.S.	1	1.00	1.18	0.96	0.88	0.85	0.00	0.96	1.23
time (sec)	N/A	0.051	0.061	0.236	0.282	0.353	0.000	7.248	6.732

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	33	31	0	35	48
N.S.	1	1.00	1.00	0.97	0.97	0.91	0.00	1.03	1.41
time (sec)	N/A	0.030	0.015	0.120	0.286	0.356	0.000	6.181	6.362

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	76	77	118	0	114	144
N.S.	1	1.00	0.77	0.90	0.92	1.40	0.00	1.36	1.71
time (sec)	N/A	0.062	0.111	0.286	0.368	0.363	0.000	8.086	6.632

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	115	137	139	271	0	201	281
N.S.	1	1.00	0.78	0.93	0.94	1.83	0.00	1.36	1.90
time (sec)	N/A	0.101	0.702	0.373	0.389	0.372	0.000	8.129	6.409

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	195	214	0	476	0	241	372
N.S.	1	1.00	1.10	1.21	0.00	2.69	0.00	1.36	2.10
time (sec)	N/A	0.177	0.951	0.367	0.000	0.397	0.000	4.785	9.547

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	152	112	0	305	0	107	148
N.S.	1	1.00	1.58	1.17	0.00	3.18	0.00	1.11	1.54
time (sec)	N/A	0.075	0.137	0.273	0.000	0.366	0.000	8.308	6.369

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	108	109	0	314	0	129	204
N.S.	1	1.00	1.35	1.36	0.00	3.92	0.00	1.61	2.55
time (sec)	N/A	0.173	0.171	0.306	0.000	0.412	0.000	8.954	7.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	350	223	0	633	0	273	654
N.S.	1	1.00	2.27	1.45	0.00	4.11	0.00	1.77	4.25
time (sec)	N/A	0.294	6.129	0.405	0.000	0.480	0.000	4.708	7.144

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	504	390	0	1079	0	490	1099
N.S.	1	1.00	1.64	1.27	0.00	3.51	0.00	1.60	3.58
time (sec)	N/A	0.722	0.947	0.500	0.000	0.672	0.000	4.398	7.130

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	204	205	505	555	0	494	755
N.S.	1	1.00	0.84	0.85	2.09	2.29	0.00	2.04	3.12
time (sec)	N/A	0.534	4.251	0.588	0.400	0.550	0.000	7.516	7.782

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	145	143	274	388	0	248	351
N.S.	1	1.00	0.90	0.89	1.70	2.41	0.00	1.54	2.18
time (sec)	N/A	0.238	0.529	0.460	0.325	0.454	0.000	6.885	7.245

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	162	98	124	195	0	156	158
N.S.	1	1.00	1.49	0.90	1.14	1.79	0.00	1.43	1.45
time (sec)	N/A	0.067	0.206	0.362	0.296	0.398	0.000	5.150	6.758

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	49	47	69	0	51	105
N.S.	1	1.00	0.79	0.92	0.89	1.30	0.00	0.96	1.98
time (sec)	N/A	0.039	0.057	0.184	0.291	0.367	0.000	4.410	6.376

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	96	105	116	259	0	165	235
N.S.	1	1.00	0.84	0.92	1.02	2.27	0.00	1.45	2.06
time (sec)	N/A	0.078	0.422	0.359	0.273	0.394	0.000	5.940	6.379

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	187	172	189	542	0	278	439
N.S.	1	1.00	0.99	0.91	1.01	2.88	0.00	1.48	2.34
time (sec)	N/A	0.127	6.117	0.428	0.290	0.394	0.000	6.489	6.900

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	341	248	0	815	0	406	722
N.S.	1	1.00	1.02	0.74	0.00	2.45	0.00	1.22	2.17
time (sec)	N/A	0.469	1.244	0.519	0.000	0.447	0.000	6.595	9.895

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	169	162	0	569	0	251	313
N.S.	1	1.00	0.84	0.81	0.00	2.84	0.00	1.26	1.56
time (sec)	N/A	0.221	0.724	0.378	0.000	0.398	0.000	4.168	8.675

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	139	156	0	768	0	218	1616
N.S.	1	1.00	1.21	1.36	0.00	6.68	0.00	1.90	14.05
time (sec)	N/A	0.294	0.521	0.420	0.000	0.450	0.000	21.818	8.317

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	403	287	0	1149	0	356	973
N.S.	1	1.00	1.69	1.21	0.00	4.83	0.00	1.50	4.09
time (sec)	N/A	0.467	6.174	0.488	0.000	0.462	0.000	17.408	6.830

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	361	465	0	2011	0	596	1424
N.S.	1	1.00	0.85	1.10	0.00	4.74	0.00	1.41	3.36
time (sec)	N/A	1.003	1.041	0.628	0.000	0.678	0.000	9.684	6.905

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	304	263	730	981	0	585	1229
N.S.	1	1.00	0.95	0.82	2.27	3.06	0.00	1.82	3.83
time (sec)	N/A	0.798	6.239	0.977	0.632	0.751	0.000	14.521	10.735

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	196	197	441	788	0	464	690
N.S.	1	1.00	0.84	0.85	1.90	3.40	0.00	2.00	2.97
time (sec)	N/A	0.398	1.470	0.625	0.314	0.533	0.000	12.511	7.464

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	213	134	228	462	0	257	304
N.S.	1	1.00	1.43	0.90	1.53	3.10	0.00	1.72	2.04
time (sec)	N/A	0.101	1.459	0.516	0.501	0.420	0.000	13.360	6.854

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	66	81	154	0	69	369
N.S.	1	1.00	0.80	0.88	1.08	2.05	0.00	0.92	4.92
time (sec)	N/A	0.044	0.185	0.276	0.547	0.369	0.000	13.900	6.537

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	121	131	156	404	0	154	334
N.S.	1	1.00	0.83	0.90	1.08	2.79	0.00	1.06	2.30
time (sec)	N/A	0.097	0.626	0.449	0.504	0.401	0.000	7.393	6.790

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	195	207	236	754	0	327	563
N.S.	1	1.00	0.88	0.94	1.07	3.41	0.00	1.48	2.55
time (sec)	N/A	0.159	4.893	0.570	0.481	0.417	0.000	8.095	7.262

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	351	354	0	1249	0	632	1099
N.S.	1	1.00	0.74	0.75	0.00	2.64	0.00	1.33	2.32
time (sec)	N/A	0.664	0.670	0.793	0.000	0.430	0.000	12.292	11.220

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	212	258	0	934	0	384	627
N.S.	1	1.00	0.61	0.74	0.00	2.67	0.00	1.10	1.79
time (sec)	N/A	0.404	2.093	0.537	0.000	0.439	0.000	6.093	10.358

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	195	299	0	1394	0	339	1762
N.S.	1	1.00	0.97	1.48	0.00	6.90	0.00	1.68	8.72
time (sec)	N/A	0.526	3.385	0.584	0.000	0.677	0.000	13.761	7.853

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	459	360	0	2027	0	451	1261
N.S.	1	1.00	1.59	1.25	0.00	7.01	0.00	1.56	4.36
time (sec)	N/A	0.709	6.168	0.671	0.000	0.648	0.000	7.209	7.311

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	448	559	0	2571	0	731	1614
N.S.	1	1.00	0.91	1.14	0.00	5.23	0.00	1.49	3.28
time (sec)	N/A	1.386	1.130	0.838	0.000	0.721	0.000	11.785	7.301

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	4791	0	0	0	0	0	-1
N.S.	1	1.00	17.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.271	14.621	1.121	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	2464	0	0	0	0	0	-1
N.S.	1	1.00	13.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	13.430	1.039	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	849	0	0	0	0	0	-1
N.S.	1	1.00	6.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	4.946	0.234	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	0	858	0	0	0	0	0	-1
N.S.	1	0.00	3.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.031	13.038	0.325	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	737	0	695	0	0	0	0	0	-1
N.S.	1	0.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.030	5.287	1.075	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	1.657	0.218	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [91] had the largest ratio of [23]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	19	0.105
2	A	3	2	1.00	19	0.105
3	A	3	2	1.00	17	0.118
4	A	3	2	1.00	17	0.118
5	A	3	2	1.00	19	0.105
6	A	3	2	1.00	19	0.105
7	A	3	2	1.00	19	0.105
8	A	9	5	1.00	19	0.263
9	A	8	5	1.00	19	0.263
10	A	5	5	1.00	19	0.263
11	A	7	6	1.00	19	0.316
12	A	9	7	1.00	19	0.368
13	A	11	7	1.00	19	0.368
14	A	3	2	1.00	21	0.095
15	A	3	2	1.00	21	0.095
16	A	3	2	1.00	19	0.105
17	A	2	2	1.00	21	0.095
18	A	3	2	1.00	21	0.095
19	A	14	9	1.00	21	0.429
20	A	4	4	1.00	21	0.190
21	A	6	5	1.00	21	0.238
22	A	1	1	1.00	12	0.083
23	A	8	6	1.00	21	0.286
24	A	12	7	1.00	21	0.333
25	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	2	1.00	21	0.095
27	A	3	2	1.00	19	0.105
28	A	3	2	1.00	21	0.095
29	A	9	7	1.00	21	0.333
30	A	10	7	1.00	21	0.333
31	A	8	6	1.00	21	0.286
32	A	7	5	1.00	12	0.417
33	A	10	7	1.00	21	0.333
34	A	3	2	1.00	21	0.095
35	A	3	2	1.00	21	0.095
36	A	3	2	1.00	19	0.105
37	A	3	2	1.00	21	0.095
38	A	13	7	1.00	21	0.333
39	A	11	6	1.00	21	0.286
40	A	10	5	1.00	12	0.417
41	A	12	6	1.00	21	0.286
42	A	17	8	1.00	21	0.381
43	A	21	8	1.00	21	0.381
44	A	8	5	1.00	21	0.238
45	A	7	5	1.00	21	0.238
46	A	6	5	1.00	21	0.238
47	A	5	5	1.57	19	0.263
48	A	4	4	1.00	19	0.210
49	A	5	4	1.00	21	0.190
50	A	5	4	1.00	21	0.190
51	A	6	5	1.00	21	0.238
52	A	6	5	1.00	21	0.238
53	A	6	5	1.00	21	0.238
54	A	6	5	1.00	21	0.238
55	A	5	4	1.00	21	0.190
56	A	1	1	1.00	12	0.083
57	A	4	4	1.00	21	0.190
58	A	5	5	1.00	21	0.238
59	A	6	5	1.00	21	0.238
60	A	7	5	1.00	21	0.238

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	3	1.00	21	0.143
62	A	4	3	1.00	21	0.143
63	A	4	3	1.00	21	0.143
64	A	4	3	1.00	19	0.158
65	A	3	2	1.00	19	0.105
66	A	3	2	1.00	21	0.095
67	A	3	2	1.00	21	0.095
68	A	3	2	1.00	21	0.095
69	A	3	2	1.00	21	0.095
70	A	3	2	1.00	21	0.095
71	A	3	2	1.00	21	0.095
72	A	4	3	1.00	21	0.143
73	A	4	3	1.00	21	0.143
74	A	4	3	1.00	19	0.158
75	A	3	2	1.00	19	0.105
76	A	3	2	1.00	21	0.095
77	A	3	2	1.00	21	0.095
78	A	3	2	1.00	21	0.095
79	A	3	2	1.00	21	0.095
80	A	3	2	1.00	21	0.095
81	A	3	2	1.00	21	0.095
82	A	4	3	1.00	21	0.143
83	A	3	2	1.00	21	0.095
84	A	4	3	1.00	19	0.158
85	A	3	2	1.00	21	0.095
86	A	3	2	1.00	21	0.095
87	A	17	5	1.00	21	0.238
88	A	14	8	1.00	21	0.381
89	A	14	8	1.00	21	0.381
90	A	16	6	1.00	21	0.286
91	A	15	10	1.20	23	0.435
92	A	4	4	1.00	23	0.174
93	A	4	4	1.00	23	0.174
94	A	7	7	1.00	23	0.304
95	A	14	9	1.17	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	23	0.130
97	A	5	5	1.00	23	0.217
98	A	8	8	1.00	23	0.348
99	A	10	7	1.38	23	0.304
100	A	4	4	1.00	23	0.174
101	A	6	5	1.00	23	0.217
102	A	10	8	1.00	23	0.348
103	A	17	9	1.61	23	0.391
104	A	4	4	1.00	23	0.174
105	A	4	4	1.00	23	0.174
106	A	11	7	1.00	23	0.304
107	A	20	9	1.10	23	0.391
108	A	5	5	1.00	23	0.217
109	A	6	5	1.00	23	0.217
110	A	10	6	1.00	23	0.261
111	A	23	9	1.26	23	0.391
112	A	6	6	1.00	23	0.261
113	A	7	6	1.00	23	0.261
114	A	16	8	1.00	23	0.348
115	A	10	9	1.00	23	0.391
116	A	4	4	1.00	23	0.174
117	A	3	3	1.00	23	0.130
118	A	3	3	1.00	23	0.130
119	A	8	7	1.00	23	0.304
120	A	4	4	1.00	23	0.174
121	A	3	3	1.00	23	0.130
122	A	3	3	1.00	23	0.130
123	A	10	6	1.00	23	0.261
124	A	8	6	1.00	23	0.261
125	A	6	5	1.00	21	0.238
126	A	4	4	1.00	23	0.174
127	A	10	6	1.00	23	0.261
128	A	13	6	1.00	23	0.261
129	A	4	3	1.00	23	0.130
130	A	4	4	1.00	21	0.190

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	3	1.00	19	0.158
132	A	2	2	1.00	19	0.105
133	A	3	3	1.00	21	0.143
134	A	4	4	1.00	21	0.190
135	A	6	6	1.00	21	0.286
136	A	5	5	1.00	21	0.238
137	A	2	2	1.00	12	0.167
138	A	3	3	1.00	21	0.143
139	A	3	3	1.00	21	0.143
140	A	6	5	1.00	19	0.263
141	A	5	4	1.00	17	0.235
142	A	3	2	1.00	17	0.118
143	A	3	2	1.00	19	0.105
144	A	3	2	1.00	19	0.105
145	A	8	5	1.00	19	0.263
146	A	7	5	1.00	19	0.263
147	A	7	6	1.00	19	0.316
148	A	9	7	1.00	19	0.368
149	A	11	7	1.00	19	0.368
150	A	7	5	1.00	21	0.238
151	A	6	4	1.00	19	0.210
152	A	3	2	1.00	19	0.105
153	A	3	2	1.00	21	0.095
154	A	3	2	1.00	21	0.095
155	A	13	9	1.00	21	0.429
156	A	11	9	1.00	21	0.429
157	A	9	7	1.00	21	0.333
158	A	13	9	1.00	21	0.429
159	A	16	10	1.00	21	0.476
160	A	7	5	1.00	21	0.238
161	A	6	4	1.00	19	0.210
162	A	3	2	1.00	19	0.105
163	A	3	2	1.00	21	0.095
164	A	3	2	1.00	21	0.095
165	A	16	9	1.00	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	14	10	1.00	21	0.476
167	A	11	9	1.00	21	0.429
168	A	17	10	1.00	21	0.476
169	A	21	10	1.00	21	0.476
170	A	5	3	1.00	21	0.143
171	A	4	3	1.00	21	0.143
172	A	3	2	1.00	19	0.105
173	A	4	4	1.00	19	0.210
174	A	3	2	1.00	21	0.095
175	A	3	2	1.00	21	0.095
176	A	13	9	1.00	21	0.429
177	A	8	7	1.00	21	0.333
178	A	7	7	1.00	21	0.333
179	A	7	7	1.00	21	0.333
180	A	9	7	1.00	21	0.333
181	A	5	3	1.00	21	0.143
182	A	4	3	1.00	21	0.143
183	A	3	2	1.00	19	0.105
184	A	3	2	1.00	19	0.105
185	A	3	2	1.00	21	0.095
186	A	3	2	1.00	21	0.095
187	A	16	8	1.00	21	0.381
188	A	12	7	1.00	21	0.333
189	A	8	7	1.00	21	0.333
190	A	8	7	1.00	21	0.333
191	A	10	7	1.00	21	0.333
192	A	5	3	1.00	21	0.143
193	A	4	3	1.00	21	0.143
194	A	3	2	1.00	19	0.105
195	A	3	2	1.00	19	0.105
196	A	3	2	1.00	21	0.095
197	A	3	2	1.00	21	0.095
198	A	22	9	1.00	21	0.429
199	A	18	8	1.00	21	0.381
200	A	9	8	1.00	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	9	7	1.00	21	0.333
202	A	11	7	1.00	21	0.333
203	A	10	6	1.00	23	0.261
204	A	8	6	1.00	23	0.261
205	A	6	5	1.00	21	0.238
206	F	0	0	N/A	0.	N/A
207	F	0	0	N/A	0.	N/A
208	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

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3.124 $\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx$ 600
3.125 $\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx$ 605
3.126 $\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx$ 609
3.127 $\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx$ 612
3.128 $\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx$ 616
3.129 $\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx$ 621
3.130 $\int (a+a\sin(e+fx))^m\tan^3(e+fx)\,dx$ 624
3.131 $\int (a+a\sin(e+fx))^m\tan(e+fx)\,dx$ 628
3.132 $\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx$ 631
3.133 $\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx$ 634
3.134 $\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx$ 637
3.135 $\int (a+a\sin(e+fx))^m\tan^4(e+fx)\,dx$ 641

3.136 $\int (a+a\sin(e+fx))^m \tan^2(e+fx)\,dx$ 645
3.137 $\int (a+a\sin(e+fx))^m\,dx$ 650
3.138 $\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx$ 653
3.139 $\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx$ 656
3.140 $\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx$ 659
3.141 $\int (a+b\sin(c+dx))\tan(c+dx)\,dx$ 665
3.142 $\int \cot(c+dx)(a+b\sin(c+dx))\,dx$ 670
3.143 $\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx$ 673
3.144 $\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx$ 676
3.145 $\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx$ 679
3.146 $\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx$ 683
3.147 $\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx$ 687
3.148 $\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$ 691
3.149 $\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx$ 695
3.150 $\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx$ 700
3.151 $\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx$ 704
3.152 $\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx$ 709
3.153 $\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$ 712
3.154 $\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$ 716
3.155 $\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx$ 720
3.156 $\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$ 725
3.157 $\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$ 731
3.158 $\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$ 735
3.159 $\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$ 740
3.160 $\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$ 746
3.161 $\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx$ 750
3.162 $\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx$ 755
3.163 $\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx$ 758
3.164 $\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx$ 762
3.165 $\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$ 766
3.166 $\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$ 771
3.167 $\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$ 776
3.168 $\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$ 781
3.169 $\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx$ 786
3.170 $\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$ 792
3.171 $\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$ 797
3.172 $\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$ 801
3.173 $\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx$ 804
3.174 $\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$ 807
3.175 $\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$ 811
3.176 $\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$ 815
3.177 $\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$ 820

3.178 $\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$ . . . 825
3.179 $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$ . . . 830
3.180 $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$ . . . 836
3.181 $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$ . . . 842
3.182 $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$ . . . 847
3.183 $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$ . . . 851
3.184 $\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$ . . . 855
3.185 $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$ . . . 858
3.186 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$ . . . 862
3.187 $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$ . . . 866
3.188 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$ . . . 872
3.189 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$ . . . 878
3.190 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$ . . . 885
3.191 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$ . . . 891
3.192 $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ . . . 898
3.193 $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ . . . 904
3.194 $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ . . . 909
3.195 $\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ . . . 913
3.196 $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ . . . 917
3.197 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ . . . 921
3.198 $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ . . . 925
3.199 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ . . . 932
3.200 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ . . . 938
3.201 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ . . . 945
3.202 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ . . . 952
3.203 $\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx$ . . . 961
3.204 $\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx$ . . . 966
3.205 $\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx$ . . . 971
3.206 $\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$ . . . 975
3.207 $\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$ . . . 978
3.208 $\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$ . . . 981

3.1 $\int (a + a\sin(c + dx))\tan^5(c + dx)\,dx$

Optimal. Leaf size=115

$$-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\log(1+\sin(c+dx))}{16d}-\frac{a\sin(c+dx)}{d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{d(a-a\sin(c}$$

```
[Out] -23/16*a*ln(1-sin(d*x+c))/d+7/16*a*ln(1+sin(d*x+c))/d-a*sin(d*x+c)/d+1/8*a^
3/d/(a-a*sin(d*x+c))^2-a^2/d/(a-a*sin(d*x+c))+1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2786, 90\}$

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\sin(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\log(\sin(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (-23*a*Log[1 - Sin[c + d*x]])/(16*d) + (7*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(d*(a - a*Sin[
c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a + a\sin(c+dx))\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-1 + \frac{a^3}{4(a-x)^3} - \frac{a^2}{(a-x)^2} + \frac{23a}{16(a-x)} - \frac{a^2}{8(a+x)^2} + \frac{7a}{16(a+x)}\right) dx, x \cdots}{d}$$

$$= -\frac{23a\log(1-\sin(c+dx))}{16d} + \frac{7a\log(1+\sin(c+dx))}{16d} - \frac{a\sin(c+dx \cdots}{d}$$

Mathematica [A]
time = 0.31, size = 123, normalized size = 1.07

$$-\frac{a\sin(c+dx)\tan^4(c+dx)}{d} - \frac{a(4\log(\cos(c+dx)) + 2\tan^2(c+dx) - \tan^4(c+dx))}{4d} - \frac{5a(6\sec^3(c+dx)\tan(c+dx) - 8\sec(c+dx)\tan^3(c+dx) - 3(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Sin[c + d*x]*Tan[c + d*x]^4)/d) - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c +
d*x]^2 - Tan[c + d*x]^4))/(4*d) - (5*a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*S
ec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(8*d)
```

Maple [A]
time = 0.20, size = 121, normalized size = 1.05

method	result
derivativedivides	$\frac{a\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4} - \frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2} - \frac{3\left(\sin^5(dx+c)\right)}{8} - \frac{5\left(\sin^3(dx+c)\right)}{8} - \frac{15\sin(dx+c)}{8} + \frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right) + a\left(\frac{\left(\tan^4(dx+ \cdots}{4}\right.}{d}$
default	$\frac{a\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4} - \frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2} - \frac{3\left(\sin^5(dx+c)\right)}{8} - \frac{5\left(\sin^3(dx+c)\right)}{8} - \frac{15\sin(dx+c)}{8} + \frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right) + a\left(\frac{\left(\tan^4(dx+ \cdots}{4}\right.}{d}$
risch	$iax + \frac{ia\,e^{i(dx+c)}}{2d} - \frac{ia\,e^{-i(dx+c)}}{2d} + \frac{2iac}{d} + \frac{i\left(2ia\,e^{2i(dx+c)} + 9a\,e^{i(dx+c)} - 2ia\,e^{4i(dx+c)} + 6a\,e^{3i(dx+c)} + 9a\,e^{5i(dx+c)}\right) \cdots}{4\left(e^{i(dx+c)} - i\right)^4\left(e^{i(dx+c)} + i\right)^2 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin
(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))+
a*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.27, size = 95, normalized size = 0.83

$$\frac{7\,a\log\left(\sin\left(dx+c\right)+1\right)-23\,a\log\left(\sin\left(dx+c\right)-1\right)-16\,a\sin\left(dx+c\right)+\frac{2\,\left(9\,a\sin(dx+c)^2-a\sin(dx+c)-6\,a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/16*(7*a*log(sin(d*x + c) + 1) - 23*a*log(sin(d*x + c) - 1) - 16*a*sin(d*x
 + c) + 2*(9*a*sin(d*x + c)^2 - a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3 - sin
(d*x + c)^2 - sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.37, size = 159, normalized size = 1.38

$$\frac{16\,a\cos\left(dx+c\right)^4+2\,a\cos\left(dx+c\right)^2+7\,\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-23\,\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(8\,a\cos\left(dx+c\right)^2+a\right)\sin\left(dx+c\right)-6\,a}{16\,\left(d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*cos(d*x + c)^4 + 2*a*cos(d*x + c)^2 + 7*(a*cos(d*x + c)^2*sin(d*
x + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 23*(a*cos(d*x + c)^2*sin
(d*x + c) - a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(8*a*cos(d*x + c)^
2 + a)*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x + c)^
2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\tan^5\left(c+dx\right)\,dx+\int\tan^5\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**5,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**5, x) + Integral(tan(c + d*x)**5, x)
)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")

[Out] Timed out

Mupad [B]
time = 6.63, size = 235, normalized size = 2.04

$$-\frac{\frac{15\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{15\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}-\frac{23\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{8\,d}+\frac{7\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{8\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^5*(a + a*sin(c + d*x)),x)

[Out] ((11*a*tan(c/2 + (d*x)/2)^2)/2 - (15*a*tan(c/2 + (d*x)/2))/4 + (11*a*tan(c/2 + (d*x)/2)^3)/4 - 5*a*tan(c/2 + (d*x)/2)^4 + (11*a*tan(c/2 + (d*x)/2)^5)/4 + (11*a*tan(c/2 + (d*x)/2)^6)/2 - (15*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(2*tan(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^4 + 2*tan(c/2 + (d*x)/2)^5 - 2*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 + 1)) - (23*a*log(tan(c/2 + (d*x)/2) - 1))/(8*d) + (7*a*log(tan(c/2 + (d*x)/2) + 1))/(8*d) + (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d

3.2 $\int (a+a\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=71

$$\frac{5a\log(1-\sin(c+dx))}{4d}-\frac{a\log(1+\sin(c+dx))}{4d}+\frac{a\sin(c+dx)}{d}+\frac{a^2}{2d(a-a\sin(c+dx))}$$

```
[Out] 5/4*a*ln(1-sin(d*x+c))/d-1/4*a*ln(1+sin(d*x+c))/d+a*sin(d*x+c)/d+1/2*a^2/d/
(a-a*sin(d*x+c))
```

Rubi [A]

time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2786, 90}

$$\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a\sin(c+dx)}{d}+\frac{5a\log(1-\sin(c+dx))}{4d}-\frac{a\log(\sin(c+dx)+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (5*a*Log[1 - Sin[c + d*x]])/(4*d) - (a*Log[1 + Sin[c + d*x]])/(4*d) + (a*Si
n[c + d*x])/d + a^2/(2*d*(a - a*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))\tan^3(c+dx)\,dx&=\frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int \left(1+\frac{a^2}{2(a-x)^2}-\frac{5a}{4(a-x)}-\frac{a}{4(a+x)}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{5a\log(1-\sin(c+dx))}{4d}-\frac{a\log(1+\sin(c+dx))}{4d}+\frac{a\sin(c+dx)}{d}+\end{aligned}$$

Mathematica [A]
time = 0.08, size = 77, normalized size = 1.08

$$-\frac{a\sin(c+dx)\tan^2(c+dx)}{d}-\frac{3a\left(\tanh^{-1}(\sin(c+dx))-\sec(c+dx)\tan(c+dx)\right)}{2d}+\frac{a(2\log(\cos(c+dx))+\tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((a*Sin[c + d*x]*Tan[c + d*x]^2)/d) - (3*a*(ArcTanh[Sin[c + d*x]] - Sec[c
+ d*x]*Tan[c + d*x]))/(2*d) + (a*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2
*d)
```

Maple [A]
time = 0.14, size = 81, normalized size = 1.14

method	result	size
derivativedivides	$\frac{a\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$	81
default	$\frac{a\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$	81
risch	$-iax-\frac{ia\,e^{i(dx+c)}}{2d}+\frac{ia\,e^{-i(dx+c)}}{2d}-\frac{2iac}{d}-\frac{ia\,e^{i(dx+c)}}{\left(e^{i(dx+c)}-i\right)^2 d}-\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{5a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*l
n(sec(d*x+c)+tan(d*x+c)))+a*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.72

$$-\frac{a\log\left(\sin\left(dx+c\right)+1\right)-5\,a\log\left(\sin\left(dx+c\right)-1\right)-4\,a\sin\left(dx+c\right)+\frac{2\,a}{\sin(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*log(sin(d*x + c) + 1) - 5*a*log(sin(d*x + c) - 1) - 4*a*sin(d*x + c
) + 2*a/(sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.37, size = 87, normalized size = 1.23

$$-\frac{4\,a\cos\left(dx+c\right)^2+\left(a\sin\left(dx+c\right)-a\right)\log\left(\sin\left(dx+c\right)+1\right)-5\left(a\sin\left(dx+c\right)-a\right)\log\left(-\sin\left(dx+c\right)+1\right)+4\,a\sin\left(dx+c\right)-2\,a}{4\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*cos(d*x + c)^2 + (a*sin(d*x + c) - a)*log(sin(d*x + c) + 1) - 5*(
a*sin(d*x + c) - a)*log(-sin(d*x + c) + 1) + 4*a*sin(d*x + c) - 2*a)/(d*sin
(d*x + c) - d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\tan^{3}\left(c+dx\right)dx+\int\tan^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x)
)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 26228 vs. $2(66) = 132$.
time = 171.23, size = 26228, normalized size = 369.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d
*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 3*a*log(2*(tan(1/2*d*x)^4*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + ta
n(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*t
an(c)^2 + 2*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*
tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 2*a*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/
2*c)^6*tan(c)^2 - 6*a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c) + 6*a*log(2*(tan(1/2*d*
```

```
x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*
c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^6*tan(1/2
*c)^6*tan(c) - 4*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*
x)*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c) - 3*a*log(2*(tan(1/2*d*x)^4*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2
*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)
^2 + 3*a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d
*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 2*a*log(4*(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 2
4*a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2
*tan(1/2*d*x)^5*tan(1/2*c)^5*tan(c)^2 + 24*a*log(2*(tan(1/2*d*x)^4*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^5*tan(1/2*c)^5*tan(c
)^2 - 16*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan
(1/2*d*x)^5*tan(1/2*c)^5*tan(c)^2 - 12*a*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*
c)^5*tan(c)^2 - 3*a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*t
an(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
+ 1))*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 3*a*log(2*(tan(1/2*
d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/
2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
- 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^4*tan
(1/2*c)^6*tan(c)^2 - 2*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
 tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*
tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 - 12*a*tan(d*x)^2*tan(1/2*d
*x)^5*tan(1/2*c)^6*tan(c)^2 + 2*a*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 -
2*a*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 16*a*tan(d*x)^2*tan(1
/2*d*x)^5*tan(1/2*c)^5*tan(c)^2 - 2*a*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^
```

```
6*tan(c)^2 + 2*a*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 3*a*log(2*(tan(1/2*
d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/
2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*t...
```

Mupad [B]
time = 6.47, size = 154, normalized size = 2.17

$$\frac{5\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{2\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{2\,d}+\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (5*a*log(tan(c/2 + (d*x)/2) - 1))/(2*d) - (a*log(tan(c/2 + (d*x)/2) + 1))/(
2*d) + (3*a*tan(c/2 + (d*x)/2) - 4*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (
d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) - 2*tan(c/2 +
(d*x)/2)^3 + tan(c/2 + (d*x)/2)^4 + 1)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))
/d
```

3.3 $\int (a+a\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=30

$$-\frac{a\log(1-\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}$$

```
[Out] -a*ln(1-sin(d*x+c))/d-a*sin(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2786, 45}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[1 - Sin[c + d*x]])/d) - (a*Sin[c + d*x])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))\tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a-x}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-1+\frac{a}{a-x}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a\log(1-\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 1.27

$$\frac{a \tanh^{-1}(\sin(c+dx))}{d} - \frac{a \log(\cos(c+dx))}{d} - \frac{a \sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x],x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d - (a*Sin[c + d*x])/d
```

Maple [A]
time = 0.13, size = 23, normalized size = 0.77

method	result	size
derivativedivides	$-\frac{a(\sin(dx+c)+\ln(\sin(dx+c)-1))}{d}$	23
default	$-\frac{a(\sin(dx+c)+\ln(\sin(dx+c)-1))}{d}$	23
risch	$iax + \frac{ia\,e^{i(dx+c)}}{2d} - \frac{ia\,e^{-i(dx+c)}}{2d} + \frac{2iac}{d} - \frac{2a\ln\left(e^{i(dx+c)}-i\right)}{d}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] -1/d*a*(sin(d*x+c)+ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.83

$$-\frac{a \log\left(\sin\left(dx+c\right)-1\right) + a \sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="maxima")
[Out] -(a*log(sin(d*x + c) - 1) + a*sin(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 27, normalized size = 0.90

$$-\frac{a \log\left(-\sin\left(dx+c\right)+1\right) + a \sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="fricas")
[Out] -(a*log(-sin(d*x + c) + 1) + a*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin (c+dx)\tan (c+dx)\,dx+\int \tan (c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x), x) + Integral(tan(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1456 vs. $2(30) = 60$.
time = 7.55, size = 1456, normalized size = 48.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*log(2*(tan(1/2*d*x)^4*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2
*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - a*log(2*(tan(1/2*d*x)
^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)
^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + a*log(4*(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2 - 4*a*tan(1/2*d*x)^2*tan
(1/2*c) + a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
```

```
n(1/2*c)^2 - a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/
2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(1/2*c)^2 + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*
x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2
*c)^2 - 4*a*tan(1/2*d*x)*tan(1/2*c)^2 + a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d
*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*
c) + 1)/(tan(1/2*c)^2 + 1)) - a*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(t
an(1/2*c)^2 + 1)) + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 4
*a*tan(1/2*d*x) + 4*a*tan(1/2*c))/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/
2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

Mupad [B]
time = 6.62, size = 43, normalized size = 1.43

$$-\frac{a\left(\sin\left(c+d\,x\right)+2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] -(a*(sin(c + d*x) + 2*log(tan(c/2 + (d*x)/2) - 1) - log(tan(c/2 + (d*x)/2)^
2 + 1)))/d
```

3.4 $\int \cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2786, 45}

$$\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a+x}{x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{a}{x}\right)\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 1.08

$$\frac{a(\log(\cos(c+dx))+\log(\tan(c+dx))+\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]] + Sin[c + d*x]))/d
```

Maple [A]
time = 0.09, size = 20, normalized size = 0.83

method	result	size
derivativedivides	$\frac{a(\sin(dx+c)+\ln(\sin(dx+c)))}{d}$	20
default	$\frac{a(\sin(dx+c)+\ln(\sin(dx+c)))}{d}$	20
risch	$-iax-\frac{2iac}{d}+\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{a\sin(dx+c)}{d}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] a/d*(sin(d*x+c)+ln(sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.92

$$\frac{a\log\left(\sin\left(dx+c\right)\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] (a*log(sin(d*x + c)) + a*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 24, normalized size = 1.00

$$\frac{a\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] (a*log(1/2*sin(d*x + c)) + a*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin\left(c+dx\right)\cot\left(c+dx\right)dx+\int \cot\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x), x) + Integral(cot(c + d*x), x))
```

Giac [A]
time = 6.04, size = 23, normalized size = 0.96

$$\frac{a\log\left(\left|\sin\left(dx+c\right)\right|\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(sin(d*x + c))) + a*sin(d*x + c))/d
```

Mupad [B]
time = 6.58, size = 38, normalized size = 1.58

$$\frac{a\left(\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\sin\left(c+d\,x\right)-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*(log(tan(c/2 + (d*x)/2)) + sin(c + d*x) - log(tan(c/2 + (d*x)/2)^2 + 1))
)/d
```

3.5 $\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}$$

```
[Out] -a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-a*ln(sin(d*x+c))/d-a*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2786, 76}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (a*Sin[c + d*x])/d
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-1+\frac{a^3}{x^3}+\frac{a^2}{x^2}-\frac{a}{x}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 1.11

$$-\frac{a\csc(c+dx)}{d}-\frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d}-\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
[Out] -((a*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Ta
n[c + d*x]]))/(2*d) - (a*Sin[c + d*x])/d
```

Maple [A]
time = 0.18, size = 67, normalized size = 1.24

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	67
default	$\frac{a\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	67
risch	$iax+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}-\frac{2ia\left(ie^{2i(dx+c)}+e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/sin(d*x+c)*cos(d*x+c)^4-(2+cos(d*x+c)^2)*sin(d*x+c))+a*(-1/2*cot
(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.27, size = 45, normalized size = 0.83

$$-\frac{2\,a\log\left(\sin\left(dx+c\right)\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a\sin(dx+c)+a}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(2*a*log(sin(d*x + c)) + 2*a*sin(d*x + c) + (2*a*sin(d*x + c) + a)/sin
(d*x + c)^2)/d
```

Fricas [A]
time = 0.36, size = 69, normalized size = 1.28

$$-\frac{2\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+2\left(a\cos\left(dx+c\right)^2-2\,a\right)\sin\left(dx+c\right)-a}{2\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(a*cos(d*x + c)^2
- 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin(c+dx)\cot^3(c+dx)\,dx + \int \cot^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x)
)
```

Giac [A]
time = 4.89, size = 60, normalized size = 1.11

$$-\frac{2\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+2\,a\sin\left(dx+c\right)-\frac{3\,a\sin(dx+c)^2-2\,a\sin(dx+c)-a}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(sin(d*x + c))) + 2*a*sin(d*x + c) - (3*a*sin(d*x + c)^2 -
 2*a*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

Mupad [B]
time = 6.54, size = 146, normalized size = 2.70

$$\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a*tan(c/2 + (d*x)/2))/(2*d) - (a/2 +
 2*a*tan(c/2 + (d*x)/2) + (a*tan(c/2 + (d*x)/2)^2)/2 + 10*a*tan(c/2 + (d*x)
/2)^3)/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) - (a*tan(c/2 +
 (d*x)/2)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2)))/d
```

3.6 $\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{2a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] 2*a*csc(d*x+c)/d+a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d
+a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$,
Rules used = {2786, 90}

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^5}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{a^5}{x^5}+\frac{a^4}{x^4}-\frac{2a^3}{x^3}-\frac{2a^2}{x^2}+\frac{a}{x}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{2a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\end{aligned}$$

Mathematica [A]
time = 0.14, size = 87, normalized size = 1.07

$$\frac{2a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx)))}{4d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (a*Sin[c
 + d*x])/d
```

Maple [A]
time = 0.21, size = 101, normalized size = 1.25

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(s\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(s\right.}{d}$
risch	$-iax-\frac{ia\,e^{i(dx+c)}}{2d}+\frac{ia\,e^{-i(dx+c)}}{2d}-\frac{2iac}{d}+\frac{4ia\left(3ie^{6i(dx+c)}+3\,e^{7i(dx+c)}-3ie^{4i(dx+c)}-7\,e^{5i(dx+c)}+3ie^{2i(dx+c)}\right.}{3d\left(e^{2i(dx+c)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3/sin(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d
*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2
+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.85

$$\frac{12\,a\log\left(\sin\left(dx+c\right)\right)+12\,a\sin\left(dx+c\right)+\frac{24\,a\sin(dx+c)^3+12\,a\sin(dx+c)^2-4\,a\sin(dx+c)-3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*a*sin(d*x + c) + (24*a*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*a*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.36, size = 110, normalized size = 1.36

$$-\frac{12\,a\cos\left(dx+c\right)^2-12\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\left(3\,a\cos\left(dx+c\right)^4-12\,a\cos\left(dx+c\right)^2+8\,a\right)\sin\left(dx+c\right)-9\,a}{12\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*a*cos(d*x + c)^4 - 12*a*cos(d*x + c)^2 + 8*a)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin\left(c+dx\right)\cot^{5}\left(c+dx\right)dx+\int \cot^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**5, x) + Integral(cot(c + d*x)**5, x)
)
```

Giac [A]
time = 4.61, size = 82, normalized size = 1.01

$$\frac{12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,a\sin\left(dx+c\right)-\frac{25\,a\sin(dx+c)^{4}-24\,a\sin(dx+c)^{3}-12\,a\sin(dx+c)^{2}+4\,a\sin(dx+c)+3\,a}{\sin(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*a*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*a*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

Mupad [B]
time = 6.70, size = 207, normalized size = 2.56

$$\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{46\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{40\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a}{4}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + a*sin(c + d*x)),x)
```

```
[Out] (7*a*tan(c/2 + (d*x)/2))/(8*d) + ((11*a*tan(c/2 + (d*x)/2)^2)/4 - (2*a*tan(
c/2 + (d*x)/2))/3 - a/4 + (40*a*tan(c/2 + (d*x)/2)^3)/3 + 3*a*tan(c/2 + (d*
x)/2)^4 + 46*a*tan(c/2 + (d*x)/2)^5)/(d*(16*tan(c/2 + (d*x)/2)^4 + 16*tan(c
/2 + (d*x)/2)^6)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (3*a*tan(c/2 + (d
*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^
4)/(64*d) + (a*log(tan(c/2 + (d*x)/2)))/d
```

3.7 $\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{3a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^4(c+dx)}{4d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^6(c+dx)}{6d}-a\log$$

```
[Out] -3*a*csc(d*x+c)/d-3/2*a*csc(d*x+c)^2/d+a*csc(d*x+c)^3/d+3/4*a*csc(d*x+c)^4/
d-1/5*a*csc(d*x+c)^5/d-1/6*a*csc(d*x+c)^6/d-a*ln(sin(d*x+c))/d-a*sin(d*x+c)
/d
```

Rubi [A]
time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 90}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d +
 (3*a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)
/(6*d) - (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^7}{x^7}+\frac{a^6}{x^6}-\frac{3a^5}{x^5}-\frac{3a^4}{x^4}+\frac{3a^3}{x^3}+\frac{3a^2}{x^2}-\frac{a}{x}\right)dx, x, a\sin(c\right.}{d}$$
$$= -\frac{3a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^4(c+d}{4d}$$

Mathematica [A]
time = 0.27, size = 111, normalized size = 0.97

$$-\frac{3a\csc(c+dx)}{d}+\frac{a\csc^3(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a(6\cot^2(c+dx)-3\cot^4(c+dx)+2\cot^6(c+dx)+12\log(\cos(c+dx))+12\log(\tan(c+dx)))}{12d}-\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^5)/(5*d) - (
a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Cos[c +
d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) - (a*Sin[c + d*x])/d
```

Maple [A]
time = 0.24, size = 143, normalized size = 1.24

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^8(dx+c)}{5\sin(dx+c)^3}-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)\right)+a\left(-\frac{(\cot^6}{}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^8(dx+c)}{5\sin(dx+c)^3}-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)\right)+a\left(-\frac{(\cot^6}{}\right.}{d}$
risch	$iax+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}-\frac{2ia\left(45ie^{10i(dx+c)}+45\,e^{11i(dx+c)}-90ie^{8i(dx+c)}-165\,e^{9i(dx+c)}+170ie^{6i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^8+1/5/sin(d*x+c)^3*cos(d*x+c)^8-1/sin(
d*x+c)*cos(d*x+c)^8-(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*s
in(d*x+c))+a*(-1/6*cot(d*x+c)^6+1/4*cot(d*x+c)^4-1/2*cot(d*x+c)^2-ln(sin(d*
x+c))))
```

Maxima [A]
time = 0.27, size = 91, normalized size = 0.79

$$-\frac{60\,a\log(\sin(dx+c))+60\,a\sin(dx+c)+\frac{180\,a\sin(dx+c)^5+90\,a\sin(dx+c)^4-60\,a\sin(dx+c)^3-45\,a\sin(dx+c)^2+12\,a\sin(dx+c)+10\,a}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c) + (180*a*sin(d*x + c)^5 +
 90*a*sin(d*x + c)^4 - 60*a*sin(d*x + c)^3 - 45*a*sin(d*x + c)^2 + 12*a*sin
(d*x + c) + 10*a)/sin(d*x + c)^6)/d
```

Fricas [A]
time = 0.38, size = 158, normalized size = 1.37

$$\frac{90\,a\cos\left(dx+c\right)^{4}-135\,a\cos\left(dx+c\right)^{2}-60\,\left(a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}+3\,a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-12\,\left(5\,a\cos\left(dx+c\right)^{6}-30\,a\cos\left(dx+c\right)^{4}+40\,a\cos\left(dx+c\right)^{2}-16\,a\right)\sin\left(dx+c\right)+55\,a}{60\,\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(90*a*cos(d*x + c)^4 - 135*a*cos(d*x + c)^2 - 60*(a*cos(d*x + c)^6 - 3
*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) - 12*(5*a
*cos(d*x + c)^6 - 30*a*cos(d*x + c)^4 + 40*a*cos(d*x + c)^2 - 16*a)*sin(d*x
 + c) + 55*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\cot^{7}\left(c+dx\right)\,dx+\int\cot^{7}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**7, x) + Integral(cot(c + d*x)**7, x)
)
```

Giac [A]
time = 3.46, size = 104, normalized size = 0.90

$$-\frac{60\,a\log\left(|\sin\left(dx+c\right)|\right)+60\,a\sin\left(dx+c\right)-\frac{147\,a\sin(dx+c)^{6}-180\,a\sin(dx+c)^{5}-90\,a\sin(dx+c)^{4}+60\,a\sin(dx+c)^{3}+45\,a\sin(dx+c)^{2}-12\,a\sin(dx+c)-10\,a}{\sin(dx+c)^{6}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c) - (147*a*sin(d*x + c
)^6 - 180*a*sin(d*x + c)^5 - 90*a*sin(d*x + c)^4 + 60*a*sin(d*x + c)^3 + 45
*a*sin(d*x + c)^2 - 12*a*sin(d*x + c) - 10*a)/sin(d*x + c)^6)/d
```

Mupad [B]
time = 7.37, size = 267, normalized size = 2.32

$$\frac{3\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{32\,d}-\frac{29\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{128\,d}-\frac{19\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{32\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{160\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{384\,d}-\frac{a\,\left(1920\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-1920\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\right)}{1920\,d}-\frac{{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{51\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{16}+\frac{29\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{128}+\frac{35\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{32}+\frac{25\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{128}-\frac{7\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{80}-\frac{11\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{384}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{160}+\frac{a}{384}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + a*sin(c + d*x)),x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2)^3)/(32*d) - (29*a*tan(c/2 + (d*x)/2)^2)/(128*d) - (
19*a*tan(c/2 + (d*x)/2))/(16*d) + (a*tan(c/2 + (d*x)/2)^4)/(32*d) - (a*tan(
c/2 + (d*x)/2)^5)/(160*d) - (a*tan(c/2 + (d*x)/2)^6)/(384*d) - (a*(1920*log
(tan(c/2 + (d*x)/2)) - 1920*log(tan(c/2 + (d*x)/2)^2 + 1)))/(1920*d) - (cot
(c/2 + (d*x)/2)^6*(a/384 + (a*tan(c/2 + (d*x)/2))/160 - (11*a*tan(c/2 + (d*
x)/2)^2)/384 - (7*a*tan(c/2 + (d*x)/2)^3)/80 + (25*a*tan(c/2 + (d*x)/2)^4)/
128 + (35*a*tan(c/2 + (d*x)/2)^5)/32 + (29*a*tan(c/2 + (d*x)/2)^6)/128 + (5
1*a*tan(c/2 + (d*x)/2)^7)/16))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.8 $\int (a+a\sin(c+dx))\tan^6(c+dx)\,dx$

Optimal. Leaf size=101

$$-ax+\frac{a\cos(c+dx)}{d}+\frac{3a\sec(c+dx)}{d}-\frac{a\sec^3(c+dx)}{d}+\frac{a\sec^5(c+dx)}{5d}+\frac{a\tan(c+dx)}{d}-\frac{a\tan^3(c+dx)}{3d}+\frac{a\,\mathrm{ta}}{}$$

```
[Out] -a*x+a*cos(d*x+c)/d+3*a*sec(d*x+c)/d-a*sec(d*x+c)^3/d+1/5*a*sec(d*x+c)^5/d+
a*tan(d*x+c)/d-1/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2789, 3554, 8, 2670, 276}

$$\frac{a\cos(c+dx)}{d}+\frac{a\tan^5(c+dx)}{5d}-\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^5(c+dx)}{5d}-\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec(c+dx)}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (3*a*Sec[c + d*x])/d - (a*Sec[c + d*x]^3)/d +
 (a*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d - (a*Tan[c + d*x]^3)/(3*d) +
 (a*Tan[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(c+dx))\tan^6(c+dx)\,dx &= \int \left(a\tan^6(c+dx)+a\sin(c+dx)\tan^6(c+dx)\right)\,dx \\
&= a\int \tan^6(c+dx)\,dx + a\int \sin(c+dx)\tan^6(c+dx)\,dx \\
&= \frac{a\tan^5(c+dx)}{5d} - a\int \tan^4(c+dx)\,dx - \frac{a\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x, \cos(\right.}{d} \\
&= -\frac{a\tan^3(c+dx)}{3d} + \frac{a\tan^5(c+dx)}{5d} + a\int \tan^2(c+dx)\,dx - \frac{a\text{Subst}(}{} \\
&= \frac{a\cos(c+dx)}{d} + \frac{3a\sec(c+dx)}{d} - \frac{a\sec^3(c+dx)}{d} + \frac{a\sec^5(c+dx)}{5d} + \\
&= -ax + \frac{a\cos(c+dx)}{d} + \frac{3a\sec(c+dx)}{d} - \frac{a\sec^3(c+dx)}{d} + \frac{a\sec^5(c+}{5d}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 110, normalized size = 1.09

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{3a\sec(c+dx)}{d}-\frac{a\sec^3(c+dx)}{d}+\frac{a\sec^5(c+dx)}{5d}+\frac{a\tan(c+dx)}{d}-\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^6,x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (3*a*Sec[c + d*x])/d -
 (a*Sec[c + d*x]^3)/d + (a*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d - (a*
Tan[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^5)/(5*d)
```

Maple [A]
time = 0.19, size = 135, normalized size = 1.34

method	result
derivativedivides	$\frac{a\left(\frac{\sin^8(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^8(dx+c)}{5\cos(dx+c)^3}+\frac{\sin^8(dx+c)}{\cos(dx+c)}+\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)\right)+a\left(\frac{\left(\tan^5(dx\right.}{5}\right.}{d}$

default	$\frac{a\left(\frac{\sin^8(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^8(dx+c)}{5\cos(dx+c)^3}+\frac{\sin^8(dx+c)}{\cos(dx+c)}+\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)\right)+a\left(\frac{\tan^5(}{5}\right.}{d}$
risch	$-ax+\frac{a\,e^{i(dx+c)}}{2d}+\frac{a\,e^{-i(dx+c)}}{2d}+\frac{-\frac{182ia\,e^{2i(dx+c)}}{15}+\frac{2a\,e^{i(dx+c)}}{15}+\frac{42a\,e^{3i(dx+c)}}{5}-\frac{46ia}{15}-14ia\,e^{4i(dx+c)}+10a\,e^{5i(d}}{\left(e^{i(dx+c)}+i\right)^3\left(e^{i(dx+c)}-i\right)^5 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+c)^3+sin(d*x
+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(
d*x+c))+a*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 87, normalized size = 0.86

$$\frac{\left(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a+3\,a\left(\frac{15\,\cos(dx+c)^4-5\,\cos(dx+c)^2+1}{\cos(dx+c)^5}+5\,\cos\left(dx+c\right)\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c
))*a + 3*a*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*c
os(d*x + c)))/d
```

Fricas [A]
time = 0.35, size = 116, normalized size = 1.15

$$\frac{15\,adx\cos\left(dx+c\right)^3-38\,a\cos\left(dx+c\right)^4-11\,a\cos\left(dx+c\right)^2-\left(15\,adx\cos\left(dx+c\right)^3-15\,a\cos\left(dx+c\right)^4-22\,a\cos\left(dx+c\right)^2+4\,a\right)\sin\left(dx+c\right)+a}{15\,\left(d\cos\left(dx+c\right)^3\sin\left(dx+c\right)-d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a*d*x*cos(d*x + c)^3 - 38*a*cos(d*x + c)^4 - 11*a*cos(d*x + c)^2 -
 (15*a*d*x*cos(d*x + c)^3 - 15*a*cos(d*x + c)^4 - 22*a*cos(d*x + c)^2 + 4*a
)*sin(d*x + c) + a)/(d*cos(d*x + c)^3*sin(d*x + c) - d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\tan^6\left(c+dx\right)dx+\int\tan^6\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**6,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**6, x) + Integral(tan(c + d*x)**6, x)
)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 11.22, size = 351, normalized size = 3.48

$$\frac{\left(\frac{a\,\left(30\,c+30\,d\,x-30\right)}{15}-2\,a\,\left(c+d\,x\right)\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+\left(\frac{a\,\left(15\,c+15\,d\,x+60\right)}{15}-a\,\left(c+d\,x\right)\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+\left(4\,a\,\left(c+d\,x\right)-\frac{a\,\left(60\,c+60\,d\,x-40\right)}{15}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+\left(\frac{a\,\left(30\,c+30\,d\,x-140\right)}{15}-2\,a\,\left(c+d\,x\right)\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+\frac{44\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{15}+\left(2\,a\,\left(c+d\,x\right)-\frac{a\,\left(30\,c+30\,d\,x-52\right)}{15}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\left(\frac{a\,\left(60\,c+60\,d\,x-344\right)}{15}-4\,a\,\left(c+d\,x\right)\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(a\,\left(c+d\,x\right)-\frac{a\,\left(15\,c+15\,d\,x-156\right)}{15}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\left(2\,a\,\left(c+d\,x\right)-\frac{a\,\left(30\,c+30\,d\,x-162\right)}{15}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\,\left(15\,c+15\,d\,x-96\right)}{15}-a\,\left(c+d\,x\right)}{d\,{\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^5\,{\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}^3\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*(15*c + 15*d*x - 96))/15 - tan(c/2 + (d*x)/2)*((a*(30*c + 30*d*x - 162)
)/15 - 2*a*(c + d*x)) - a*(c + d*x) + tan(c/2 + (d*x)/2)^8*((a*(15*c + 15*d
*x + 60))/15 - a*(c + d*x)) + tan(c/2 + (d*x)/2)^9*((a*(30*c + 30*d*x - 30)
)/15 - 2*a*(c + d*x)) - tan(c/2 + (d*x)/2)^4*((a*(30*c + 30*d*x - 52))/15 -
 2*a*(c + d*x)) - tan(c/2 + (d*x)/2)^7*((a*(60*c + 60*d*x - 40))/15 - 4*a*(
c + d*x)) - tan(c/2 + (d*x)/2)^2*((a*(15*c + 15*d*x - 156))/15 - a*(c + d*x
)) + tan(c/2 + (d*x)/2)^6*((a*(30*c + 30*d*x - 140))/15 - 2*a*(c + d*x)) +
tan(c/2 + (d*x)/2)^3*((a*(60*c + 60*d*x - 344))/15 - 4*a*(c + d*x)) + (44*a
*tan(c/2 + (d*x)/2)^5)/15)/(d*(tan(c/2 + (d*x)/2) - 1)^5*(tan(c/2 + (d*x)/2
) + 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)) - a*x
```

3.9 $\int (a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=72

$$ax-\frac{a\cos(c+dx)}{d}-\frac{2a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] a*x-a*cos(d*x+c)/d-2*a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d-a*tan(d*x+c)/d+1/3
*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2789, 3554, 8, 2670, 276}

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))\tan^4(c+dx)\,dx &= \int \left(a\tan^4(c+dx)+a\sin(c+dx)\tan^4(c+dx)\right)\,dx\\ &= a\int \tan^4(c+dx)\,dx+a\int \sin(c+dx)\tan^4(c+dx)\,dx\\ &= \frac{a\tan^3(c+dx)}{3d}-a\int \tan^2(c+dx)\,dx-\frac{a\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,\cos(\right.}{d}\\ &= -\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}+a\int 1\,dx-\frac{a\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\right.\right.}{}\\ &= ax-\frac{a\cos(c+dx)}{d}-\frac{2a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\tan(c+d}{d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 81, normalized size = 1.12

$$\frac{a\tan^{-1}(\tan(c+dx))}{d}-\frac{a\cos(c+dx)}{d}-\frac{2a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a
*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.18, size = 98, normalized size = 1.36

method	result	s
derivativedivides	$\frac{a\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)}{d}$	9
default	$\frac{a\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)}{d}$	9

risch	$ax - \frac{a\,e^{i(dx+c)}}{2d} - \frac{a\,e^{-i(dx+c)}}{2d} - \frac{4\left(-2ia+a\,e^{i(dx+c)}-3ia\,e^{2i(dx+c)}+3a\,e^{3i(dx+c)}\right)}{3\left(e^{i(dx+c)}+i\right)\left(e^{i(dx+c)}-i\right)^{3}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+
c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.49, size = 65, normalized size = 0.90

$$\frac{\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a-a\left(\frac{6\,\cos\left(dx+c\right)^{2}-1}{\cos\left(dx+c\right)^{3}}+3\,\cos\left(dx+c\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - a*((6*cos(d*x + c)
^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [A]
time = 0.35, size = 88, normalized size = 1.22

$$-\frac{3\,adx\cos\left(dx+c\right)-7\,a\cos\left(dx+c\right)^{2}-\left(3\,adx\cos\left(dx+c\right)-3\,a\cos\left(dx+c\right)^{2}-2\,a\right)\sin\left(dx+c\right)-a}{3\,\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*d*x*cos(d*x + c) - 7*a*cos(d*x + c)^2 - (3*a*d*x*cos(d*x + c) - 3
*a*cos(d*x + c)^2 - 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)*sin(d*x + c) - d
*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\tan^{4}\left(c+dx\right)\,dx+\int\tan^{4}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**4, x) + Integral(tan(c + d*x)**4, x)
)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.42, size = 231, normalized size = 3.21

$$a\,x+\frac{\left(\frac{2\,a\,\left(3\,c+3\,d\,x\right)}{3}-\frac{a\,\left(6\,c+6\,d\,x-6\right)}{3}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(\frac{a\,\left(3\,c+3\,d\,x-12\right)}{3}-\frac{a\,\left(3\,c+3\,d\,x\right)}{3}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\frac{4\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{3}+\left(\frac{a\,\left(3\,c+3\,d\,x\right)}{3}-\frac{a\,\left(3\,c+3\,d\,x-4\right)}{3}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\left(\frac{a\,\left(6\,c+6\,d\,x-26\right)}{3}-\frac{2\,a\,\left(3\,c+3\,d\,x\right)}{3}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\,\left(3\,c+3\,d\,x\right)}{3}-\frac{a\,\left(3\,c+3\,d\,x-16\right)}{3}}{d\,{\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] a*x + ((a*(3*c + 3*d*x))/3 - (a*(3*c + 3*d*x - 16))/3 + tan(c/2 + (d*x)/2)^
2*((a*(3*c + 3*d*x))/3 - (a*(3*c + 3*d*x - 4))/3) - tan(c/2 + (d*x)/2)^4*((
a*(3*c + 3*d*x))/3 - (a*(3*c + 3*d*x - 12))/3) + tan(c/2 + (d*x)/2)^5*((2*a
*(3*c + 3*d*x))/3 - (a*(6*c + 6*d*x - 6))/3) - tan(c/2 + (d*x)/2)*((2*a*(3*
c + 3*d*x))/3 - (a*(6*c + 6*d*x - 26))/3) + (4*a*tan(c/2 + (d*x)/2)^3)/3)/(
d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2
 + 1))
```

3.10 $\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=39

$$-ax+\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)}{d(1-\sin(c+dx))}$$

```
[Out] -a*x+a*cos(d*x+c)/d+a*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2787, 2825, 12, 2814, 2727}

$$\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)}{d(1-\sin(c+dx))}-ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (a*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2787

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx &= a^2\int \frac{\sin^2(c+dx)}{a-a\sin(c+dx)}\,dx \\
&= \frac{a\cos(c+dx)}{d}+a\int \frac{a\sin(c+dx)}{a-a\sin(c+dx)}\,dx \\
&= \frac{a\cos(c+dx)}{d}+a^2\int \frac{\sin(c+dx)}{a-a\sin(c+dx)}\,dx \\
&= -ax+\frac{a\cos(c+dx)}{d}+a^2\int \frac{1}{a-a\sin(c+dx)}\,dx \\
&= -ax+\frac{a\cos(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d(a-a\sin(c+dx))}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 1.21

$$
-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

Maple [A]
time = 0.13, size = 59, normalized size = 1.51

method	result	size
risch	$-ax+\frac{a\,e^{i(dx+c)}}{2d}+\frac{a\,e^{-i(dx+c)}}{2d}+\frac{2a}{d\left(e^{i(dx+c)}-i\right)}$	56
derivativedivides	$\frac{a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+a(\tan(dx+c)-dx-c)}{d}$	59
default	$\frac{a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+a(\tan(dx+c)-dx-c)}{d}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a*(tan(d*x+c)-
d*x-c))
```

Maxima [A]
time = 0.51, size = 39, normalized size = 1.00

$$-\frac{(dx+c-\tan(dx+c))a-a\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(38) = 76$.
time = 0.34, size = 80, normalized size = 2.05

$$-\frac{adx-a\cos(dx+c)^2+(adx-2a)\cos(dx+c)-(adx-a\cos(dx+c)+a)\sin(dx+c)-a}{d\cos(dx+c)-d\sin(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(a*d*x - a*cos(d*x + c)^2 + (a*d*x - 2*a)*cos(d*x + c) - (a*d*x - a*cos(d*
x + c) + a)*sin(d*x + c) - a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin(c+dx)\tan^2(c+dx)\,dx+\int\tan^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x)
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(38) = 76$.
time = 7.40, size = 1008, normalized size = 25.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")

[Out] -(a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - a*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + a*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4 + a*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c)^3 + 2*a*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 8*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - a*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 4*a*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + a*d*x*tan(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c)^3 - 8*a*tan(1/2*d*x)^3*tan(1/2*c)^3 + a*d*x*tan(1/2*c)^4 - 2*a*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 24*a*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 8*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 2*a*tan(d*x)*tan(1/2*c)^4*tan(c) - a*tan(d*x)*tan(1/2*d*x)^4 - 4*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3 - a*tan(d*x)*tan(1/2*c)^4 - a*tan(1/2*d*x)^4*tan(c) - 4*a*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - a*tan(1/2*c)^4*tan(c) + 2*a*tan(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*a*tan(1/2*d*x)^3*tan(1/2*c) + 24*a*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*tan(1/2*d*x)*tan(1/2*c)^3 + 2*a*tan(1/2*c)^4 + a*d*x*tan(d*x)*tan(c) + 8*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c) - 4*a*tan(1/2*d*x)*tan(1/2*c)*tan(c) - a*d*x - 8*a*tan(1/2*d*x)*tan(1/2*c) - 2*a*tan(d*x)*tan(c) + a*tan(d*x) + a*tan(c) + 2*a)/(d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - d*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3 - d*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - d*tan(d*x)*tan(1/2*c)^4*tan(c) + d*tan(1/2*d*x)^4 + 4*d*tan(1/2*d*x)^3*tan(1/2*c) + 4*d*tan(1/2*d*x)*tan(1/2*c)^3 + d*tan(1/2*c)^4 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) + 4*d*tan(1/2*d*x)*tan(1/2*c) + d*tan(d*x)*tan(c) - d)

Mupad [B]
time = 6.77, size = 111, normalized size = 2.85

$$\frac{\left(a\,\left(c+d\,x-2\right)-a\,\left(c+d\,x\right)\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\left(a\,\left(c+d\,x\right)-a\,\left(c+d\,x-2\right)\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\left(c+d\,x\right)+a\,\left(c+d\,x-4\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^2*(a + a*sin(c + d*x)),x)

[Out] (tan(c/2 + (d*x)/2)*(a*(c + d*x) - a*(c + d*x - 2)) - tan(c/2 + (d*x)/2)^2*

```
(a*(c + d*x) - a*(c + d*x - 2)) - a*(c + d*x) + a*(c + d*x - 4))/(d*(tan(c/
2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)) - a*x
```

3.11 $\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$-ax-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}$$

```
[Out] -a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d-a*cot(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.316$, Rules used = {2789, 2672, 327, 212, 3554, 8}

$$\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx &= \int \left(a\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)\,dx \\
&= a\int \cos(c+dx)\cot(c+dx)\,dx+a\int \cot^2(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)}{d}-a\int 1\,dx-\frac{a\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d} \\
&= -ax+\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c\right.}{d} \\
&= -ax-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 75, normalized size = 1.83

$$\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/2]])/d
```

Maple [A]
time = 0.12, size = 49, normalized size = 1.20

method	result	size
derivativedivides	$\frac{a(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a(-\cot(dx+c)-dx-c)}{d}$	49
default	$\frac{a(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a(-\cot(dx+c)-dx-c)}{d}$	49
risch	$-ax+\frac{a\,e^{i(dx+c)}}{2d}+\frac{a\,e^{-i(dx+c)}}{2d}-\frac{2ia}{d\left(e^{2i(dx+c)}-1\right)}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{d}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.51, size = 54, normalized size = 1.32

$$-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a-a(2\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - a*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(41)=82$.
time = 0.37, size = 84, normalized size = 2.05

$$-\frac{a\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-a\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+2\,a\cos(dx+c)+2\,(adx-a\cos(dx+c))\sin(dx+c)}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - a*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - a*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin(c+dx)\cot^2(c+dx)\,dx+\int\cot^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x)
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(41) = 82.
time = 4.23, size = 108, normalized size = 2.63

$$-\frac{6\,(dx+c)a-6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*a*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*a*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

Mupad [B]
time = 6.91, size = 108, normalized size = 2.63

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)+a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\cos\left(c+d\,x\right)-\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{2}}{d\,\sin\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] (2*a*atan((2^(1/2)*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(2*cos(c/2 -
pi/4 + (d*x)/2))) + a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*co
s(c + d*x) - (a*sin(2*c + 2*d*x))/2)/(d*sin(c + d*x))
```

3.12 $\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$ax+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}$$

```
[Out] a*x+3/2*a*arctanh(cos(d*x+c))/d-3/2*a*cos(d*x+c)/d+a*cot(d*x+c)/d-1/2*a*cos
(d*x+c)*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2789, 2672, 294, 327, 212, 3554, 8}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx &= \int \left(a\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)\,dx \\
&= a\int \cos(c+dx)\cot^3(c+dx)\,dx + a\int \cot^4(c+dx)\,dx \\
&= -\frac{a\cot^3(c+dx)}{3d} - a\int \cot^2(c+dx)\,dx - \frac{a\mathrm{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \mathrm{c}\right.}{d} \\
&= \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + a\int \\
&= ax - \frac{3a\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} \\
&= ax + \frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} -
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.04, size = 125, normalized size = 1.52

$$-\frac{a\cos(c+dx)}{d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d}+\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*a*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2
)/(8*d)
```

Maple [A]
time = 0.12, size = 86, normalized size = 1.05

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$
risch	$ax-\frac{a\,e^{i(dx+c)}}{2d}-\frac{a\,e^{-i(dx+c)}}{2d}+\frac{a\left(12ie^{4i(dx+c)}+3\,e^{5i(dx+c)}-12ie^{2i(dx+c)}+8i-3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{3a\ln\left(e^{i(dx+c)}+1\right)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*
ln(csc(d*x+c)-cot(d*x+c)))+a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.50, size = 92, normalized size = 1.12

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+3\,a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*a*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(74) = 148$.
time = 0.39, size = 160, normalized size = 1.95

$$\frac{16\,a\cos\left(dx+c\right)^3+9\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\left(a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-12\,a\cos\left(dx+c\right)+6\left(2\,adx\cos\left(dx+c\right)^2-2\,a\cos\left(dx+c\right)^3-2\,adx+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*a*cos(d*x
+ c)^3 - 2*a*d*x + 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin\left(c + dx\right)\cot^{4}\left(c + dx\right)\,dx + \int \cot^{4}\left(c + dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4, x)
)
```

Giac [A]
time = 3.72, size = 141, normalized size = 1.72

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,(dx+c)a-36\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{48\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+\frac{66\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*a*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*a/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*a*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 6.60, size = 228, normalized size = 2.78

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+17\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{6\,a^2+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^2+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a/3 + a*tan(c/2 + (d*x)/2) - (14*a*tan(c/
2 + (d*x)/2)^2)/3 + 17*a*tan(c/2 + (d*x)/2)^3 - 5*a*tan(c/2 + (d*x)/2)^4)/(
d*(8*tan(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) - (5*a*tan(c/2 + (d*x)
/2))/(8*d) + (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*a*log(tan(c/2 + (d*x)/2))
)/(2*d) - (2*a*atan((4*a^2)/(6*a^2 + 4*a^2*tan(c/2 + (d*x)/2)) - (6*a^2*tan
(c/2 + (d*x)/2))/(6*a^2 + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

3.13 $\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$-ax-\frac{15a\tanh^{-1}(\cos(c+dx))}{8d}+\frac{15a\cos(c+dx)}{8d}-\frac{a\cot(c+dx)}{d}+\frac{5a\cos(c+dx)\cot^2(c+dx)}{8d}+\frac{a\cot^3(c+dx)}{3d}$$

```
[Out] -a*x-15/8*a*arctanh(cos(d*x+c))/d+15/8*a*cos(d*x+c)/d-a*cot(d*x+c)/d+5/8*a*
cos(d*x+c)*cot(d*x+c)^2/d+1/3*a*cot(d*x+c)^3/d-1/4*a*cos(d*x+c)*cot(d*x+c)^
4/d-1/5*a*cot(d*x+c)^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2789, 2672, 294, 327, 212, 3554, 8}

$$\frac{15a\cos(c+dx)}{8d}-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{15a\tanh^{-1}(\cos(c+dx))}{8d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d) - (
a*Cot[c + d*x])/d + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*
x]^3)/(3*d) - (a*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx = \int \left(a\cos(c+dx)\cot^5(c+dx)+a\cot^6(c+dx)\right)\,dx$$

$$= a\int \cos(c+dx)\cot^5(c+dx)\,dx + a\int \cot^6(c+dx)\,dx$$

$$= -\frac{a\cot^5(c+dx)}{5d} - a\int \cot^4(c+dx)\,dx - \frac{a\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \text{c}\right.}{d}$$

$$= \frac{a\cot^3(c+dx)}{3d} - \frac{a\cos(c+dx)\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + a\int$$

$$= -\frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3(c+dx)}{3d} - $$

$$= -ax + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+}{8d}$$

$$= -ax - \frac{15a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 164, normalized size = 1.34

$$\frac{a\cos(c+dx)}{d}+\frac{9a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d}-\frac{15a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{15a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{9a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d + (9*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^
4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x
]^2])/(5*d) - (15*a*Log[Cos[(c + d*x)/2]])/(8*d) + (15*a*Log[Sin[(c + d*x)/
2]])/(8*d) - (9*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d
)
```

Maple [A]
time = 0.16, size = 129, normalized size = 1.06

method	result
derivativedivides	$\frac{a(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3(\cos^7(dx+c))}{8\sin(dx+c)^2}+\frac{3(\cos^5(dx+c))}{8}+\frac{5(\cos^3(dx+c))}{8}+\frac{15\cos(dx+c)}{8}+\frac{15\ln(\csc(dx+c)-\cot(dx+c))}{8})+a(-(\cot^5}{d}$
default	$\frac{a(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3(\cos^7(dx+c))}{8\sin(dx+c)^2}+\frac{3(\cos^5(dx+c))}{8}+\frac{5(\cos^3(dx+c))}{8}+\frac{15\cos(dx+c)}{8}+\frac{15\ln(\csc(dx+c)-\cot(dx+c))}{8})+a(-(\cot^5}{d}$
risch	$-ax+\frac{a\,e^{i(dx+c)}}{2d}+\frac{a\,e^{-i(dx+c)}}{2d}-\frac{a(360ie^{8i(dx+c)}+135\,e^{9i(dx+c)}-720ie^{6i(dx+c)}-150\,e^{7i(dx+c)}+1120ie^{4i(dx+c)}-}{60d(e^{2i(dx+c)}-1)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*co
s(d*x+c)^5+5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))
+a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 125, normalized size = 1.02

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+15\,a\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos\left(dx+c\right)+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a + 15*a*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4
```

```
 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*
log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(110) = 220$.
time = 0.36, size = 222, normalized size = 1.82

$$-\frac{368\,a\cos\left(dx+c\right)^{5}-560\,a\cos\left(dx+c\right)^{3}+225\,\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-225\,\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+240\,a\cos\left(dx+c\right)+30\,\left(8\,adx\cos\left(dx+c\right)^{4}-8\,a\cos\left(dx+c\right)^{5}-16\,adx\cos\left(dx+c\right)^{2}+25\,a\cos\left(dx+c\right)^{3}+8\,adx-15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*a*cos(d*x + c)^5 - 560*a*cos(d*x + c)^3 + 225*(a*cos(d*x + c)^4
 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(
a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 240*a*cos(d*x + c) + 30*(8*a*d*x*cos(d*x + c)^4 - 8*a*cos(d*x +
 c)^5 - 16*a*d*x*cos(d*x + c)^2 + 25*a*cos(d*x + c)^3 + 8*a*d*x - 15*a*cos(
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\cot^{6}\left(c+dx\right)\,dx+\int\cot^{6}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**6, x) + Integral(cot(c + d*x)**6, x)
)
```

Giac [A]
time = 4.80, size = 199, normalized size = 1.63

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-960\,(dx+c)a+1800\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+660\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{1920\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}-\frac{4110\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+660\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*a*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 1800*
a*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*a/(tan
(1/2*d*x + 1/2*c)^2 + 1) - (4110*a*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*d
*x + 1/2*c)^4 - 240*a*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
+ 15*a*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 6.67, size = 291, normalized size = 2.39

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{22\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-72\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{59\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{32\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a}{5}}{d\,\left(32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}-\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{15\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}+\frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{\frac{15\,a^2}{2}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{15\,a^2}{2}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a*sin(c + d*x)),x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(16*d) - (a/5 + (a*tan(c/2 + (d*x)/2))/2 - (32*a*
tan(c/2 + (d*x)/2)^2)/15 - (15*a*tan(c/2 + (d*x)/2)^3)/2 + (59*a*tan(c/2 +
(d*x)/2)^4)/3 - 72*a*tan(c/2 + (d*x)/2)^5 + 22*a*tan(c/2 + (d*x)/2)^6)/(d*(
32*tan(c/2 + (d*x)/2)^5 + 32*tan(c/2 + (d*x)/2)^7)) - (a*tan(c/2 + (d*x)/2)
^2)/(4*d) - (7*a*tan(c/2 + (d*x)/2)^3)/(96*d) + (a*tan(c/2 + (d*x)/2)^4)/(6
4*d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (15*a*log(tan(c/2 + (d*x)/2)))/(8
*d) + (2*a*atan((4*a^2)/((15*a^2)/2 + 4*a^2*tan(c/2 + (d*x)/2)) - (15*a^2*t
an(c/2 + (d*x)/2))/(2*((15*a^2)/2 + 4*a^2*tan(c/2 + (d*x)/2)))))/d
```

3.14 $\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=119

$$-\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^4}{4d(a-a\sin(c+d$$

[Out] -31/8*a^2*ln(1-sin(d*x+c))/d-1/8*a^2*ln(1+sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/2*a^2*sin(d*x+c)^2/d+1/4*a^4/d/(a-a*sin(d*x+c))^2-9/4*a^3/d/(a-a*sin(d*x+c))

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]

[Out] (-31*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) - (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) - (9*a^3)/(4*d*(a - a*Sin[c + d*x]))

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^4}{2(a-x)^3}-\frac{9a^3}{4(a-x)^2}+\frac{31a^2}{8(a-x)}-x-\frac{a^2}{8(a+x)}\right)dx, x, a\,\text{si}}{d}$$
$$= -\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}-\frac{2a^2\sin(c+}{d}$$

Mathematica [A]
time = 0.16, size = 75, normalized size = 0.63

$$-\frac{a^2\left(31\log(1-\sin(c+dx))+\log(1+\sin(c+dx))-\frac{2}{(-1+\sin(c+dx))^2}-\frac{18}{-1+\sin(c+dx)}+16\sin(c+dx)+4\sin^2(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -1/8*(a^2*(31*Log[1 - Sin[c + d*x]] + Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c
 + d*x])^2 - 18/(-1 + Sin[c + d*x]) + 16*Sin[c + d*x] + 4*Sin[c + d*x]^2))/
d
```

Maple [A]
time = 0.18, size = 206, normalized size = 1.73

method	result
risch	$4ia^2x+\frac{a^2e^{2i(dx+c)}}{8d}+\frac{ia^2e^{i(dx+c)}}{d}-\frac{ia^2e^{-i(dx+c)}}{d}+\frac{a^2e^{-2i(dx+c)}}{8d}+\frac{8ia^2c}{d}+\frac{i\left(-9a^2e^{i(dx+c)}-16ia^2e^{2i(dx+c}}{2\left(e^{i(dx+c)}-i\right)^4}$
derivativedivides	$\frac{a^2\left(\frac{\sin^8(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}-\frac{\left(\sin^6(dx+c)\right)}{2}-\frac{3\left(\sin^4(dx+c)\right)}{4}-\frac{3\left(\sin^2(dx+c)\right)}{2}-3\ln(\cos(dx+c))\right)+2a^2\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3(s}{8c}}{}$
default	$\frac{a^2\left(\frac{\sin^8(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}-\frac{\left(\sin^6(dx+c)\right)}{2}-\frac{3\left(\sin^4(dx+c)\right)}{4}-\frac{3\left(\sin^2(dx+c)\right)}{2}-3\ln(\cos(dx+c))\right)+2a^2\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3(s}{8c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4*sin(d*x+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*s
in(d*x+c)^6-3/4*sin(d*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x+c)))+2*a^2*(1/4*
sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-5/
8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+a^2*(1/4*tan
(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 96, normalized size = 0.81

$$-\frac{4\,a^2\sin\left(dx+c\right)^2+a^2\log\left(\sin\left(dx+c\right)+1\right)+31\,a^2\log\left(\sin\left(dx+c\right)-1\right)+16\,a^2\sin\left(dx+c\right)-\frac{2\,\left(9\,a^2\sin(dx+c)-8\,a^2\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(4*a^2*sin(d*x + c)^2 + a^2*log(sin(d*x + c) + 1) + 31*a^2*log(sin(d*x
 + c) - 1) + 16*a^2*sin(d*x + c) - 2*(9*a^2*sin(d*x + c) - 8*a^2)/(sin(d*x
+ c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.37, size = 168, normalized size = 1.41

$$\frac{4\,a^2\cos\left(dx+c\right)^4+22\,a^2\cos\left(dx+c\right)^2-12\,a^2-\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)-31\,\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(4\,a^2\cos\left(dx+c\right)^2-5\,a^2\right)\sin\left(dx+c\right)}{8\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*cos(d*x + c)^4 + 22*a^2*cos(d*x + c)^2 - 12*a^2 - (a^2*cos(d*x +
 c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(sin(d*x + c) + 1) - 31*(a^2*cos(d*x
 + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(-sin(d*x + c) + 1) - 2*(4*a^2*cos
(d*x + c)^2 - 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\tan^5\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\tan^5\left(c+dx\right)dx+\int\tan^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**5,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**5, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**5, x) + Integral(tan(c + d*x)**5, x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.57, size = 283, normalized size = 2.38

$$\frac{4a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}-\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{4d}-\frac{\frac{15a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{2}-22a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\frac{61a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{2}-36a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{61a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2}-22a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{15a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}-\frac{31a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (4*a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a^2*log(tan(c/2 + (d*x)/2) + 1))
/(4*d) - ((61*a^2*tan(c/2 + (d*x)/2)^3)/2 - 22*a^2*tan(c/2 + (d*x)/2)^2 - 3
6*a^2*tan(c/2 + (d*x)/2)^4 + (61*a^2*tan(c/2 + (d*x)/2)^5)/2 - 22*a^2*tan(c
/2 + (d*x)/2)^6 + (15*a^2*tan(c/2 + (d*x)/2)^7)/2 + (15*a^2*tan(c/2 + (d*x)
/2))/2)/(d*(8*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 12*tan(c/2 + (d
*x)/2)^3 + 14*tan(c/2 + (d*x)/2)^4 - 12*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 +
(d*x)/2)^6 - 4*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 + 1)) - (31*a^2*
log(tan(c/2 + (d*x)/2) - 1))/(4*d)
```

3.15 $\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{3a^2\log(1-\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^3}{d(a-a\sin(c+dx))}$$

```
[Out] 3*a^2*ln(1-sin(d*x+c))/d+2*a^2*sin(d*x+c)/d+1/2*a^2*sin(d*x+c)^2/d+a^3/d/(a
-a*sin(d*x+c))
```

Rubi [A]

time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2786, 45}

$$\frac{a^3}{d(a-a\sin(c+dx))}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (3*a^2*Log[1 - Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x
]^2)/(2*d) + a^3/(d*(a - a*Sin[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx&=\frac{\text{Subst}\left(\int\frac{x^3}{(a-x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(2a+\frac{a^3}{(a-x)^2}-\frac{3a^2}{a-x}+x\right)dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{3a^2\log(1-\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{}{d(}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 0.75

$$\frac{a^2\left(6\log(1-\sin(c+dx))+\frac{2}{1-\sin(c+dx)}+4\sin(c+dx)+\sin^2(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(6*Log[1 - Sin[c + d*x]] + 2/(1 - Sin[c + d*x]) + 4*Sin[c + d*x] + Sin
[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.17, size = 136, normalized size = 1.89

method	result
risch	$-3ia^2x-\frac{ia^2 e^{i(dx+c)}}{d}+\frac{ia^2 e^{-i(dx+c)}}{d}-\frac{6ia^2c}{d}-\frac{2ia^2 e^{i(dx+c)}}{\left(e^{i(dx+c)}-i\right)^2 d}+\frac{6a^2\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{a^2\cos(2dx+2c)}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+2a^2\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(c}{}\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+2a^2\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(c}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(
cos(d*x+c)))+2*a^2*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(
d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))
)
```

Maxima [A]
time = 0.29, size = 58, normalized size = 0.81

$$\frac{a^2\sin\left(dx+c\right)^2+6\,a^2\log\left(\sin\left(dx+c\right)-1\right)+4\,a^2\sin\left(dx+c\right)-\frac{2\,a^2}{\sin(dx+c)-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*sin(d*x + c)^2 + 6*a^2*log(sin(d*x + c) - 1) + 4*a^2*sin(d*x + c)
- 2*a^2/(sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.35, size = 90, normalized size = 1.25

$$-\frac{6\,a^2\cos\left(dx+c\right)^2-3\,a^2-12\left(a^2\sin\left(dx+c\right)-a^2\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,a^2\cos\left(dx+c\right)^2+7\,a^2\right)\sin\left(dx+c\right)}{4\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^2*cos(d*x + c)^2 - 3*a^2 - 12*(a^2*sin(d*x + c) - a^2)*log(-sin(d
*x + c) + 1) + (2*a^2*cos(d*x + c)^2 + 7*a^2)*sin(d*x + c))/(d*sin(d*x + c)
 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\tan^3\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\tan^3\left(c+dx\right)dx+\int\tan^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**3, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 7.11, size = 204, normalized size = 2.83

$$\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-6\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+8\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-6\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+6\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}+\frac{6\,a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{d}-\frac{3\,a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (8*a^2*tan(c/2 + (d*x)/2)^3 - 6*a^2*tan(c/2 + (d*x)/2)^2 - 6*a^2*tan(c/2 +
(d*x)/2)^4 + 6*a^2*tan(c/2 + (d*x)/2)^5 + 6*a^2*tan(c/2 + (d*x)/2))/(d*(3*t
an(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^3 + 3*tan
(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^6 + 1)) + (
6*a^2*log(tan(c/2 + (d*x)/2) - 1))/d - (3*a^2*log(tan(c/2 + (d*x)/2)^2 + 1)
)/d
```

3.16 $\int (a+a\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=52

$$-\frac{2a^2\log(1-\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\sin^2(c+dx)}{2d}$$

```
[Out] -2*a^2*ln(1-sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/2*a^2*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 78}

$$-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{2a^2\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (-2*a^2*Log[1 - Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*
x]^2)/(2*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{2a^2}{a-x}-x\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{2a^2\log(1-\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\sin^2(c+dx)}{2d}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.77

$$-\frac{a^2(4\log(1-\sin(c+dx))+4\sin(c+dx)+\sin^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
[Out] -1/2*(a^2*(4*Log[1 - Sin[c + d*x]] + 4*Sin[c + d*x] + Sin[c + d*x]^2))/d
```

Maple [A]
time = 0.12, size = 70, normalized size = 1.35

method	result	size
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-a^2\ln(\cos(dx+c))}{d}$	70
default	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-a^2\ln(\cos(dx+c))}{d}$	70
risch	$2ia^2x+\frac{ia^2e^{i(dx+c)}}{d}-\frac{ia^2e^{-i(dx+c)}}{d}+\frac{4ia^2c}{d}-\frac{4a^2\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a^2\cos(2dx+2c)}{4d}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+2*a^2*(-sin(d*x+c)+ln(sec(d*x+c
)+tan(d*x+c)))-a^2*ln(cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.83

$$-\frac{a^2\sin\left(dx+c\right)^2+4\,a^2\log\left(\sin\left(dx+c\right)-1\right)+4\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*sin(d*x + c)^2 + 4*a^2*log(sin(d*x + c) - 1) + 4*a^2*sin(d*x + c)
)/d
```

Fricas [A]
time = 0.36, size = 45, normalized size = 0.87

$$\frac{a^2\cos(dx+c)^2 - 4a^2\log(-\sin(dx+c)+1) - 4a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cos(d*x + c)^2 - 4*a^2*log(-sin(d*x + c) + 1) - 4*a^2*sin(d*x + c)
)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin(c+dx)\tan(c+dx)\,dx + \int \sin^2(c+dx)\tan(c+dx)\,dx + \int \tan(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x), x) + Integral(sin(c + d*x)**2*t
an(c + d*x), x) + Integral(tan(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6695 vs. $2(50) = 100$.
time = 5.26, size = 6695, normalized size = 128.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
n(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2
 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*ta
```

n(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
+ 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(c)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan
(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(
d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - a^2*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(ta
n(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^2*log(2*(tan(
1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*ta
n(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan
(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2
*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 4*a^2*lo
g(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*
d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1
/2*d*x)^2*tan(c)^2 + 4*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
 + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)
)*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 16*a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan
(1/2*c)*tan(c)^2 + 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*
c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
 tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*
a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*c)
^2*tan(c)^2 - 16*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 + 4*a^2*
log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/
2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1

```
/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(ta
n(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a^2*log(4*(tan(d*
x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*
tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 +
 a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*tan(d*x)*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(c) - a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - a^2*tan(d*x
)^2*tan(1/2*c)^2*tan(c)^2 + a^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a^
2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(
1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*t
an(1/2*d*x)^2 - 4*a^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4
*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + t...
```

Mupad [B]
time = 6.67, size = 178, normalized size = 3.42

$$-\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(4\,a^2\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)-2\,a^2\left(4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)+1\right)\right)+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}-\frac{2\,a^2\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (4*a^2*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*(4*a^2*(2*log(tan(c/2
+ (d*x)/2) - 1) - log(tan(c/2 + (d*x)/2)^2 + 1)) - 2*a^2*(4*log(tan(c/2 + (
d*x)/2) - 1) - 2*log(tan(c/2 + (d*x)/2)^2 + 1) + 1)) + 4*a^2*tan(c/2 + (d*x
)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (2*a^2*(2*log(tan(c/2 + (d*x)/2) -
 1) - log(tan(c/2 + (d*x)/2)^2 + 1)))/d
```

3.17 $\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^2(c+dx)(a+a\sin(c+dx))^4}{2a^2d}$$

```
[Out] -1/2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4/a^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 75}

$$-\frac{\csc^2(c+dx)(a\sin(c+dx)+a)^4}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(Csc[c + d*x]^2*(a + a*Sin[c + d*x])^4)/(a^2*d)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{\csc^2(c+dx)(a+a\sin(c+dx))^4}{2a^2d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 0.93

$$-\frac{a^2\csc^2(c+dx)(1+\sin(c+dx))^4}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*Csc[c + d*x]^2*(1 + Sin[c + d*x])^4)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(28) = 56$.
time = 0.21, size = 94, normalized size = 3.13

method	result
derivativedivides	$\frac{a^2\left(\frac{\cos^2(dx+c)}{2}+\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+a^2\left(-\frac{\cot^2(dx+c)}{2}-\ln(\sin(dx+c\right.}{d}$
default	$\frac{a^2\left(\frac{\cos^2(dx+c)}{2}+\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+a^2\left(-\frac{\cot^2(dx+c)}{2}-\ln(\sin(dx+c\right.}{d}$
risch	$\frac{a^2 e^{2i(dx+c)}}{8d}+\frac{i a^2 e^{i(dx+c)}}{d}-\frac{i a^2 e^{-i(dx+c)}}{d}+\frac{a^2 e^{-2i(dx+c)}}{8d}-\frac{2i a^2\left(i e^{2i(dx+c)}+2\,e^{3i(dx+c)}-2\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+2*a^2*(-1/sin(d*x+c)*cos(d*x+c)^
4-(2+cos(d*x+c)^2)*sin(d*x+c))+a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.27, size = 53, normalized size = 1.77

$$-\frac{a^2\sin(dx+c)^2+4\,a^2\sin(dx+c)+\frac{4\,a^2\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*sin(d*x + c)^2 + 4*a^2*sin(d*x + c) + (4*a^2*sin(d*x + c) + a^2)/
sin(d*x + c)^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.
time = 0.35, size = 76, normalized size = 2.53

$$\frac{2\,a^2\cos(dx+c)^4-3\,a^2\cos(dx+c)^2+3\,a^2-8\left(a^2\cos(dx+c)^2-2\,a^2\right)\sin(dx+c)}{4\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*cos(d*x + c)^4 - 3*a^2*cos(d*x + c)^2 + 3*a^2 - 8*(a^2*cos(d*x +
 c)^2 - 2*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\cot^3\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\cot^3\left(c+dx\right)dx+\int\cot^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(sin(c + d*x)**
2*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x))
```

Giac [A]
time = 3.62, size = 47, normalized size = 1.57

$$-\frac{a^2\left(\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)\right)^2+4\,a^2\left(\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*(1/sin(d*x + c) + sin(d*x + c))^2 + 4*a^2*(1/sin(d*x + c) + sin(d
*x + c)))/d
```

Mupad [B]
time = 6.67, size = 56, normalized size = 1.87

$$-\frac{a^2\left(2\sin\left(c+d\,x\right)^4+8\sin\left(c+d\,x\right)^3-\sin\left(c+d\,x\right)^2+8\,\sin\left(c+d\,x\right)+2\right)}{4\,d\sin\left(c+d\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] -(a^2*(8*sin(c + d*x) - sin(c + d*x)^2 + 8*sin(c + d*x)^3 + 2*sin(c + d*x)^
4 + 2))/(4*d*sin(c + d*x)^2)
```

3.18 $\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=132

$$-\frac{6a^2\csc(c+dx)}{d}+\frac{2a^2\csc^3(c+dx)}{d}+\frac{a^2\csc^4(c+dx)}{2d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^6(c+dx)}{6d}+\frac{2a^2\log(\sin(c+dx}{d}$$

```
[Out] -6*a^2*csc(d*x+c)/d+2*a^2*csc(d*x+c)^3/d+1/2*a^2*csc(d*x+c)^4/d-2/5*a^2*csc
(d*x+c)^5/d-1/6*a^2*csc(d*x+c)^6/d+2*a^2*ln(sin(d*x+c))/d-2*a^2*sin(d*x+c)/
d-1/2*a^2*sin(d*x+c)^2/d
```

Rubi [A]

time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\csc^4(c+dx)}{2d}+\frac{2a^2\csc^3(c+dx)}{d}-\frac{6a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-6*a^2*Csc[c + d*x])/d + (2*a^2*Csc[c + d*x]^3)/d + (a^2*Csc[c + d*x]^4)/(
2*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Csc[c + d*x]^6)/(6*d) + (2*a^2*L
og[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^5}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^8}{x^7}+\frac{2a^7}{x^6}-\frac{2a^6}{x^5}-\frac{6a^5}{x^4}+\frac{6a^3}{x^2}+\frac{2a^2}{x}-x\right)dx, x, a\sin\right.}{d}$$
$$= -\frac{6a^2\csc(c+dx)}{d}+\frac{2a^2\csc^3(c+dx)}{d}+\frac{a^2\csc^4(c+dx)}{2d}-\frac{2a^2\csc^5}{5}$$

Mathematica [A]
time = 0.15, size = 86, normalized size = 0.65

$$-\frac{a^2(180\csc(c+dx)-60\csc^3(c+dx)-15\csc^4(c+dx)+12\csc^5(c+dx)+5\csc^6(c+dx)-60\log(\sin(c+dx))+60\sin(c+dx)+15\sin^2(c+dx))}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/30*(a^2*(180*Csc[c + d*x] - 60*Csc[c + d*x]^3 - 15*Csc[c + d*x]^4 + 12*C
sc[c + d*x]^5 + 5*Csc[c + d*x]^6 - 60*Log[Sin[c + d*x]] + 60*Sin[c + d*x] +
 15*Sin[c + d*x]^2))/d
```

Maple [A]
time = 0.23, size = 228, normalized size = 1.73

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^8(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^8(dx+c)}{2\sin(dx+c)^2}+\frac{\left(\cos^6(dx+c)\right)}{2}+\frac{3\left(\cos^4(dx+c)\right)}{4}+\frac{3\left(\cos^2(dx+c)\right)}{2}+3\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^8(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^8(dx+c)}{2\sin(dx+c)^2}+\frac{\left(\cos^6(dx+c)\right)}{2}+\frac{3\left(\cos^4(dx+c)\right)}{4}+\frac{3\left(\cos^2(dx+c)\right)}{2}+3\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}\right.}{}$
risch	$-2ia^2x+\frac{a^2e^{2i(dx+c)}}{8d}+\frac{ia^2e^{i(dx+c)}}{d}-\frac{ia^2e^{-i(dx+c)}}{d}+\frac{a^2e^{-2i(dx+c)}}{8d}-\frac{4ia^2c}{d}-\frac{4ia^2\left(45\,e^{11i(dx+c)}+30ie^{8i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^8+1/2/sin(d*x+c)^2*cos(d*x+c)^8+1/2*
cos(d*x+c)^6+3/4*cos(d*x+c)^4+3/2*cos(d*x+c)^2+3*ln(sin(d*x+c)))+2*a^2*(-1/
5/sin(d*x+c)^5*cos(d*x+c)^8+1/5/sin(d*x+c)^3*cos(d*x+c)^8-1/sin(d*x+c)*cos(
d*x+c)^8-(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+
a^2*(-1/6*cot(d*x+c)^6+1/4*cot(d*x+c)^4-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 107, normalized size = 0.81

$$-\frac{15\,a^2\sin\left(dx+c\right)^2-60\,a^2\log\left(\sin\left(dx+c\right)\right)+60\,a^2\sin\left(dx+c\right)+\frac{180\,a^2\sin(dx+c)^5-60\,a^2\sin(dx+c)^3-15\,a^2\sin(dx+c)^2+12\,a^2\sin(dx+c)+5\,a^2}{\sin(dx+c)^6}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*a^2*sin(d*x + c)^2 - 60*a^2*log(sin(d*x + c)) + 60*a^2*sin(d*x +
c) + (180*a^2*sin(d*x + c)^5 - 60*a^2*sin(d*x + c)^3 - 15*a^2*sin(d*x + c)^
2 + 12*a^2*sin(d*x + c) + 5*a^2)/sin(d*x + c)^6)/d
```

Fricas [A]
time = 0.40, size = 206, normalized size = 1.56

$$\frac{30\,a^2\cos\left(dx+c\right)^8-105\,a^2\cos\left(dx+c\right)^6+135\,a^2\cos\left(dx+c\right)^4-45\,a^2\cos\left(dx+c\right)^2-5\,a^2+120\,\left(a^2\cos\left(dx+c\right)^6-3\,a^2\cos\left(dx+c\right)^4+3\,a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-24\,\left(5\,a^2\cos\left(dx+c\right)^6-30\,a^2\cos\left(dx+c\right)^4+40\,a^2\cos\left(dx+c\right)^2-16\,a^2\right)\sin\left(dx+c\right)}{60\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(30*a^2*cos(d*x + c)^8 - 105*a^2*cos(d*x + c)^6 + 135*a^2*cos(d*x + c)
^4 - 45*a^2*cos(d*x + c)^2 - 5*a^2 + 120*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*
x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*x + c)) - 24*(5*a^2*co
s(d*x + c)^6 - 30*a^2*cos(d*x + c)^4 + 40*a^2*cos(d*x + c)^2 - 16*a^2)*sin(
d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\cot^7\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\cot^7\left(c+dx\right)dx+\int\cot^7\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**7, x) + Integral(sin(c + d*x)**
2*cot(c + d*x)**7, x) + Integral(cot(c + d*x)**7, x))
```

Giac [A]
time = 4.02, size = 121, normalized size = 0.92

$$-\frac{15\,a^2\sin\left(dx+c\right)^2-60\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a^2\sin\left(dx+c\right)+\frac{147\,a^2\sin(dx+c)^6+180\,a^2\sin(dx+c)^5-60\,a^2\sin(dx+c)^3-15\,a^2\sin(dx+c)^2+12\,a^2\sin(dx+c)+5\,a^2}{\sin(dx+c)^6}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*a^2*sin(d*x + c)^2 - 60*a^2*log(abs(sin(d*x + c))) + 60*a^2*sin(d
*x + c) + (147*a^2*sin(d*x + c)^6 + 180*a^2*sin(d*x + c)^5 - 60*a^2*sin(d*x
 + c)^3 - 15*a^2*sin(d*x + c)^2 + 12*a^2*sin(d*x + c) + 5*a^2)/sin(d*x + c)
^6)/d
```

Mupad [B]
time = 11.18, size = 392, normalized size = 2.97

$$-\frac{a^2\left(24\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-312\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-220\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3864\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-360\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+21000\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+3510\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+21000\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-360\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+3864\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-220\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-312\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+24\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+3840\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)+7680\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)+3840\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)-3840\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-7680\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-3840\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\right)}{1920\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(a + a*sin(c + d*x))^2,x)
```

```
[Out] -(a^2*(24*tan(c/2 + (d*x)/2) - 20*tan(c/2 + (d*x)/2)^2 - 312*tan(c/2 + (d*x
)/2)^3 - 220*tan(c/2 + (d*x)/2)^4 + 3864*tan(c/2 + (d*x)/2)^5 - 360*tan(c/2
 + (d*x)/2)^6 + 21000*tan(c/2 + (d*x)/2)^7 + 3510*tan(c/2 + (d*x)/2)^8 + 21
000*tan(c/2 + (d*x)/2)^9 - 360*tan(c/2 + (d*x)/2)^10 + 3864*tan(c/2 + (d*x)
/2)^11 - 220*tan(c/2 + (d*x)/2)^12 - 312*tan(c/2 + (d*x)/2)^13 - 20*tan(c/2
 + (d*x)/2)^14 + 24*tan(c/2 + (d*x)/2)^15 + 5*tan(c/2 + (d*x)/2)^16 + 3840*
tan(c/2 + (d*x)/2)^6*log(tan(c/2 + (d*x)/2)^2 + 1) + 7680*tan(c/2 + (d*x)/2
)^8*log(tan(c/2 + (d*x)/2)^2 + 1) + 3840*tan(c/2 + (d*x)/2)^10*log(tan(c/2
+ (d*x)/2)^2 + 1) - 3840*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 - 768
0*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^8 - 3840*log(tan(c/2 + (d*x)/2
))*tan(c/2 + (d*x)/2)^10 + 5))/(1920*d*tan(c/2 + (d*x)/2)^6*(tan(c/2 + (d*x
)/2)^2 + 1)^2)
```

3.19 $\int (a+a\sin(c+dx))^2\tan^6(c+dx)\,dx$

Optimal. Leaf size=149

$$-\frac{9a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{6a^2\sec(c+dx)}{d}-\frac{2a^2\sec^3(c+dx)}{d}+\frac{2a^2\sec^5(c+dx)}{5d}+\frac{9a^2\tan(c+dx)}{2d}-\frac{3a^2\tan^3(c}{2d}$$

```
[Out] -9/2*a^2*x+2*a^2*cos(d*x+c)/d+6*a^2*sec(d*x+c)/d-2*a^2*sec(d*x+c)^3/d+2/5*a
^2*sec(d*x+c)^5/d+9/2*a^2*tan(d*x+c)/d-3/2*a^2*tan(d*x+c)^3/d+9/10*a^2*tan(
d*x+c)^5/d-1/2*a^2*sin(d*x+c)^2*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2789, 3554, 8, 2670, 276, 2671, 294, 308, 209}

$$\frac{2a^2\cos(c+dx)}{d}+\frac{9a^2\tan^5(c+dx)}{10d}-\frac{3a^2\tan^3(c+dx)}{2d}+\frac{9a^2\tan(c+dx)}{2d}+\frac{2a^2\sec^5(c+dx)}{5d}-\frac{2a^2\sec^3(c+dx)}{d}+\frac{6a^2\sec(c+dx)}{d}-\frac{a^2\sin^2(c+dx)\tan^5(c+dx)}{2d}-\frac{9a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] (-9*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (6*a^2*Sec[c + d*x])/d - (2*a^2*Sec
[c + d*x]^3)/d + (2*a^2*Sec[c + d*x]^5)/(5*d) + (9*a^2*Tan[c + d*x])/(2*d)
- (3*a^2*Tan[c + d*x]^3)/(2*d) + (9*a^2*Tan[c + d*x]^5)/(10*d) - (a^2*Sin[c
 + d*x]^2*Tan[c + d*x]^5)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sin(c+dx))^2\tan^6(c+dx)\,dx &= \int \left(a^2\tan^6(c+dx)+2a^2\sin(c+dx)\tan^6(c+dx)+a^2\sin^2(c+dx)\right.\\
&= a^2\int \tan^6(c+dx)\,dx+a^2\int \sin^2(c+dx)\tan^6(c+dx)\,dx+\left(2a^2\right)\\
&= \frac{a^2\tan^5(c+dx)}{5d}-a^2\int \tan^4(c+dx)\,dx+\frac{a^2\text{Subst}\left(\int \frac{x^8}{(1+x^2)^2}\,dx,x,\right.}{d}\\
&= -\frac{a^2\tan^3(c+dx)}{3d}+\frac{a^2\tan^5(c+dx)}{5d}-\frac{a^2\sin^2(c+dx)\tan^5(c+dx)}{2d}\\
&= \frac{2a^2\cos(c+dx)}{d}+\frac{6a^2\sec(c+dx)}{d}-\frac{2a^2\sec^3(c+dx)}{d}+\frac{2a^2\sec^5(c}{5d}\\
&= -a^2x+\frac{2a^2\cos(c+dx)}{d}+\frac{6a^2\sec(c+dx)}{d}-\frac{2a^2\sec^3(c+dx)}{d}+\frac{2a}{}\\
&= -\frac{9a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{6a^2\sec(c+dx)}{d}-\frac{2a^2\sec^3(c+dx)}{d}+
\end{aligned}
$$

Mathematica [A]
time = 0.57, size = 174, normalized size = 1.17

$$-\frac{a^2\sec^5(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4(-500+10(103+90c+90dx)\cos(c+dx)-544\cos(2(c+dx))-206\cos(3(c+dx))-180c\cos(3(c+dx))-180dx\cos(3(c+dx))+20\cos(4(c+dx))+250\sin(c+dx)-824\sin(2(c+dx))-720c\sin(2(c+dx))-720dx\sin(2(c+dx))+351\sin(3(c+dx))+5\sin(5(c+dx)))}{160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -1/160*(a^2*Sec[c + d*x]^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-500 +
10*(103 + 90*c + 90*d*x)*Cos[c + d*x] - 544*Cos[2*(c + d*x)] - 206*Cos[3*(c
 + d*x)] - 180*c*Cos[3*(c + d*x)] - 180*d*x*Cos[3*(c + d*x)] + 20*Cos[4*(c
+ d*x)] + 250*Sin[c + d*x] - 824*Sin[2*(c + d*x)] - 720*c*Sin[2*(c + d*x)]
- 720*d*x*Sin[2*(c + d*x)] + 351*Sin[3*(c + d*x)] + 5*Sin[5*(c + d*x)]))/d
```

Maple [A]
time = 0.24, size = 251, normalized size = 1.68

method	result
risch	$-\frac{9a^2x}{2}-\frac{ia^2e^{2i(dx+c)}}{8d}+\frac{a^2e^{i(dx+c)}}{d}+\frac{a^2e^{-i(dx+c)}}{d}+\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{-\frac{156a^2e^{i(dx+c)}}{5}-24ia^2e^{2i(dx+c)}+\frac{54ia^2}{5}}{\left(e^{i(dx+c)}+\right.}$
derivativedivides	$\frac{a^2\left(\frac{\sin^9(dx+c)}{5\cos(dx+c)^5}-\frac{4\left(\sin^9(dx+c)\right)}{15\cos(dx+c)^3}+\frac{8\left(\sin^9(dx+c)\right)}{5\cos(dx+c)}+\frac{8\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{5}-\right.}{}$

default	$\frac{a^2\left(\frac{\sin^9(dx+c)}{5\cos(dx+c)^5}-\frac{4\left(\sin^9(dx+c)\right)}{15\cos(dx+c)^3}+\frac{8\left(\sin^9(dx+c)\right)}{5\cos(dx+c)}+\frac{8\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c}{5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/5*sin(d*x+c)^9/cos(d*x+c)^5-4/15*sin(d*x+c)^9/cos(d*x+c)^3+8/5*
sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)
^3+35/16*sin(d*x+c))*cos(d*x+c)-7/2*d*x-7/2*c)+2*a^2*(1/5*sin(d*x+c)^8/cos(
d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*
x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/5*tan(d*x+c)^5
-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.51, size = 152, normalized size = 1.02

$$\frac{\left(6\tan(dx+c)^5-20\tan(dx+c)^3-105\,dx-105\,c+\frac{15\tan(dx+c)}{\tan(dx+c)^2+1}+90\tan(dx+c)\right)a^2+2\left(3\tan(dx+c)^5-5\tan(dx+c)^3-15\,dx-15\,c+15\tan(dx+c)\right)a^2+12\,a^2\left(\frac{15\cos(dx+c)^4-5\cos(dx+c)^2+1}{\cos(dx+c)^5}+5\cos(dx+c)\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/30*((6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan(d*x
+ c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*a^2 + 2*(3*tan(d*x + c)^5 - 5*
tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^2 + 12*a^2*((15*cos(d*x
 + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*cos(d*x + c)))/d
```

Fricas [A]
time = 0.37, size = 152, normalized size = 1.02

$$-\frac{45\,a^2dx\cos(dx+c)^3-10\,a^2\cos(dx+c)^4-90\,a^2dx\cos(dx+c)+78\,a^2\cos(dx+c)^2-4\,a^2-\left(5\,a^2\cos(dx+c)^4-90\,a^2dx\cos(dx+c)+84\,a^2\cos(dx+c)^2-6\,a^2\right)\sin(dx+c)}{10\left(d\cos(dx+c)^3+2\,d\cos(dx+c)\sin(dx+c)-2\,d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/10*(45*a^2*d*x*cos(d*x + c)^3 - 10*a^2*cos(d*x + c)^4 - 90*a^2*d*x*cos(d
*x + c) + 78*a^2*cos(d*x + c)^2 - 4*a^2 - (5*a^2*cos(d*x + c)^4 - 90*a^2*d*
x*cos(d*x + c) + 84*a^2*cos(d*x + c)^2 - 6*a^2)*sin(d*x + c))/(d*cos(d*x +
c)^3 + 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin(c+dx)\tan^6(c+dx)\,dx+\int\sin^2(c+dx)\tan^6(c+dx)\,dx+\int\tan^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**6,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**6, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**6, x) + Integral(tan(c + d*x)**6, x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 10.92, size = 392, normalized size = 2.63

$$-\frac{9\,a^2\,x}{2}-\frac{\frac{9\,a^2\,\left(c+d\,x\right)}{2}-\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(18\,a^2\,\left(c+d\,x\right)-\frac{a^2\,\left(180\,c+180\,d\,x-422\right)}{10}\right)+\frac{174\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{5}-\frac{a^2\,\left(45\,c+45\,d\,x-128\right)}{10}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(18\,a^2\,\left(c+d\,x\right)-\frac{a^2\,\left(180\,c+180\,d\,x-90\right)}{10}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(27\,a^2\,\left(c+d\,x\right)-\frac{a^2\,\left(270\,c+270\,d\,x-168\right)}{10}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{63\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(315\,c+315\,d\,x-360\right)}{10}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(27\,a^2\,\left(c+d\,x\right)-\frac{a^2\,\left(270\,c+270\,d\,x-600\right)}{10}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(36\,a^2\,\left(c+d\,x\right)-\frac{a^2\,\left(360\,c+360\,d\,x-424\right)}{10}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{63\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(315\,c+315\,d\,x-536\right)}{10}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(36\,a^2\,\left(c+d\,x\right)-\frac{a^2\,\left(360\,c+360\,d\,x-600\right)}{10}\right)}{d\,{\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^5\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6*(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (9*a^2*x)/2 - ((9*a^2*(c + d*x))/2 - tan(c/2 + (d*x)/2)*(18*a^2*(c + d*x)
 - (a^2*(180*c + 180*d*x - 422))/10) + (174*a^2*tan(c/2 + (d*x)/2)^5)/5 - (
a^2*(45*c + 45*d*x - 128))/10 + tan(c/2 + (d*x)/2)^9*(18*a^2*(c + d*x) - (a
^2*(180*c + 180*d*x - 90))/10) + tan(c/2 + (d*x)/2)^4*(27*a^2*(c + d*x) - (
a^2*(270*c + 270*d*x - 168))/10) - tan(c/2 + (d*x)/2)^8*((63*a^2*(c + d*x))
/2 - (a^2*(315*c + 315*d*x - 360))/10) - tan(c/2 + (d*x)/2)^6*(27*a^2*(c +
d*x) - (a^2*(270*c + 270*d*x - 600))/10) - tan(c/2 + (d*x)/2)^3*(36*a^2*(c
+ d*x) - (a^2*(360*c + 360*d*x - 424))/10) + tan(c/2 + (d*x)/2)^2*((63*a^2*
(c + d*x))/2 - (a^2*(315*c + 315*d*x - 536))/10) + tan(c/2 + (d*x)/2)^7*(36
*a^2*(c + d*x) - (a^2*(360*c + 360*d*x - 600))/10))/(d*(tan(c/2 + (d*x)/2)
- 1)^5*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.20 $\int (a+a\sin(c+dx))^2 \tan^4(c+dx)\,dx$

Optimal. Leaf size=120

$$\frac{7a^2x}{2}-\frac{16a^2\cos(c+dx)}{3d}-\frac{7a^2\cos(c+dx)\sin(c+dx)}{2d}-\frac{8a^2\cos(c+dx)\sin^2(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2}$$

```
[Out] 7/2*a^2*x-16/3*a^2*cos(d*x+c)/d-7/2*a^2*cos(d*x+c)*sin(d*x+c)/d-8/3*a^2*cos
(d*x+c)*sin(d*x+c)^2/d/(1-sin(d*x+c))+1/3*a^4*cos(d*x+c)*sin(d*x+c)^3/d/(a-
a*sin(d*x+c))^2
```

Rubi [A]
time = 0.14, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2787, 2844, 3056, 2813}

$$\frac{a^4\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{16a^2\cos(c+dx)}{3d}-\frac{8a^2\sin^2(c+dx)\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{7a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (7*a^2*x)/2 - (16*a^2*Cos[c + d*x])/(3*d) - (7*a^2*Cos[c + d*x]*Sin[c + d*x
])/(2*d) - (8*a^2*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(1 - Sin[c + d*x])) + (
a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 2787

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
```

```
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx &= a^4\int \frac{\sin^4(c+dx)}{(a-a\sin(c+dx))^2}\,dx\\ &= \frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{1}{3}a^2\int \frac{\sin^2(c+dx)(-3a-5a\sin(c+}{a-a\sin(c+dx)}\\ &= -\frac{8a^2\cos(c+dx)\sin^2(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{1}{3}\int\\ &= \frac{7a^2x}{2}-\frac{16a^2\cos(c+dx)}{3d}-\frac{7a^2\cos(c+dx)\sin(c+dx)}{2d}-\frac{8a^2\cos(c}{3d(1}\end{aligned}$$

Mathematica [A]
time = 0.85, size = 159, normalized size = 1.32

$$-\frac{a^2\left(-21(7+12c+12dx)\cos\left(\frac{1}{2}(c+dx)\right)+(239+84c+84dx)\cos\left(\frac{3}{2}(c+dx)\right)+3\left(-5\cos\left(\frac{5}{2}(c+dx)\right)+\cos\left(\frac{7}{2}(c+dx)\right)+2(50+56c+56dx+(-27+28c+28dx)\cos(c+dx)-6\cos(2(c+dx))-\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{48d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -1/48*(a^2*(-21*(7 + 12*c + 12*d*x)*Cos[(c + d*x)/2] + (239 + 84*c + 84*d*x
)*Cos[(3*(c + d*x))/2] + 3*(-5*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2]
+ 2*(50 + 56*c + 56*d*x + (-27 + 28*c + 28*d*x)*Cos[c + d*x] - 6*Cos[2*(c +
 d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2])))/(d*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^3)
```

Maple [A]
time = 0.24, size = 186, normalized size = 1.55

method	result
risch	$\frac{7a^2x}{2}+\frac{ia^2\mathrm{e}^{2i(dx+c)}}{8d}-\frac{a^2\mathrm{e}^{i(dx+c)}}{d}-\frac{a^2\mathrm{e}^{-i(dx+c)}}{d}-\frac{ia^2\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{2\left(-21ia^2\mathrm{e}^{i(dx+c)}+12a^2\mathrm{e}^{2i(dx+c)}-11a^2\right)}{3\left(\mathrm{e}^{i(dx+c)}-i\right)^3d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+2a^2\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+2a^2\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(si
n(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+2*a^
2*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+
4/3*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.51, size = 120, normalized size = 1.00

$$\frac{\left(2\tan(dx+c)^3+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^2+1}-12\tan(dx+c)\right)a^2+2\left(\tan(dx+c)^3+3\,dx+3\,c-3\tan(dx+c)\right)a^2-4\,a^2\left(\frac{6\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\cos(dx+c)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a^2 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c)
)*a^2 - 4*a^2*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [A]
time = 0.37, size = 196, normalized size = 1.63

$$\frac{3\,a^2\cos(dx+c)^4-6\,a^2\cos(dx+c)^3-42\,a^2dx+(21\,a^2dx+31\,a^2)\cos(dx+c)^2-2\,a^2-(21\,a^2dx-38\,a^2)\cos(dx+c)-\left(3\,a^2\cos(dx+c)^3-42\,a^2dx+9\,a^2\cos(dx+c)^2+2\,a^2-(21\,a^2dx-40\,a^2)\cos(dx+c)\right)\sin(dx+c)}{6\left(d\cos(dx+c)^2-d\cos(dx+c)+(d\cos(dx+c)+2\,d)\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^4 - 6*a^2*cos(d*x + c)^3 - 42*a^2*d*x + (21*a^2*d*x
 + 31*a^2)*cos(d*x + c)^2 - 2*a^2 - (21*a^2*d*x - 38*a^2)*cos(d*x + c) - (3
*a^2*cos(d*x + c)^3 - 42*a^2*d*x + 9*a^2*cos(d*x + c)^2 + 2*a^2 - (21*a^2*d
```

```
*x - 40*a^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c)
 + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int\tan^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**4, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**4, x) + Integral(tan(c + d*x)**4, x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 10.07, size = 287, normalized size = 2.39

$$\frac{7\,a^2\,x}{2}+\frac{\frac{7\,a^2\,\left(c+d\,x\right)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{21\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(63\,c+63\,d\,x-150\right)}{6}\right)-\frac{a^2\,\left(21\,c+21\,d\,x-64\right)}{6}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{21\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(63\,c+63\,d\,x-42\right)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{35\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(105\,c+105\,d\,x-126\right)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{35\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(105\,c+105\,d\,x-194\right)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{49\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(147\,c+147\,d\,x-196\right)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{49\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(147\,c+147\,d\,x-252\right)}{6}\right)}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (7*a^2*x)/2 + ((7*a^2*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((21*a^2*(c + d*x))
/2 - (a^2*(63*c + 63*d*x - 150))/6) - (a^2*(21*c + 21*d*x - 64))/6 + tan(c/
2 + (d*x)/2)^6*((21*a^2*(c + d*x))/2 - (a^2*(63*c + 63*d*x - 42))/6) - tan(
c/2 + (d*x)/2)^5*((35*a^2*(c + d*x))/2 - (a^2*(105*c + 105*d*x - 126))/6) +
 tan(c/2 + (d*x)/2)^2*((35*a^2*(c + d*x))/2 - (a^2*(105*c + 105*d*x - 194))
/6) + tan(c/2 + (d*x)/2)^4*((49*a^2*(c + d*x))/2 - (a^2*(147*c + 147*d*x -
196))/6) - tan(c/2 + (d*x)/2)^3*((49*a^2*(c + d*x))/2 - (a^2*(147*c + 147*d
*x - 252))/6))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.21 $\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=71

$$-\frac{5a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] -5/2*a^2*x+2*a^2*cos(d*x+c)/d+2*a^2*cos(d*x+c)/d/(1-sin(d*x+c))+1/2*a^2*cos
(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {2788, 2727, 2718, 2715, 8}

$$\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{5a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-5*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c
+ d*x])) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx &= a^2\int \left(-2-\frac{2}{-1+\sin(c+dx)}-2\sin(c+dx)-\sin^2(c+dx)\right)dx\\ &= -2a^2x-a^2\int \sin^2(c+dx)\,dx-\left(2a^2\right)\int \frac{1}{-1+\sin(c+dx)}\,dx-(2\\ &= -2a^2x+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin}{2d}\\ &= -\frac{5a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin}{2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 145 vs. 2(71) = 142.
time = 0.29, size = 145, normalized size = 2.04

$$-\frac{a^2(1+\sin(c+dx))^2\left(\cos\left(\frac{1}{2}(c+dx)\right)(10(c+dx)-8\cos(c+dx)-\sin(2(c+dx)))+\sin\left(\frac{1}{2}(c+dx)\right)(-2(8+5c+5dx)+8\cos(c+dx)+\sin(2(c+dx)))\right)}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -1/4*(a^2*(1 + Sin[c + d*x])^2*(Cos[(c + d*x)/2]*(10*(c + d*x) - 8*Cos[c +
d*x] - Sin[2*(c + d*x)]) + Sin[(c + d*x)/2]*(-2*(8 + 5*c + 5*d*x) + 8*Cos[c
 + d*x] + Sin[2*(c + d*x)])))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.20, size = 117, normalized size = 1.65

method	result
risch	$-\frac{5a^2x}{2}+\frac{a^2e^{i(dx+c)}}{d}+\frac{a^2e^{-i(dx+c)}}{d}+\frac{4a^2}{d\left(e^{i(dx+c)}-i\right)}+\frac{a^2\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+a^2(\mathrm{t}}{d}$

default	$\frac{a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+a^2}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-
3/2*d*x-3/2*c)+2*a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+
a^2*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 84, normalized size = 1.18

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)a^2+2\left(dx+c-\tan\left(dx+c\right)\right)a^2-4\,a^2\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^
2 + 2*(d*x + c - tan(d*x + c))*a^2 - 4*a^2*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

Fricas [A]
time = 0.36, size = 125, normalized size = 1.76

$$\frac{a^2\cos\left(dx+c\right)^3-5\,a^2dx+4\,a^2\cos\left(dx+c\right)^2+4\,a^2-\left(5\,a^2dx-7\,a^2\right)\cos\left(dx+c\right)+\left(5\,a^2dx+a^2\cos\left(dx+c\right)^2-3\,a^2\cos\left(dx+c\right)+4\,a^2\right)\sin\left(dx+c\right)}{2\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cos(d*x + c)^3 - 5*a^2*d*x + 4*a^2*cos(d*x + c)^2 + 4*a^2 - (5*a^2
*d*x - 7*a^2)*cos(d*x + c) + (5*a^2*d*x + a^2*cos(d*x + c)^2 - 3*a^2*cos(d*
x + c) + 4*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int\tan^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**2, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5370 vs. $2(65) = 130$.

time = 12.03, size = 5370, normalized size = 75.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 5*a^2*d*x
*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 5*a^2*d*x*tan(d*x)^2*tan(1
/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(
1/2*c)^3*tan(c)^3 + 5*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3
 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 5*a^2*tan(d*x)^3
*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 5*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan
(1/2*c)^4*tan(c)^3 - 5*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 20*
a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 5*a^2*d*x*tan(d*x)*
tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/
2*c)^4*tan(c) + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2
- 5*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 8*a^2*tan(d*x)^2*tan(1/2
*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 5*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^
3 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 20*a^2*d*x*t
an(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 20*a^2*d*x*tan(d*x)*tan(1/2*
d*x)^3*tan(1/2*c)^3*tan(c)^3 + 32*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^
3*tan(c)^3 - 5*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 8*a^2*tan(d*x)*ta
n(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 4*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/
2*c)^4 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 20*a^2*tan(d
*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*tan(d*x)*tan(1/2*d*x)^4*
tan(1/2*c)^4*tan(c)^2 - 20*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c
)^3 + 4*a^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 20*a^2*d*x*tan(d*x)^2*ta
n(1/2*d*x)^3*tan(1/2*c)^3 - 5*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 8*a^2*t
an(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 5*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4
*tan(c) - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 20*a^2*d
*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 20*a^2*d*x*tan(d*x)*tan(1/
2*d*x)^3*tan(1/2*c)^3*tan(c) + 32*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^
3*tan(c) - 5*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c) - 8*a^2*tan(d*x)*tan(1/
2*d*x)^4*tan(1/2*c)^4*tan(c) + 5*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2
 + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 20*a^2*d*x*ta
n(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 20*a^2*d*x*tan(1/2*d*x)^3*tan
(1/2*c)^3*tan(c)^2 - 32*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2
 + 5*a^2*d*x*tan(d*x)^2*tan(1/2*c)^4*tan(c)^2 + 8*a^2*tan(1/2*d*x)^4*tan(1/
2*c)^4*tan(c)^2 - 5*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*tan(d*
x)^3*tan(1/2*d*x)^4*tan(c)^3 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c
)*tan(c)^3 - 20*a^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 32*a^
2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 96*a^2*tan(d*x)^3*tan(1/2
```

```
*d*x)^2*tan(1/2*c)^2*tan(c)^3 - 20*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)
^3*tan(c)^3 - 32*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 + 32*a^2
*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 5*a^2*d*x*tan(d*x)*tan(1/2
*c)^4*tan(c)^3 - 8*a^2*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 16*a^2*tan(d*x)^3
*tan(1/2*d*x)^3*tan(1/2*c)^3 + 5*a^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4 -
 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 5*a^2*tan(1/2*d*x)^4
*tan(1/2*c)^4*tan(c) - 5*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^2 - 20*a^2*ta
n(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 20*a^2*tan(d*x)^3*tan(1/2*d*x
)*tan(1/2*c)^3*tan(c)^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)
^2 - 5*a^2*tan(d*x)^3*tan(1/2*c)^4*tan(c)^2 - 5*a^2*tan(d*x)^2*tan(1/2*d*x)
^4*tan(c)^3 - 20*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 20*a^2
*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 16*a^2*tan(1/2*d*x)^3*tan(
1/2*c)^3*tan(c)^3 - 5*a^2*tan(d*x)^2*tan(1/2*c)^4*tan(c)^3 + 5*a^2*d*x*tan(
d*x)^2*tan(1/2*d*x)^4 + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c) + 2
0*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3 + 20*a^2*d*x*tan(1/2*d*x)^3*
tan(1/2*c)^3 - 32*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 5*a^2*d*x*ta
n(d*x)^2*tan(1/2*c)^4 + 8*a^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 5*a^2*d*x*tan(d
*x)*tan(1/2*d*x)^4*tan(c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 20*a^2
*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 20*a^2*d*x*tan(d*x)*tan(1/
2*d*x)^3*tan(1/2*c)*tan(c) - 32*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*ta
n(c) - 96*a^2*tan(d*x)^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 20*a^2*d*x*ta
n(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 32*a^2*tan(d*x)^3*tan(1/2*d*x)*ta
n(1/2*c)^3*tan(c) + 32*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 5*
a^2*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 8*a^2*tan(d*x)^3*tan(1/2*c)^4*tan(c)
 + 5*a^2*d*x*tan(1/2*d*x)^4*tan(c)^2 + 8*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(
c)^2 + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 20*a^2*d*x*
tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 32*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1
/2*c)*tan(c)^2 + 96*a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2
0*a^2*d*x*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 32*a^2*tan(d*x)^2*tan(1/2*d*
x)*tan(1/2*c)^3*tan(c)^2 - 32*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 5*
a^2*d*x*tan(1/2*c)^4*tan(c)^2 + 8*a^2*tan(d*x)^...
```

Mupad [B]
time = 8.69, size = 213, normalized size = 3.00

$$-\frac{5\,a^2\,x}{2}-\frac{\frac{5\,a^2\,\left(c+d\,x\right)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(5\,c+5\,d\,x-6\right)}{2}\right)-\frac{a^2\,\left(5\,c+5\,d\,x-16\right)}{2}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{5\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(5\,c+5\,d\,x-10\right)}{2}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(5\,a^2\,\left(c+d\,x\right)-\frac{a^2\,\left(10\,c+10\,d\,x-10\right)}{2}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(5\,a^2\,\left(c+d\,x\right)-\frac{a^2\,\left(10\,c+10\,d\,x-22\right)}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
[Out] - (5*a^2*x)/2 - ((5*a^2*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((5*a^2*(c + d*x)
)/2 - (a^2*(5*c + 5*d*x - 6))/2) - (a^2*(5*c + 5*d*x - 16))/2 + tan(c/2 + (
d*x)/2)^4*((5*a^2*(c + d*x))/2 - (a^2*(5*c + 5*d*x - 10))/2) - tan(c/2 + (d
*x)/2)^3*(5*a^2*(c + d*x) - (a^2*(10*c + 10*d*x - 10))/2) + tan(c/2 + (d*x)
/2)^2*(5*a^2*(c + d*x) - (a^2*(10*c + 10*d*x - 22))/2))/(d*(tan(c/2 + (d*x)
/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.22 $\int (a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=45

$$\frac{3a^2x}{2}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 3/2*a^2*x-2*a^2*cos(d*x+c)/d-1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {2723}

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/2 - (2*a^2*Cos[c + d*x])/d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d
)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\,dx=\frac{3a^2x}{2}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A]
time = 0.13, size = 34, normalized size = 0.76

$$-\frac{a^2(-6(c+dx)+8\cos(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*(a^2*(-6*(c + d*x) + 8*Cos[c + d*x] + Sin[2*(c + d*x)]))/d
```

Maple [A]
time = 0.08, size = 52, normalized size = 1.16

method	result
risch	$\frac{3a^2x}{2}-\frac{2a^2\cos(dx+c)}{d}-\frac{a^2\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-2a^2\cos(dx+c)+a^2(dx+c)}{d}$
default	$\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-2a^2\cos(dx+c)+a^2(dx+c)}{d}$
norman	$\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a^2x}{2}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+3a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{3a^2x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{4a^2\left(\tan^2\left(\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-2*a^2*cos(d*x+c)+a^2*(d
*x+c))
```

Maxima [A]
time = 0.29, size = 47, normalized size = 1.04

$$a^2x+\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2}{4\,d}-\frac{2\,a^2\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] a^2*x + 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d - 2*a^2*cos(d*x + c)/d
```

Fricas [A]
time = 0.36, size = 41, normalized size = 0.91

$$\frac{3\,a^2dx-a^2\cos(dx+c)\sin(dx+c)-4\,a^2\cos(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*(3*a^2*d*x - a^2*cos(d*x + c)*sin(d*x + c) - 4*a^2*cos(d*x + c))/d
```

Sympy [A]
time = 0.08, size = 78, normalized size = 1.73

$$\begin{cases}\frac{a^2x\sin^2(c+dx)}{2}+\frac{a^2x\cos^2(c+dx)}{2}+a^2x-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\cos(c+dx)}{d} & \text{for } d\neq 0\\ x(a\sin(c)+a)^2 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*x - a
**2*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**2*cos(c + d*x)/d, Ne(d, 0)), (x*
(a*sin(c) + a)**2, True))
```

Giac [A]
time = 5.66, size = 38, normalized size = 0.84

$$\frac{3}{2}\,a^2x-\frac{2\,a^2\cos\left(dx+c\right)}{d}-\frac{a^2\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/2*a^2*x - 2*a^2*cos(d*x + c)/d - 1/4*a^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 6.58, size = 123, normalized size = 2.73

$$\frac{3\,a^2\,x}{2}-\frac{a^2\,\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)-a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-a^2\,\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}-4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2\,\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)-a^2\,\left(3\,c+3\,d\,x-4\right)\right)+a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/2 - (a^2*((3*c)/2 + (3*d*x)/2) - a^2*tan(c/2 + (d*x)/2)^3 - a^2*(
(3*c)/2 + (3*d*x)/2 - 4) + tan(c/2 + (d*x)/2)^2*(2*a^2*((3*c)/2 + (3*d*x)/2
) - a^2*(3*c + 3*d*x - 4)) + a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)
^2 + 1)^2)
```

3.23 $\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=74

$$-\frac{a^2x}{2}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] -1/2*a^2*x-2*a^2*arctanh(cos(d*x+c))/d+2*a^2*cos(d*x+c)/d-a^2*cot(d*x+c)/d+
1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2788, 3855, 3852, 8, 2718, 2715}

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*x) - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d - (
a^2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int (2a^4\csc(c+dx)+a^4\csc^2(c+dx)-2a^4\sin(c+dx)-a^4\sin^2(c+}{a^2}$$

$$= a^2\int \csc^2(c+dx)\,dx - a^2\int \sin^2(c+dx)\,dx + \left(2a^2\right)\int \csc(c+dx)$$

$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c}{2d}$$

$$= -\frac{a^2x}{2} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} - \frac{a^2\cot(c+dx}{d}$$

Mathematica [A]

time = 0.40, size = 94, normalized size = 1.27

$$-\frac{a^2\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(7\cos(c+dx)+\cos(3(c+dx))+4\left(c+dx-4\cos(c+dx)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/16*(a^2*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(7*Cos[c + d*x] + Cos[3*(c + d
*x)] + 4*(c + d*x - 4*Cos[c + d*x] + 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c
 + d*x)/2]])*Sin[c + d*x]))/d
```

Maple [A]

time = 0.13, size = 80, normalized size = 1.08

method	result
derivativedivides	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a^2(-\cot(dx+c)-dx-c)}{d}$
default	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a^2(-\cot(dx+c)-dx-c)}{d}$

risch	$-\frac{a^2x}{2}-\frac{ia^2e^{2i(dx+c)}}{8d}+\frac{a^2e^{i(dx+c)}}{d}+\frac{a^2e^{-i(dx+c)}}{d}+\frac{ia^2e^{-2i(dx+c)}}{8d}-\frac{2ia^2}{d(e^{2i(dx+c)}-1)}+\frac{2a^2\ln(e^{i(dx+c)}-1)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*(cos(d*x+c)+ln(csc
(d*x+c)-cot(d*x+c)))+a^2*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 79, normalized size = 1.07

$$\frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})a^2-4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2+4\,a^2(2\,\cos{(dx+c)}-\log{(\cos{(dx+c)}+1)}+\log{(\cos{(dx+c)}-1)})}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 4*(d*x + c + 1/tan(d*x + c))*a^
2 + 4*a^2*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))
/d
```

Fricas [A]
time = 0.36, size = 105, normalized size = 1.42

$$-\frac{a^2\cos{(dx+c)}^3+2\,a^2\log{\left(\frac{1}{2}\,\cos{(dx+c)}+\frac{1}{2}\right)}\sin{(dx+c)}-2\,a^2\log{\left(-\frac{1}{2}\,\cos{(dx+c)}+\frac{1}{2}\right)}\sin{(dx+c)}+a^2\cos{(dx+c)}+(a^2dx-4\,a^2\cos{(dx+c)})\sin{(dx+c)}}{2\,d\sin{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)^3 + 2*a^2*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a^2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + a^2*cos(d*x + c) + (a^2*
d*x - 4*a^2*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin{(c+dx)}\cot^2{(c+dx)}\,dx+\int\sin^2{(c+dx)}\cot^2{(c+dx)}\,dx+\int\cot^2{(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(sin(c + d*x)**
2*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(70) = 140.
time = 7.51, size = 143, normalized size = 1.93

$$-\frac{(dx+c)a^2 - 4\,a^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \frac{4\,a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + \frac{2\left(a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 4\,a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - a^2*tan(1/2*d*
x + 1/2*c) + (4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) + 2*(a
^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x
+ 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 6.52, size = 201, normalized size = 2.72

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{-3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) - a^2}{d\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)} + \frac{a^2\,\mathrm{atan}\left(\frac{a^4}{4\,a^4+a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} - \frac{4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^4+a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (8*a^2*tan(c/2 + (d*x)/2)^3 - 3*a^2*tan
(c/2 + (d*x)/2)^4 - a^2 + 8*a^2*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2
) + 4*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5)) + (a^2*atan(a^4/(4*a^
4 + a^4*tan(c/2 + (d*x)/2)) - (4*a^4*tan(c/2 + (d*x)/2))/(4*a^4 + a^4*tan(c
/2 + (d*x)/2))))/d + (a^2*tan(c/2 + (d*x)/2))/(2*d)
```

3.24 $\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=98

$$-\frac{a^2x}{2}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}-\frac{a^2\cos(c+$$

```
[Out] -1/2*a^2*x+3*a^2*arctanh(cos(d*x+c))/d-2*a^2*cos(d*x+c)/d-1/3*a^2*cot(d*x+c
)^3/d-a^2*cot(d*x+c)*csc(d*x+c)/d-1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2788, 3855, 3852, 8, 3853, 2718, 2715}

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}-\frac{a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*x) + (3*a^2*ArcTanh[Cos[c + d*x]])/d - (2*a^2*Cos[c + d*x])/d - (
a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (a^2*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
```

```
p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-a^6-4a^6\csc(c+dx)-a^6\csc^2(c+dx)+2a^6\csc^3(c+dx)+a^6\right.}{a^4}$$
$$= -a^2x-a^2\int \csc^2(c+dx)\,dx+a^2\int \csc^4(c+dx)\,dx+a^2\int \sin^2(c$$
$$= -a^2x+\frac{4a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx}{d}$$
$$= -\frac{a^2x}{2}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+d}{3d}$$

Mathematica [A]
time = 4.01, size = 191, normalized size = 1.95

$$\frac{a^2(1+\sin(c+dx))^2\left(-12(c+dx)-48\cos(c+dx)+4\cot\left(\frac{1}{2}(c+dx)\right)-6\csc^2\left(\frac{1}{2}(c+dx)\right)+72\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-72\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6\sec^2\left(\frac{1}{2}(c+dx)\right)+8\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-\frac{1}{2}\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-6\sin(2(c+dx))-4\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(-12*(c + d*x) - 48*Cos[c + d*x] + 4*Cot[(c + d*x
)/2] - 6*Csc[(c + d*x)/2]^2 + 72*Log[Cos[(c + d*x)/2]] - 72*Log[Sin[(c + d*
x)/2]] + 6*Sec[(c + d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (Csc[
```

```
(c + d*x)/2]^4*Sin[c + d*x])/2 - 6*Sin[2*(c + d*x)] - 4*Tan[(c + d*x)/2]))/
(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.20, size = 146, normalized size = 1.49

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\right.}{d}$
risch	$-\frac{a^2x}{2}+\frac{ia^2e^{2i(dx+c)}}{8d}-\frac{a^2e^{i(dx+c)}}{d}-\frac{a^2e^{-i(dx+c)}}{d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{2a^2\left(3ie^{4i(dx+c)}+3e^{5i(dx+c)}+i-3e^{i(dx}\right.}{3d\left(e^{2i(dx+c)}-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+
c)-3/2*d*x-3/2*c)+2*a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/
2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+a^2*(-1/3*cot(d*x+c)^3+cot(d*x+
c)+d*x+c))
```

Maxima [A]
time = 0.51, size = 139, normalized size = 1.42

$$-\frac{3\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^2+2}{\tan\left(dx+c\right)^3+\tan\left(dx+c\right)}\right)a^2-2\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^2-1}{\tan\left(dx+c\right)^3}\right)a^2-3\,a^2\left(\frac{2\,\cos\left(dx+c\right)}{\cos\left(dx+c\right)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a^2 - 2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*a
^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x +
c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(92) = 184.
time = 0.39, size = 192, normalized size = 1.96

$$\frac{3\,a^2\cos\left(dx+c\right)^5-4\,a^2\cos\left(dx+c\right)^3+3\,a^2\cos\left(dx+c\right)+9\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\left(a^2dx\cos\left(dx+c\right)^2+4\,a^2\cos\left(dx+c\right)^3-a^2dx-6\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^5 - 4*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c) + 9*(
a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a^2
*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a^2*d
*x*cos(d*x + c)^2 + 4*a^2*cos(d*x + c)^3 - a^2*d*x - 6*a^2*cos(d*x + c))*si
n(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin(c+dx)\cot^4(c+dx)\,dx+\int \sin^2(c+dx)\cot^4(c+dx)\,dx+\int \cot^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**4, x) + Integral(sin(c + d*x)**
2*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(92) = 184.
time = 6.02, size = 209, normalized size = 2.13

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,(dx+c)a^2-72\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{24\,\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}+\frac{132\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x +
 c)*a^2 - 72*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a^2*tan(1/2*d*x + 1/2*c
) + 24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan
(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a^2*tan(1/
2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*tan(1/2*d*x + 1/2*c
) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 6.52, size = 293, normalized size = 2.99

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{3\,a^2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\,\mathrm{atan}\left(\frac{a^4}{6\,a^4-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{6\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^4-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{-9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+34\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{19\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+36\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^2}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(4*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*a
^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*atan(a^4/(6*a^4 - a^4*tan(c/2 + (d*x)/
2)) + (6*a^4*tan(c/2 + (d*x)/2))/(6*a^4 - a^4*tan(c/2 + (d*x)/2))))/d - (36
*a^2*tan(c/2 + (d*x)/2)^3 - (a^2*tan(c/2 + (d*x)/2)^2)/3 + (19*a^2*tan(c/2
+ (d*x)/2)^4)/3 + 34*a^2*tan(c/2 + (d*x)/2)^5 - 9*a^2*tan(c/2 + (d*x)/2)^6
+ a^2/3 + 2*a^2*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 16*tan(c/2
 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/2)^7)) - (a^2*tan(c/2 + (d*x)/2))/(8*d)
```

3.25 $\int (a + a\sin(c + dx))^3 \tan^7(c + dx)\, dx$

Optimal. Leaf size=160

$$\frac{209a^3\log(1-\sin(c+dx))}{16d} - \frac{a^3\log(1+\sin(c+dx))}{16d} + \frac{7a^3\sin(c+dx)}{d} + \frac{3a^3\sin^2(c+dx)}{2d} + \frac{a^3\sin^3(c+dx)}{3d} +$$

```
[Out] 209/16*a^3*ln(1-sin(d*x+c))/d-1/16*a^3*ln(1+sin(d*x+c))/d+7*a^3*sin(d*x+c)/
d+3/2*a^3*sin(d*x+c)^2/d+1/3*a^3*sin(d*x+c)^3/d+1/6*a^6/d/(a-a*sin(d*x+c))^
3-13/8*a^5/d/(a-a*sin(d*x+c))^2+71/8*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$\frac{a^6}{6d(a-a\sin(c+dx))^3} - \frac{13a^5}{8d(a-a\sin(c+dx))^2} + \frac{71a^4}{8d(a-a\sin(c+dx))} + \frac{a^3\sin^3(c+dx)}{3d} + \frac{3a^3\sin^2(c+dx)}{2d} + \frac{7a^3\sin(c+dx)}{d} + \frac{209a^3\log(1-\sin(c+dx))}{16d} - \frac{a^3\log(\sin(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (209*a^3*Log[1 - Sin[c + d*x]])/(16*d) - (a^3*Log[1 + Sin[c + d*x]])/(16*d)
 + (7*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x
]^3)/(3*d) + a^6/(6*d*(a - a*Sin[c + d*x])^3) - (13*a^5)/(8*d*(a - a*Sin[c
+ d*x])^2) + (71*a^4)/(8*d*(a - a*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^7}{(a-x)^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(7a^2+\frac{a^6}{2(a-x)^4}-\frac{13a^5}{4(a-x)^3}+\frac{71a^4}{8(a-x)^2}-\frac{209a^3}{16(a-x)}+3ax+x^2-\right.\right.}{d}$$
$$= \frac{209a^3\log(1-\sin(c+dx))}{16d}-\frac{a^3\log(1+\sin(c+dx))}{16d}+\frac{7a^3\sin(c+}{d}$$

Mathematica [A]
time = 0.36, size = 99, normalized size = 0.62

$$\frac{a^3\left(627\log(1-\sin(c+dx))-3\log(1+\sin(c+dx))-\frac{8}{(-1+\sin(c+dx))^3}-\frac{78}{(-1+\sin(c+dx))^2}-\frac{426}{-1+\sin(c+dx)}+336\sin(c+dx)+72\sin^2(c+dx)+16\sin^3(c+dx)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (a^3*(627*Log[1 - Sin[c + d*x]] - 3*Log[1 + Sin[c + d*x]] - 8/(-1 + Sin[c +
 d*x])^3 - 78/(-1 + Sin[c + d*x])^2 - 426/(-1 + Sin[c + d*x]) + 336*Sin[c +
 d*x] + 72*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3))/(48*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(146) = 292$.
time = 0.23, size = 392, normalized size = 2.45

method	result
risch	$-13ia^3x+\frac{ia^3e^{3i(dx+c)}}{24d}-\frac{3a^3e^{2i(dx+c)}}{8d}-\frac{29ia^3e^{i(dx+c)}}{8d}+\frac{29ia^3e^{-i(dx+c)}}{8d}-\frac{3a^3e^{-2i(dx+c)}}{8d}-\frac{ia^3e^{-3i(dx+c}}{24d}$
derivativedivides	$\frac{a^3\left(\frac{\sin^{11}(dx+c)}{6\cos(dx+c)^6}-\frac{5\left(\sin^{11}(dx+c)\right)}{24\cos(dx+c)^4}+\frac{35\left(\sin^{11}(dx+c)\right)}{48\cos(dx+c)^2}+\frac{35\left(\sin^9(dx+c)\right)}{48}+\frac{15\left(\sin^7(dx+c)\right)}{16}+\frac{21\left(\sin^5(dx+c)\right)}{16}+\frac{35\left(\sin^3(dx+c)\right)}{16}+\right.}{}$
default	$\frac{a^3\left(\frac{\sin^{11}(dx+c)}{6\cos(dx+c)^6}-\frac{5\left(\sin^{11}(dx+c)\right)}{24\cos(dx+c)^4}+\frac{35\left(\sin^{11}(dx+c)\right)}{48\cos(dx+c)^2}+\frac{35\left(\sin^9(dx+c)\right)}{48}+\frac{15\left(\sin^7(dx+c)\right)}{16}+\frac{21\left(\sin^5(dx+c)\right)}{16}+\frac{35\left(\sin^3(dx+c)\right)}{16}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/6*sin(d*x+c)^11/cos(d*x+c)^6-5/24*sin(d*x+c)^11/cos(d*x+c)^4+35
/48*sin(d*x+c)^11/cos(d*x+c)^2+35/48*sin(d*x+c)^9+15/16*sin(d*x+c)^7+21/16*
sin(d*x+c)^5+35/16*sin(d*x+c)^3+105/16*sin(d*x+c)-105/16*ln(sec(d*x+c)+tan(
d*x+c)))+3*a^3*(1/6*sin(d*x+c)^10/cos(d*x+c)^6-1/6*sin(d*x+c)^10/cos(d*x+c)
^4+1/2*sin(d*x+c)^10/cos(d*x+c)^2+1/2*sin(d*x+c)^8+2/3*sin(d*x+c)^6+sin(d*x
```

```
+c)^4+2*sin(d*x+c)^2+4*ln(cos(d*x+c)))+3*a^3*(1/6*sin(d*x+c)^9/cos(d*x+c)^6
-1/8*sin(d*x+c)^9/cos(d*x+c)^4+5/16*sin(d*x+c)^9/cos(d*x+c)^2+5/16*sin(d*x+
c)^7+7/16*sin(d*x+c)^5+35/48*sin(d*x+c)^3+35/16*sin(d*x+c)-35/16*ln(sec(d*x
+c)+tan(d*x+c)))+a^3*(1/6*tan(d*x+c)^6-1/4*tan(d*x+c)^4+1/2*tan(d*x+c)^2+ln
(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 133, normalized size = 0.83

$$\frac{16\,a^3\sin\left(dx+c\right)^3+72\,a^3\sin\left(dx+c\right)^2-3\,a^3\log\left(\sin\left(dx+c\right)+1\right)+627\,a^3\log\left(\sin\left(dx+c\right)-1\right)+336\,a^3\sin\left(dx+c\right)-\frac{2\left(213\,a^3\sin(dx+c)^2-387\,a^3\sin(dx+c)+178\,a^3\right)}{\sin(dx+c)^3-3\sin(dx+c)^2+3\sin(dx+c)-1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/48*(16*a^3*sin(d*x + c)^3 + 72*a^3*sin(d*x + c)^2 - 3*a^3*log(sin(d*x + c
) + 1) + 627*a^3*log(sin(d*x + c) - 1) + 336*a^3*sin(d*x + c) - 2*(213*a^3*
sin(d*x + c)^2 - 387*a^3*sin(d*x + c) + 178*a^3)/(sin(d*x + c)^3 - 3*sin(d*
x + c)^2 + 3*sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.40, size = 240, normalized size = 1.50

$$-\frac{16\,a^3\cos\left(dx+c\right)^6-216\,a^3\cos\left(dx+c\right)^4+1002\,a^3\cos\left(dx+c\right)^2-482\,a^3+3\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3-\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-627\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3-\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(12\,a^3\cos\left(dx+c\right)^4+398\,a^3\cos\left(dx+c\right)^2-245\,a^3\right)\sin\left(dx+c\right)}{48\left(3\,d\cos\left(dx+c\right)^2-\left(d\cos\left(dx+c\right)^2-4\,d\right)\sin\left(dx+c\right)-4\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] -1/48*(16*a^3*cos(d*x + c)^6 - 216*a^3*cos(d*x + c)^4 + 1002*a^3*cos(d*x +
c)^2 - 482*a^3 + 3*(3*a^3*cos(d*x + c)^2 - 4*a^3 - (a^3*cos(d*x + c)^2 - 4*
a^3)*sin(d*x + c))*log(sin(d*x + c) + 1) - 627*(3*a^3*cos(d*x + c)^2 - 4*a^
3 - (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 2*(
12*a^3*cos(d*x + c)^4 + 398*a^3*cos(d*x + c)^2 - 245*a^3)*sin(d*x + c))/(3*
d*cos(d*x + c)^2 - (d*cos(d*x + c)^2 - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\tan^7\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\tan^7\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\tan^7\left(c+dx\right)dx+\int \tan^7\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**7,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**7, x) + Integral(3*sin(c + d*x)
**2*tan(c + d*x)**7, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**7, x) + In
tegral(tan(c + d*x)**7, x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 6.47, size = 398, normalized size = 2.49

$$\frac{\frac{105\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{4}-\frac{263\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+\frac{1301\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4}-582\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{1657\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2}-\frac{2767\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}+\frac{1657\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{2}-582\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{1301\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{4}-\frac{263\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{2}+\frac{105\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+18\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-38\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+63\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-84\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+92\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-84\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+63\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-38\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+18\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}+\frac{209\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{8\,d}-\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{8\,d}-\frac{13\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((1301*a^3*tan(c/2 + (d*x)/2)^3)/4 - (263*a^3*tan(c/2 + (d*x)/2)^2)/2 - 582
*a^3*tan(c/2 + (d*x)/2)^4 + (1657*a^3*tan(c/2 + (d*x)/2)^5)/2 - (2767*a^3*t
an(c/2 + (d*x)/2)^6)/3 + (1657*a^3*tan(c/2 + (d*x)/2)^7)/2 - 582*a^3*tan(c/
2 + (d*x)/2)^8 + (1301*a^3*tan(c/2 + (d*x)/2)^9)/4 - (263*a^3*tan(c/2 + (d*
x)/2)^10)/2 + (105*a^3*tan(c/2 + (d*x)/2)^11)/4 + (105*a^3*tan(c/2 + (d*x)/
2))/4)/(d*(18*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2) - 38*tan(c/2 + (d
*x)/2)^3 + 63*tan(c/2 + (d*x)/2)^4 - 84*tan(c/2 + (d*x)/2)^5 + 92*tan(c/2 +
 (d*x)/2)^6 - 84*tan(c/2 + (d*x)/2)^7 + 63*tan(c/2 + (d*x)/2)^8 - 38*tan(c/
2 + (d*x)/2)^9 + 18*tan(c/2 + (d*x)/2)^10 - 6*tan(c/2 + (d*x)/2)^11 + tan(c
/2 + (d*x)/2)^12 + 1)) + (209*a^3*log(tan(c/2 + (d*x)/2) - 1))/(8*d) - (a^3
*log(tan(c/2 + (d*x)/2) + 1))/(8*d) - (13*a^3*log(tan(c/2 + (d*x)/2)^2 + 1)
)/d
```

3.26 $\int (a+a\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{7a^3\log(1-\sin(c+dx))}{d}+\frac{5a^3\sin(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin^3(c+dx)}{3d}+\frac{2a^4}{d(a-a\sin(c+dx))}$$

```
[Out] 7*a^3*ln(1-sin(d*x+c))/d+5*a^3*sin(d*x+c)/d+3/2*a^3*sin(d*x+c)^2/d+1/3*a^3*
sin(d*x+c)^3/d+2*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 78}

$$\frac{2a^4}{d(a-a\sin(c+dx))}+\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{5a^3\sin(c+dx)}{d}+\frac{7a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (7*a^3*Log[1 - Sin[c + d*x]])/d + (5*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d
*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d) + (2*a^4)/(d*(a - a*Sin[c + d*x])
)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(5a^2+\frac{2a^4}{(a-x)^2}-\frac{7a^3}{a-x}+3ax+x^2\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{7a^3\log(1-\sin(c+dx))}{d}+\frac{5a^3\sin(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3}{}$$

Mathematica [A]
time = 0.11, size = 66, normalized size = 0.73

$$\frac{a^3\left(42\log(1-\sin(c+dx))+\frac{12}{1-\sin(c+dx)}+30\sin(c+dx)+9\sin^2(c+dx)+2\sin^3(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (a^3*(42*Log[1 - Sin[c + d*x]] + 12/(1 - Sin[c + d*x]) + 30*Sin[c + d*x] +
9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/(6*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(87) = 174$.
time = 0.17, size = 204, normalized size = 2.24

method	result
risch	$-7ia^3x-\frac{21ia^3e^{i(dx+c)}}{8d}+\frac{21ia^3e^{-i(dx+c)}}{8d}-\frac{14ia^3c}{d}-\frac{4ia^3e^{i(dx+c)}}{\left(e^{i(dx+c)}-i\right)^2d}+\frac{14a^3\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{a^3\sin(3dx+}{12d}$
derivativedivides	$\frac{a^3\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\frac{5\left(\sin^3(dx+c)\right)}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c\right.}{2}\right.}{}$
default	$\frac{a^3\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\frac{5\left(\sin^3(dx+c)\right)}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c\right.}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sin(d*x+c)^7/cos(d*x+c)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5
/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(1/2*sin(d*x+c)^6/cos(d*
x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c)))+3*a^3*(1/2*sin(d*x+c
)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+
c)))+a^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 72, normalized size = 0.79

$$\frac{2\,a^{3}\sin\left(dx+c\right)^{3}+9\,a^{3}\sin\left(dx+c\right)^{2}+42\,a^{3}\log\left(\sin\left(dx+c\right)-1\right)+30\,a^{3}\sin\left(dx+c\right)-\frac{12\,a^{3}}{\sin(dx+c)-1}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 42*a^3*log(sin(d*x + c)
- 1) + 30*a^3*sin(d*x + c) - 12*a^3/(sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.37, size = 104, normalized size = 1.14

$$\frac{4\,a^{3}\cos\left(dx+c\right)^{4}-50\,a^{3}\cos\left(dx+c\right)^{2}+31\,a^{3}+84\,\left(a^{3}\sin\left(dx+c\right)-a^{3}\right)\log\left(-\sin\left(dx+c\right)+1\right)-\left(14\,a^{3}\cos\left(dx+c\right)^{2}+55\,a^{3}\right)\sin\left(dx+c\right)}{12\,\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^4 - 50*a^3*cos(d*x + c)^2 + 31*a^3 + 84*(a^3*sin(d
*x + c) - a^3)*log(-sin(d*x + c) + 1) - (14*a^3*cos(d*x + c)^2 + 55*a^3)*si
n(d*x + c))/(d*sin(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\sin\left(c+dx\right)\tan^{3}\left(c+dx\right)\,dx+\int 3\sin^{2}\left(c+dx\right)\tan^{3}\left(c+dx\right)\,dx+\int \sin^{3}\left(c+dx\right)\tan^{3}\left(c+dx\right)\,dx+\int \tan^{3}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**3, x) + Integral(3*sin(c + d*x)
**2*tan(c + d*x)**3, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**3, x) + In
tegral(tan(c + d*x)**3, x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 7.46, size = 262, normalized size = 2.88

$$\frac{14\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}+\frac{14\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-14\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{98\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{100\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{98\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-14\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+14\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}-\frac{7\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (14*a^3*log(tan(c/2 + (d*x)/2) - 1))/d + ((98*a^3*tan(c/2 + (d*x)/2)^3)/3 -
 14*a^3*tan(c/2 + (d*x)/2)^2 - (100*a^3*tan(c/2 + (d*x)/2)^4)/3 + (98*a^3*t
an(c/2 + (d*x)/2)^5)/3 - 14*a^3*tan(c/2 + (d*x)/2)^6 + 14*a^3*tan(c/2 + (d*
x)/2)^7 + 14*a^3*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2
 + (d*x)/2) - 6*tan(c/2 + (d*x)/2)^3 + 6*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 +
 (d*x)/2)^5 + 4*tan(c/2 + (d*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (
d*x)/2)^8 + 1)) - (7*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.27 $\int (a + a\sin(c + dx))^3 \tan(c + dx)\, dx$

Optimal. Leaf size=70

$$-\frac{4a^3\log(1-\sin(c+dx))}{d}-\frac{4a^3\sin(c+dx)}{d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}$$

```
[Out] -4*a^3*ln(1-sin(d*x+c))/d-4*a^3*sin(d*x+c)/d-3/2*a^3*sin(d*x+c)^2/d-1/3*a^3
*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 78}

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{4a^3\sin(c+dx)}{d}-\frac{4a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (-4*a^3*Log[1 - Sin[c + d*x]])/d - (4*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c +
d*x]^2)/(2*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sin(c+dx))^3\tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+x)^2}{a-x}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-4a^2+\frac{4a^3}{a-x}-3ax-x^2\right)\,dx, x, a\sin(c+dx)\right)}{d} \\
&= -\frac{4a^3\log(1-\sin(c+dx))}{d}-\frac{4a^3\sin(c+dx)}{d}-\frac{3a^3\sin^2(c+dx)}{2d}-a
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.74

$$
-\frac{a^3(24\log(1-\sin(c+dx))+24\sin(c+dx)+9\sin^2(c+dx)+2\sin^3(c+dx))}{6d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -1/6*(a^3*(24*Log[1 - Sin[c + d*x]] + 24*Sin[c + d*x] + 9*Sin[c + d*x]^2 +
2*Sin[c + d*x]^3))/d
```

Maple [A]
time = 0.16, size = 108, normalized size = 1.54

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a^3(-\sin(dx+c)+}{d}$
default	$\frac{a^3\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a^3(-\sin(dx+c)+}{d}$
risch	$4ia^3x+\frac{17ia^3e^{i(dx+c)}}{8d}-\frac{17ia^3e^{-i(dx+c)}}{8d}+\frac{8ia^3c}{d}-\frac{8a^3\ln(e^{i(dx+c)}-i)}{d}+\frac{a^3\sin(3dx+3c)}{12d}+\frac{3a^3\cos(2dx+2c)}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(-1
/2*sin(d*x+c)^2-ln(cos(d*x+c)))+3*a^3*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)
))-a^3*ln(cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.81

$$
-\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2+24\,a^3\log\left(\sin\left(dx+c\right)-1\right)+24\,a^3\sin\left(dx+c\right)}{6\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 24*a^3*log(sin(d*x + c)
 - 1) + 24*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 61, normalized size = 0.87

$$\frac{9\,a^3\cos\left(dx+c\right)^2-24\,a^3\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(a^3\cos\left(dx+c\right)^2-13\,a^3\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(9*a^3*cos(d*x + c)^2 - 24*a^3*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x
+ c)^2 - 13*a^3)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\tan\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\tan\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\tan\left(c+dx\right)dx+\int \tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c),x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x), x) + Integral(3*sin(c + d*x)**2
*tan(c + d*x), x) + Integral(sin(c + d*x)**3*tan(c + d*x), x) + Integral(ta
n(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 28789 vs. 2(66) = 132.
time = 32.14, size = 28789, normalized size = 411.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/12*(24*a^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2
*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*
tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 24*a^3*log(2*(tan(1/2*d*x
)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c
)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2
```

*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 24*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 9*a^3*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 24*a^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 - 24*a^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 + 24*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 + 72*a^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 72*a^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 72*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 96*a^3*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^5*tan(c)^2 + 72*a^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 - 72*a^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 72*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 - 96*a^3*tan(d*x)^2*tan(1/2*d*x)^5*tan(1/2*c)^6*tan(c)^2 + 24*a^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(

```
1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 24*a^3*log(2*(tan(1/2
*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6*
tan(c)^2 + 24*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*
x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2
*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 9*a^3*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^
6 + 36*a^3*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c) - 27*a^3*tan(d*x)^2*
tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 27*a^3*tan(d*x)^2*tan(1/2*d*x)^4*tan
(1/2*c)^6*tan(c)^2 + 9*a^3*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 72*a^3*lo
g(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*
d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2...
```

Mupad [B]
time = 7.26, size = 281, normalized size = 4.01

$$-\frac{\frac{56\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^3\left(12\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)-\frac{2\,a^3\left(36\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-18\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)+9\right)}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(2\,a^3\left(12\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)-\frac{2\,a^3\left(36\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-18\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)+9\right)}{3}\right)+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}^3}-\frac{2\,a^3\left(12\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] - ((56*a^3*tan(c/2 + (d*x)/2)^3)/3 + 8*a^3*tan(c/2 + (d*x)/2)^5 - tan(c/2 +
 (d*x)/2)^2*(2*a^3*(12*log(tan(c/2 + (d*x)/2) - 1) - 6*log(tan(c/2 + (d*x)/
2)^2 + 1)) - (2*a^3*(36*log(tan(c/2 + (d*x)/2) - 1) - 18*log(tan(c/2 + (d*x
)/2)^2 + 1) + 9))/3) - tan(c/2 + (d*x)/2)^4*(2*a^3*(12*log(tan(c/2 + (d*x)/
2) - 1) - 6*log(tan(c/2 + (d*x)/2)^2 + 1)) - (2*a^3*(36*log(tan(c/2 + (d*x)
/2) - 1) - 18*log(tan(c/2 + (d*x)/2)^2 + 1) + 9))/3) + 8*a^3*tan(c/2 + (d*x
)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (2*a^3*(12*log(tan(c/2 + (d*x)/2)
- 1) - 6*log(tan(c/2 + (d*x)/2)^2 + 1)))/(3*d)
```

3.28 $\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=98

$$-\frac{3a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{2a^3\log(\sin(c+dx))}{d}-\frac{2a^3\sin(c+dx)}{d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}$$

```
[Out] -3*a^3*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d+2*a^3*ln(sin(d*x+c))/d-2*a^3*sin
(d*x+c)/d-3/2*a^3*sin(d*x+c)^2/d-1/3*a^3*sin(d*x+c)^3/d
```

Rubi [A]

time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 76}

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{2a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{2a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (2*a^3*Log[Sin[c + d
*x]])/d - (2*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c + d*x]^2)/(2*d) - (a^3*Sin[
c + d*x]^3)/(3*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^4}{x^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-2a^2+\frac{a^5}{x^3}+\frac{3a^4}{x^2}+\frac{2a^3}{x}-3ax-x^2\right)dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{3a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{2a^3\log(\sin(c+dx))}{d}-\frac{2a^3\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 67, normalized size = 0.68

$$-\frac{a^3(30+18\csc(c+dx)+3\csc^2(c+dx)-12\log(\sin(c+dx))+12\sin(c+dx)+9\sin^2(c+dx)+2\sin^3(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*(a^3*(30 + 18*Csc[c + d*x] + 3*Csc[c + d*x]^2 - 12*Log[Sin[c + d*x]] +
 12*Sin[c + d*x] + 9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/d
```

Maple [A]
time = 0.20, size = 116, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{a^3(2+\cos^2(dx+c))\sin(dx+c)}{3}+3a^3\left(\frac{(\cos^2(dx+c))}{2}+\ln(\sin(dx+c))\right)+3a^3\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+a}{d}$
default	$\frac{\frac{a^3(2+\cos^2(dx+c))\sin(dx+c)}{3}+3a^3\left(\frac{(\cos^2(dx+c))}{2}+\ln(\sin(dx+c))\right)+3a^3\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+a}{d}$
risch	$-2ia^3x-\frac{ia^3e^{3i(dx+c)}}{24d}+\frac{3a^3e^{2i(dx+c)}}{8d}+\frac{9ia^3e^{i(dx+c)}}{8d}-\frac{9ia^3e^{-i(dx+c)}}{8d}+\frac{3a^3e^{-2i(dx+c)}}{8d}+\frac{ia^3e^{-3i(dx+c)}}{24d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)^2+ln(sin(d*x
+c)))+3*a^3*(-1/sin(d*x+c)*cos(d*x+c)^4-(2+cos(d*x+c)^2)*sin(d*x+c))+a^3*(-
1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.29, size = 80, normalized size = 0.82

$$-\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2-12\,a^3\log\left(\sin\left(dx+c\right)\right)+12\,a^3\sin\left(dx+c\right)+\frac{3\,(6\,a^3\sin(dx+c)+a^3)}{\sin(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 - 12*a^3*log(sin(d*x + c)
) + 12*a^3*sin(d*x + c) + 3*(6*a^3*sin(d*x + c) + a^3)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.38, size = 118, normalized size = 1.20

$$\frac{18\,a^3\cos\left(dx+c\right)^4-27\,a^3\cos\left(dx+c\right)^2+15\,a^3+24\,\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\,\left(a^3\cos\left(dx+c\right)^4-8\,a^3\cos\left(dx+c\right)^2+16\,a^3\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a^3*cos(d*x + c)^4 - 27*a^3*cos(d*x + c)^2 + 15*a^3 + 24*(a^3*cos(
d*x + c)^2 - a^3)*log(1/2*sin(d*x + c)) + 4*(a^3*cos(d*x + c)^4 - 8*a^3*cos
(d*x + c)^2 + 16*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\sin\left(c+dx\right)\cot^{3}\left(c+dx\right)dx+\int 3\sin^{2}\left(c+dx\right)\cot^{3}\left(c+dx\right)dx+\int\sin^{3}\left(c+dx\right)\cot^{3}\left(c+dx\right)dx+\int\cot^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(3*sin(c + d*x)
**2*cot(c + d*x)**3, x) + Integral(sin(c + d*x)**3*cot(c + d*x)**3, x) + In
tegral(cot(c + d*x)**3, x))
```

Giac [A]
time = 8.26, size = 94, normalized size = 0.96

$$-\frac{2\,a^{3}\sin\left(dx+c\right)^{3}+9\,a^{3}\sin\left(dx+c\right)^{2}-12\,a^{3}\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,a^{3}\sin\left(dx+c\right)+\frac{3\left(6\,a^{3}\sin(dx+c)^{2}+6\,a^{3}\sin(dx+c)+a^{3}\right)}{\sin(dx+c)^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 - 12*a^3*log(abs(sin(d*x
+ c))) + 12*a^3*sin(d*x + c) + 3*(6*a^3*sin(d*x + c)^2 + 6*a^3*sin(d*x + c)
 + a^3)/sin(d*x + c)^2)/d
```

Mupad [B]
time = 6.76, size = 253, normalized size = 2.58

$$\frac{2\,a^{3}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8\,d}-\frac{22\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+\frac{49\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{2}+\frac{182\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{3}+\frac{51\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{2}+34\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\frac{3\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{2}+6\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^{3}}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\right)}-\frac{3\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{2\,a^{3}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (2*a^3*log(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - ((3*
a^3*tan(c/2 + (d*x)/2)^2)/2 + 34*a^3*tan(c/2 + (d*x)/2)^3 + (51*a^3*tan(c/2
 + (d*x)/2)^4)/2 + (182*a^3*tan(c/2 + (d*x)/2)^5)/3 + (49*a^3*tan(c/2 + (d*
x)/2)^6)/2 + 22*a^3*tan(c/2 + (d*x)/2)^7 + a^3/2 + 6*a^3*tan(c/2 + (d*x)/2)
)/(d*(4*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)
/2)^6 + 4*tan(c/2 + (d*x)/2)^8)) - (3*a^3*tan(c/2 + (d*x)/2))/(2*d) - (2*a^
3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.29 $\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx$

Optimal. Leaf size=180

$$-\frac{23a^3x}{2}+\frac{136a^3\cos(c+dx)}{5d}-\frac{136a^3\cos^3(c+dx)}{15d}+\frac{23a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^6\cos(c+dx)\sin^5(c+dx}{5d(a-a\sin(c+dx))^3}$$

```
[Out] -23/2*a^3*x+136/5*a^3*cos(d*x+c)/d-136/15*a^3*cos(d*x+c)^3/d+23/2*a^3*cos(d
*x+c)*sin(d*x+c)/d+1/5*a^6*cos(d*x+c)*sin(d*x+c)^5/d/(a-a*sin(d*x+c))^3-13/
15*a^5*cos(d*x+c)*sin(d*x+c)^4/d/(a-a*sin(d*x+c))^2+23/3*a^6*cos(d*x+c)*sin
(d*x+c)^3/d/(a^3-a^3*sin(d*x+c))
```

Rubi [A]
time = 0.25, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {2787, 2844, 3056, 2827, 2715, 8, 2713}

$$\frac{a^6\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{13a^5\sin^4(c+dx)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}-\frac{136a^3\cos^3(c+dx)}{15d}+\frac{136a^3\cos(c+dx)}{5d}+\frac{23a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{23a^3x}{2}+\frac{23a^6\sin^3(c+dx)\cos(c+dx)}{3d\left(a^3-a^3\sin(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] (-23*a^3*x)/2 + (136*a^3*Cos[c + d*x])/(5*d) - (136*a^3*Cos[c + d*x]^3)/(15
*d) + (23*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^6*Cos[c + d*x]*Sin[c +
d*x]^5)/(5*d*(a - a*Sin[c + d*x])^3) - (13*a^5*Cos[c + d*x]*Sin[c + d*x]^4)
/(15*d*(a - a*Sin[c + d*x])^2) + (23*a^6*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*
(a^3 - a^3*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2787

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx = a^6\int \frac{\sin^6(c+dx)}{(a-a\sin(c+dx))^3}\,dx$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3} + \frac{1}{5}a^4\int \frac{\sin^4(c+dx)(-5a-8a\sin(c}{(a-a\sin(c+dx))^2}$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3} - \frac{13a^5\cos(c+dx)\sin^4(c+dx)}{15d(a-a\sin(c+dx))^2} - \frac{1}{15}$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3} - \frac{13a^5\cos(c+dx)\sin^4(c+dx)}{15d(a-a\sin(c+dx))^2} + 23a$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3} - \frac{13a^5\cos(c+dx)\sin^4(c+dx)}{15d(a-a\sin(c+dx))^2} + 23a$$
$$= \frac{23a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3} - 13a^5$$
$$= -\frac{23a^3x}{2} + \frac{136a^3\cos(c+dx)}{5d} - \frac{136a^3\cos^3(c+dx)}{15d} + 23a^3\cos(c+$$

Mathematica [A]
time = 3.24, size = 243, normalized size = 1.35

$$\frac{(a+a\sin(c+dx))^3\left(-690(c+dx)+405\cos(c+dx)-5\cos(3(c+dx))+\frac{12}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}-\frac{112}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{24\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}-\frac{224\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{1576\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+45\sin(2(c+dx))\right)}{60d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-690*(c + d*x) + 405*Cos[c + d*x] - 5*Cos[3*(c + d
*x)] + 12/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4 - 112/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2 + (24*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^5 - (224*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3
+ (1576*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 45*Sin[2*
(c + d*x)]))/(60*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(166) = 332$.
time = 0.27, size = 359, normalized size = 1.99

method	result
risch	$-\frac{23a^3x}{2}-\frac{a^3e^{3i(dx+c)}}{24d}-\frac{3ia^3e^{2i(dx+c)}}{8d}+\frac{27a^3e^{i(dx+c)}}{8d}+\frac{27a^3e^{-i(dx+c)}}{8d}+\frac{3ia^3e^{-2i(dx+c)}}{8d}-\frac{a^3e^{-3i(dx+c)}}{24d}$

derivativedivides	$$\frac{a^3\left(\frac{\sin^{10}(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^{10}(dx+c)}{3\cos(dx+c)^3}+\frac{7\left(\sin^{10}(dx+c)\right)}{3\cos(dx+c)}+\frac{7\left(\frac{128}{35}+\sin^8(dx+c)+\frac{8\left(\sin^6(dx+c)\right)}{7}+\frac{48\left(\sin^4(dx+c)\right)}{35}+\frac{64\left(\sin^2(dx+c)\right)}{35}\right)\cos}{3}\right.}{}$$
default	$$\frac{a^3\left(\frac{\sin^{10}(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^{10}(dx+c)}{3\cos(dx+c)^3}+\frac{7\left(\sin^{10}(dx+c)\right)}{3\cos(dx+c)}+\frac{7\left(\frac{128}{35}+\sin^8(dx+c)+\frac{8\left(\sin^6(dx+c)\right)}{7}+\frac{48\left(\sin^4(dx+c)\right)}{35}+\frac{64\left(\sin^2(dx+c)\right)}{35}\right)\cos}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/5*sin(d*x+c)^10/cos(d*x+c)^5-1/3*sin(d*x+c)^10/cos(d*x+c)^3+7/3
*sin(d*x+c)^10/cos(d*x+c)+7/3*(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*s
in(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/5*sin(d*x+c)^9/cos(d*x
+c)^5-4/15*sin(d*x+c)^9/cos(d*x+c)^3+8/5*sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d
*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-7/
2*d*x-7/2*c)+3*a^3*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+
c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*
x+c)^2)*cos(d*x+c))+a^3*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c
))
```

Maxima [A]
time = 0.50, size = 209, normalized size = 1.16

$$\frac{3\left(6\tan(dx+c)^5-20\tan(dx+c)^3-105\,dx-105\,c+\frac{15\tan(dx+c)}{\tan(dx+c)^2+1}+90\tan(dx+c)\right)a^3+2\left(3\tan(dx+c)^5-5\tan(dx+c)^3-15\,dx-15\,c+15\tan(dx+c)\right)a^3-2\left(5\cos(dx+c)^3-\frac{90\cos(dx+c)^4-20\cos(dx+c)^2+3}{\cos(dx+c)^5}-60\cos(dx+c)\right)a^3+18\,a^3\left(\frac{15\cos(dx+c)^4-5\cos(dx+c)^2+1}{\cos(dx+c)^5}+5\cos(dx+c)\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/30*(3*(6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan(d*
x + c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*a^3 + 2*(3*tan(d*x + c)^5 -
5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^3 - 2*(5*cos(d*x + c)
^3 - (90*cos(d*x + c)^4 - 20*cos(d*x + c)^2 + 3)/cos(d*x + c)^5 - 60*cos(d*
x + c))*a^3 + 18*a^3*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x +
c)^5 + 5*cos(d*x + c)))/d
```

Fricas [A]
time = 0.36, size = 289, normalized size = 1.61

$$-\frac{10\,a^3\cos(dx+c)^6-15\,a^3\cos(dx+c)^5-140\,a^3\cos(dx+c)^4-1380\,a^3dx+\left(345\,a^3dx-839\,a^3\right)\cos(dx+c)^3+6\,a^3+\left(1035\,a^3dx+668\,a^3\right)\cos(dx+c)^2-6\left(115\,a^3dx-233\,a^3\right)\cos(dx+c)-\left(10\,a^3\cos(dx+c)^5+25\,a^3\cos(dx+c)^4-115\,a^3\cos(dx+c)^3-1380\,a^3dx-6\,a^3+\left(345\,a^3dx+724\,a^3\right)\cos(dx+c)^2-6\left(115\,a^3dx-232\,a^3\right)\cos(dx+c)\right)\sin(dx+c)}{30\left(d\cos(dx+c)^3+3\,d\cos(dx+c)^2-2\,d\cos(dx+c)-\left(d\cos(dx+c)^2-2\,d\cos(dx+c)-4\,d\right)\sin(dx+c)-4\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="fricas")
```

[Out] -1/30*(10*a^3*cos(d*x + c)^6 - 15*a^3*cos(d*x + c)^5 - 140*a^3*cos(d*x + c)^4 - 1380*a^3*d*x + (345*a^3*d*x - 839*a^3)*cos(d*x + c)^3 + 6*a^3 + (1035*a^3*d*x + 668*a^3)*cos(d*x + c)^2 - 6*(115*a^3*d*x - 233*a^3)*cos(d*x + c) - (10*a^3*cos(d*x + c)^5 + 25*a^3*cos(d*x + c)^4 - 115*a^3*cos(d*x + c)^3 - 1380*a^3*d*x - 6*a^3 + (345*a^3*d*x + 724*a^3)*cos(d*x + c)^2 - 6*(115*a^3*d*x - 232*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - (d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - 4*d)*sin(d*x + c) - 4*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\sin\left(c+dx\right)\tan^{6}\left(c+dx\right)\,dx+\int 3\sin^{2}\left(c+dx\right)\tan^{6}\left(c+dx\right)\,dx+\int \sin^{3}\left(c+dx\right)\tan^{6}\left(c+dx\right)\,dx+\int \tan^{6}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**6,x)

[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**6, x) + Integral(3*sin(c + d*x)**2*tan(c + d*x)**6, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**6, x) + Integral(tan(c + d*x)**6, x))

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="giac")

[Out] Timed out

Mupad [B]
time = 11.05, size = 438, normalized size = 2.43

$$-\frac{23\,a^3\,x}{2}-\frac{\frac{23\,a^3\,\left(c+d\,x\right)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{115\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(1725\,c+1725\,d\,x-4750\right)}{30}\right)-\frac{a^3\,\left(345\,c+345\,d\,x-1088\right)}{30}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(\frac{115\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(1725\,c+1725\,d\,x-690\right)}{30}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(\frac{299\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(4485\,c+4485\,d\,x-3450\right)}{30}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{299\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(4485\,c+4485\,d\,x-10694\right)}{30}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{575\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(8625\,c+8625\,d\,x-8740\right)}{30}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{575\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(8625\,c+8625\,d\,x-18460\right)}{30}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(437\,a^3\,\left(c+d\,x\right)-\frac{a^3\,\left(13110\,c+13110\,d\,x-16100\right)}{30}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(437\,a^3\,\left(c+d\,x\right)-\frac{a^3\,\left(13110\,c+13110\,d\,x-25244\right)}{30}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(529\,a^3\,\left(c+d\,x\right)-\frac{a^3\,\left(15870\,c+15870\,d\,x-23368\right)}{30}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(529\,a^3\,\left(c+d\,x\right)-\frac{a^3\,\left(15870\,c+15870\,d\,x-26680\right)}{30}\right)}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^5\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^6*(a + a*sin(c + d*x))^3,x)

[Out] - (23*a^3*x)/2 - ((23*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((115*a^3*(c + d*x))/2 - (a^3*(1725*c + 1725*d*x - 4750))/30) - (a^3*(345*c + 345*d*x - 1088))/30 + tan(c/2 + (d*x)/2)^10*((115*a^3*(c + d*x))/2 - (a^3*(1725*c + 1725*d*x - 690))/30) - tan(c/2 + (d*x)/2)^9*((299*a^3*(c + d*x))/2 - (a^3*(4485*c + 4485*d*x - 3450))/30) + tan(c/2 + (d*x)/2)^2*((299*a^3*(c + d*x))/2 - (a^3*(4485*c + 4485*d*x - 10694))/30) + tan(c/2 + (d*x)/2)^8*((575*a^3*(c + d*x))/2 - (a^3*(8625*c + 8625*d*x - 8740))/30) - tan(c/2 + (d*x)/2)^3*((5

```
75*a^3*(c + d*x))/2 - (a^3*(8625*c + 8625*d*x - 18460))/30) - tan(c/2 + (d*
x)/2)^7*(437*a^3*(c + d*x) - (a^3*(13110*c + 13110*d*x - 16100))/30) + tan(
c/2 + (d*x)/2)^4*(437*a^3*(c + d*x) - (a^3*(13110*c + 13110*d*x - 25244))/3
0) + tan(c/2 + (d*x)/2)^6*(529*a^3*(c + d*x) - (a^3*(15870*c + 15870*d*x -
23368))/30) - tan(c/2 + (d*x)/2)^5*(529*a^3*(c + d*x) - (a^3*(15870*c + 158
70*d*x - 26680))/30))/(d*(tan(c/2 + (d*x)/2) - 1)^5*(tan(c/2 + (d*x)/2)^2 +
 1)^3)
```

3.30 $\int (a+a\sin(c+dx))^3 \tan^4(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{17a^3x}{2}-\frac{6a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 17/2*a^3*x-6*a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d+2/3*a^3*cos(d*x+c)/d/(
1-sin(d*x+c))^2-25/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))-3/2*a^3*cos(d*x+c)*sin
(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2788, 2729, 2727, 2718, 2715, 8, 2713}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{6a^3\cos(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (17*a^3*x)/2 - (6*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) + (2*a^3
*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (25*a^3*Cos[c + d*x])/(3*d*(1 -
 Sin[c + d*x])) - (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx = a^4\int\left(\frac{7}{a}+\frac{2}{a(-1+\sin(c+dx))^2}+\frac{9}{a(-1+\sin(c+dx))}+\frac{5\sin(c}{a}\right.$$

$$= 7a^3x + a^3\int \sin^3(c+dx)\,dx + \left(2a^3\right)\int \frac{1}{(-1+\sin(c+dx))^2}\,dx + ($$

$$= 7a^3x - \frac{5a^3\cos(c+dx)}{d} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{9a^3\cos(c+dx)}{d(1-\sin(c+dx}$$

$$= \frac{17a^3x}{2} - \frac{6a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}$$

Mathematica [A]
time = 1.36, size = 177, normalized size = 1.49

$$\frac{(a+a\sin(c+dx))^3\left(102(c+dx)-69\cos(c+dx)+\cos(3(c+dx))+\frac{8}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{16\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{200\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-9\sin(2(c+dx))\right)}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]

[Out] ((a + a*Sin[c + d*x])^3*(102*(c + d*x) - 69*Cos[c + d*x] + Cos[3*(c + d*x)]
 + 8/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (16*Sin[(c + d*x)/2])/(Cos[(
```

```
c + d*x)/2] - Sin[(c + d*x)/2])^3 - (200*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]) - 9*Sin[2*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(109) = 218$.
time = 0.28, size = 266, normalized size = 2.24

method	result
risch	$\frac{17a^3x}{2} + \frac{3ia^3e^{2i(dx+c)}}{8d} - \frac{23a^3e^{i(dx+c)}}{8d} - \frac{23a^3e^{-i(dx+c)}}{8d} - \frac{3ia^3e^{-2i(dx+c)}}{8d} - \frac{2(-48ia^3e^{i(dx+c)}+27a^3e^{2i(dx+c}}{3(e^{i(dx+c)}-i)^3d}$
derivativedivides	$a^3\left(\frac{\sin^8(dx+c)}{3\cos(dx+c)^3} - \frac{5(\sin^8(dx+c))}{3\cos(dx+c)} - \frac{5\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6(\sin^4(dx+c))}{5}+\frac{8(\sin^2(dx+c))}{5}\right)\cos(dx+c)}{3}\right)+3a^3\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4}{}\right.$
default	$a^3\left(\frac{\sin^8(dx+c)}{3\cos(dx+c)^3} - \frac{5(\sin^8(dx+c))}{3\cos(dx+c)} - \frac{5\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6(\sin^4(dx+c))}{5}+\frac{8(\sin^2(dx+c))}{5}\right)\cos(dx+c)}{3}\right)+3a^3\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16
/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*s
in(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*
sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+3*a^3*(1/3*sin(d*x+
c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^
2)*cos(d*x+c))+a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.51, size = 165, normalized size = 1.39

$$\frac{2\left(\cos(dx+c)^3-\frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3}-9\cos(dx+c)\right)a^3+3\left(2\tan(dx+c)^3+15dx+15c-\frac{3\tan(dx+c)}{\tan(dx+c)^2+1}-12\tan(dx+c)\right)a^3+2\left(\tan(dx+c)^3+3dx+3c-3\tan(dx+c)\right)a^3-6a^3\left(\frac{6\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\cos(dx+c)\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x
+ c))*a^3 + 3*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x +
 c)^2 + 1) - 12*tan(d*x + c))*a^3 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan
(d*x + c))*a^3 - 6*a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x +
 c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(105) = 210$.

time = 0.37, size = 220, normalized size = 1.85

$$\frac{2\,a^3\cos\left(dx+c\right)^5+7\,a^3\cos\left(dx+c\right)^4-22\,a^3\cos\left(dx+c\right)^3-102\,a^3dx-4\,a^3+\left(51\,a^3dx+77\,a^3\right)\cos\left(dx+c\right)^2-\left(51\,a^3dx-100\,a^3\right)\cos\left(dx+c\right)+\left(2\,a^3\cos\left(dx+c\right)^4-5\,a^3\cos\left(dx+c\right)^3+102\,a^3dx-27\,a^3\cos\left(dx+c\right)^2-4\,a^3+\left(51\,a^3dx-104\,a^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^5 + 7*a^3*cos(d*x + c)^4 - 22*a^3*cos(d*x + c)^3 -
102*a^3*d*x - 4*a^3 + (51*a^3*d*x + 77*a^3)*cos(d*x + c)^2 - (51*a^3*d*x -
100*a^3)*cos(d*x + c) + (2*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^3 + 102*
a^3*d*x - 27*a^3*cos(d*x + c)^2 - 4*a^3 + (51*a^3*d*x - 104*a^3)*cos(d*x +
c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*
d)*sin(d*x + c) - 2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int \tan^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**4,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**4, x) + Integral(3*sin(c + d*x)
**2*tan(c + d*x)**4, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**4, x) + In
tegral(tan(c + d*x)**4, x))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 10.50, size = 371, normalized size = 3.12

$$\frac{17\,a^3\,x}{2}+\frac{\frac{17\,a^3\,\left(c+d\,x\right)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{51\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(153\,c+153\,d\,x-378\right)}{6}\right)-\frac{a^3\,\left(51\,c+51\,d\,x-160\right)}{6}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{51\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(153\,c+153\,d\,x-102\right)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(51\,a^3\,\left(c+d\,x\right)-\frac{a^3\,\left(306\,c+306\,d\,x-306\right)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(51\,a^3\,\left(c+d\,x\right)-\frac{a^3\,\left(306\,c+306\,d\,x-654\right)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(85\,a^3\,\left(c+d\,x\right)-\frac{a^3\,\left(510\,c+510\,d\,x-578\right)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(85\,a^3\,\left(c+d\,x\right)-\frac{a^3\,\left(510\,c+510\,d\,x-1022\right)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(102\,a^3\,\left(c+d\,x\right)-\frac{a^3\,\left(612\,c+612\,d\,x-918\right)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(102\,a^3\,\left(c+d\,x\right)-\frac{a^3\,\left(612\,c+612\,d\,x-1002\right)}{6}\right)}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (17*a^3*x)/2 + ((17*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((51*a^3*(c + d*x
))/2 - (a^3*(153*c + 153*d*x - 378))/6) - (a^3*(51*c + 51*d*x - 160))/6 + t
```

```
an(c/2 + (d*x)/2)^8*((51*a^3*(c + d*x))/2 - (a^3*(153*c + 153*d*x - 102))/6
) - tan(c/2 + (d*x)/2)^7*(51*a^3*(c + d*x) - (a^3*(306*c + 306*d*x - 306))/
6) + tan(c/2 + (d*x)/2)^2*(51*a^3*(c + d*x) - (a^3*(306*c + 306*d*x - 654))
/6) + tan(c/2 + (d*x)/2)^6*(85*a^3*(c + d*x) - (a^3*(510*c + 510*d*x - 578)
)/6) - tan(c/2 + (d*x)/2)^3*(85*a^3*(c + d*x) - (a^3*(510*c + 510*d*x - 102
2))/6) - tan(c/2 + (d*x)/2)^5*(102*a^3*(c + d*x) - (a^3*(612*c + 612*d*x -
918))/6) + tan(c/2 + (d*x)/2)^4*(102*a^3*(c + d*x) - (a^3*(612*c + 612*d*x
- 1002))/6))/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x
)/2)^3 - 1)^3)
```

3.31 $\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=89

$$-\frac{11a^3x}{2}+\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] -11/2*a^3*x+5*a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+c)^3/d+4*a^3*cos(d*x+c)/d/(1
-sin(d*x+c))+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2788, 2727, 2718, 2715, 8, 2713}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (-11*a^3*x)/2 + (5*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/(3*d) + (4*a^
3*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/
(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = a^2\int \left(-4a-\frac{4a}{-1+\sin(c+dx)}-4a\sin(c+dx)-3a\sin^2(c+dx\right.$$

$$= -4a^3x-a^3\int \sin^3(c+dx)\,dx-\left(3a^3\right)\int \sin^2(c+dx)\,dx-\left(4a^3\right)$$

$$= -4a^3x+\frac{4a^3\cos(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{3a^3\cos(c+dx)}{2d}$$

$$= -\frac{11a^3x}{2}+\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx)}$$

Mathematica [A]
time = 0.32, size = 115, normalized size = 1.29

$$\frac{(a+a\sin(c+dx))^3\left(-66(c+dx)+57\cos(c+dx)-\cos(3(c+dx))+\frac{96\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+9\sin(2(c+dx))\right)}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-66*(c + d*x) + 57*Cos[c + d*x] - Cos[3*(c + d*x)]
 + (96*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 9*Sin[2*(c
 + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.23, size = 167, normalized size = 1.88

method	result
risch	$-\frac{11a^3x}{2}+\frac{19a^3 e^{i(dx+c)}}{8d}+\frac{19a^3 e^{-i(dx+c)}}{8d}+\frac{8a^3}{d(e^{i(dx+c)}-i)}-\frac{a^3\cos(3dx+3c)}{12d}+\frac{3a^3\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^3\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+3a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)\right.}{d}$
default	$\frac{a^3\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+3a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d
*x+c))+3*a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x
+c)-3/2*d*x-3/2*c)+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+
c))+a^3*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.54, size = 117, normalized size = 1.31

$$-\frac{2\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\cos\left(dx+c\right)\right)a^3+9\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan\left(dx+c\right)\right)a^3+6\left(dx+c-\tan\left(dx+c\right)\right)a^3-18\,a^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3 + 9*(3*d*x +
 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^3 + 6*(d*x + c
 - tan(d*x + c))*a^3 - 18*a^3*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.37, size = 154, normalized size = 1.73

$$-\frac{2\,a^3\cos\left(dx+c\right)^4-7\,a^3\cos\left(dx+c\right)^3+33\,a^3dx-30\,a^3\cos\left(dx+c\right)^2-24\,a^3+3\left(11\,a^3dx-15\,a^3\right)\cos\left(dx+c\right)-\left(2\,a^3\cos\left(dx+c\right)^3+33\,a^3dx+9\,a^3\cos\left(dx+c\right)^2-21\,a^3\cos\left(dx+c\right)+24\,a^3\right)\sin\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a^3*cos(d*x + c)^4 - 7*a^3*cos(d*x + c)^3 + 33*a^3*d*x - 30*a^3*cos
(d*x + c)^2 - 24*a^3 + 3*(11*a^3*d*x - 15*a^3)*cos(d*x + c) - (2*a^3*cos(d*
x + c)^3 + 33*a^3*d*x + 9*a^3*cos(d*x + c)^2 - 21*a^3*cos(d*x + c) + 24*a^3
)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int \tan^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**2, x) + Integral(3*sin(c + d*x)
**2*tan(c + d*x)**2, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**2, x) + In
tegral(tan(c + d*x)**2, x))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 10.33, size = 288, normalized size = 3.24

$$-\frac{11\,a^3\,x}{2}-\frac{\frac{11\,a^3\,\left(c+d\,x\right)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{11\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(33\,c+33\,d\,x-38\right)}{6}\right)-\frac{a^3\,\left(33\,c+33\,d\,x-104\right)}{6}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{11\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(33\,c+33\,d\,x-66\right)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{33\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(99\,c+99\,d\,x-66\right)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{33\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(99\,c+99\,d\,x-120\right)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{33\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(99\,c+99\,d\,x-192\right)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{33\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(99\,c+99\,d\,x-246\right)}{6}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (11*a^3*x)/2 - ((11*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((11*a^3*(c + d
*x))/2 - (a^3*(33*c + 33*d*x - 38))/6) - (a^3*(33*c + 33*d*x - 104))/6 + ta
n(c/2 + (d*x)/2)^6*((11*a^3*(c + d*x))/2 - (a^3*(33*c + 33*d*x - 66))/6) -
tan(c/2 + (d*x)/2)^5*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 66))/6)
- tan(c/2 + (d*x)/2)^3*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 120))/
6) + tan(c/2 + (d*x)/2)^4*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 192
))/6) + tan(c/2 + (d*x)/2)^2*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x -
246))/6))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

3.32 $\int (a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=63

$$\frac{5a^3x}{2}-\frac{4a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 5/2*a^3*x-4*a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d-3/2*a^3*cos(d*x+c)*sin(
d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2724, 2718, 2715, 8, 2713}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{4a^3\cos(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/2 - (4*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) - (3*a^3*
Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^3\,dx &= \int \left(a^3+3a^3\sin(c+dx)+3a^3\sin^2(c+dx)+a^3\sin^3(c+dx)\right)\,dx\\ &= a^3x+a^3\int \sin^3(c+dx)\,dx+\left(3a^3\right)\int \sin(c+dx)\,dx+\left(3a^3\right)\int \sin^2(c+dx)\,dx\\ &= a^3x-\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(3a^3\right)\int 1\,dx-\frac{a^3\mathrm{Sub}}{}\\ &= \frac{5a^3x}{2}-\frac{4a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 44, normalized size = 0.70

$$\frac{a^3(30c+30dx-45\cos(c+dx)+\cos(3(c+dx))-9\sin(2(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3,x]
[Out] (a^3*(30*c + 30*d*x - 45*Cos[c + d*x] + Cos[3*(c + d*x)] - 9*Sin[2*(c + d*x
)]))/(12*d)
```

Maple [A]
time = 0.15, size = 74, normalized size = 1.17

method	result
risch	$\frac{5a^3x}{2}-\frac{15a^3\cos(dx+c)}{4d}+\frac{a^3\cos(3dx+3c)}{12d}-\frac{3a^3\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{-\frac{a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+3a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-3a^3\cos(dx+c)+a^3(dx+c)}{d}$
default	$\frac{-\frac{a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+3a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-3a^3\cos(dx+c)+a^3(dx+c)}{d}$
norman	$\frac{\frac{5a^3x}{2}-\frac{22a^3}{3d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{3a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{15a^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{15a^3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{5a^3x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a^3*(-1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)-3*a^3*cos(d*x+c)+a^3*(d*x+c))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 1.14

$$a^3x+\frac{\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)a^3}{3\,d}+\frac{3\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)a^3}{4\,d}-\frac{3\,a^3\cos\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3/d + 3/4*(2*d*x + 2*c - si
n(2*d*x + 2*c))*a^3/d - 3*a^3*cos(d*x + c)/d
```

Fricas [A]
time = 0.36, size = 54, normalized size = 0.86

$$\frac{2\,a^3\cos\left(dx+c\right)^3+15\,a^3dx-9\,a^3\cos\left(dx+c\right)\sin\left(dx+c\right)-24\,a^3\cos\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 + 15*a^3*d*x - 9*a^3*cos(d*x + c)*sin(d*x + c) -
24*a^3*cos(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(58) = 116.
time = 0.13, size = 121, normalized size = 1.92

$$\begin{cases}\frac{3a^3x\sin^2\left(c+dx\right)}{2}+\frac{3a^3x\cos^2\left(c+dx\right)}{2}+a^3x-\frac{a^3\sin^2\left(c+dx\right)\cos\left(c+dx\right)}{d}-\frac{3a^3\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{2a^3\cos^3\left(c+dx\right)}{3d}-\frac{3a^3\cos\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\left(a\sin\left(c\right)+a\right)^3 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**2/2 + a**3*x
 - a**3*sin(c + d*x)**2*cos(c + d*x)/d - 3*a**3*sin(c + d*x)*cos(c + d*x)/(
2*d) - 2*a**3*cos(c + d*x)**3/(3*d) - 3*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*
(a*sin(c) + a)**3, True))
```

Giac [A]
time = 6.71, size = 55, normalized size = 0.87

$$\frac{5}{2}\,a^3x+\frac{a^3\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{15\,a^3\cos\left(dx+c\right)}{4\,d}-\frac{3\,a^3\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 5/2*a^3*x + 1/12*a^3*cos(3*d*x + 3*c)/d - 15/4*a^3*cos(d*x + c)/d - 3/4*a^3
*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 8.92, size = 156, normalized size = 2.48

$$\frac{5\,a^3\,x}{2}-\frac{\frac{5\,a^3\,\left(c+d\,x\right)}{2}-3\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5-\frac{a^3\,\left(15\,c+15\,d\,x-44\right)}{6}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{15\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(45\,c+45\,d\,x-36\right)}{6}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{15\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(45\,c+45\,d\,x-96\right)}{6}\right)+3\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*a^3*x)/2 - ((5*a^3*(c + d*x))/2 - 3*a^3*tan(c/2 + (d*x)/2)^5 - (a^3*(15*
c + 15*d*x - 44))/6 + tan(c/2 + (d*x)/2)^4*((15*a^3*(c + d*x))/2 - (a^3*(45
*c + 45*d*x - 36))/6) + tan(c/2 + (d*x)/2)^2*((15*a^3*(c + d*x))/2 - (a^3*(
45*c + 45*d*x - 96))/6) + 3*a^3*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^
2 + 1)^3)
```

3.33 $\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=92

$$\frac{a^3x}{2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a^3*x-3*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+c)
^3/d-a^3*cot(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2788, 3855, 3852, 8, 2718, 2715, 2713}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*x)/2 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d - (a^3
*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c +
 d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(2a^5+3a^5\csc(c+dx)+a^5\csc^2(c+dx)-2a^5\sin(c+dx)-3a^5\right.}{a^2}$$

$$= 2a^3x + a^3\int \csc^2(c+dx)\,dx - a^3\int \sin^3(c+dx)\,dx - \left(2a^3\right)\int \sin$$

$$= 2a^3x - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^3\cos(c+dx)}{d} + \frac{3a^3\cos(c+}{}$$

$$= \frac{a^3x}{2} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a^3\cos^3(c+d}{3d}$$

Mathematica [A]
time = 0.74, size = 106, normalized size = 1.15

$$-\frac{a^3\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)(15-66\sin(c+dx))-12\left(c+dx-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)+\cos(3(c+dx))(9+2\sin(c+dx))\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/48*(a^3*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x]*(15 - 66*Sin[c +
 d*x]) - 12*(c + d*x - 6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]])*S
in[c + d*x] + Cos[3*(c + d*x)]*(9 + 2*Sin[c + d*x])))/d
```

Maple [A]
time = 0.12, size = 94, normalized size = 1.02

method	result
derivativedivides	$\frac{-\frac{\left(\cos^3(dx+c)\right)a^3}{3}+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a^3(-\cot(dx+c)-dx-}{d}$
default	$\frac{-\frac{\left(\cos^3(dx+c)\right)a^3}{3}+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a^3(-\cot(dx+c)-dx-}{d}$
risch	$\frac{a^3x}{2}-\frac{3ia^3e^{2i(dx+c)}}{8d}+\frac{11a^3e^{i(dx+c)}}{8d}+\frac{11a^3e^{-i(dx+c)}}{8d}+\frac{3ia^3e^{-2i(dx+c)}}{8d}-\frac{2ia^3}{d\left(e^{2i(dx+c)}-1\right)}+\frac{3a^3\ln\left(e^{i(dx+c)}-\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*cos(d*x+c)^3*a^3+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+
3*a^3*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a^3*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 93, normalized size = 1.01

$$-\frac{4\,a^3\cos\left(dx+c\right)^3-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^3+12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3-18\,a^3\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*a^3*cos(d*x + c)^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(
d*x + c + 1/tan(d*x + c))*a^3 - 18*a^3*(2*cos(d*x + c) - log(cos(d*x + c) +
 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.40, size = 121, normalized size = 1.32

$$-\frac{9\,a^3\cos\left(dx+c\right)^3+9\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\,a^3\cos\left(dx+c\right)+\left(2\,a^3\cos\left(dx+c\right)^3-3\,a^3dx-18\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(9*a^3*cos(d*x + c)^3 + 9*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
 - 9*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*a^3*cos(d*x + c) + (
2*a^3*cos(d*x + c)^3 - 3*a^3*d*x - 18*a^3*cos(d*x + c))*sin(d*x + c))/(d*si
n(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int\sin^3\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int\cot^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(3*sin(c + d*x)
**2*cot(c + d*x)**2, x) + Integral(sin(c + d*x)**3*cot(c + d*x)**2, x) + In
tegral(cot(c + d*x)**2, x))
```

Giac [A]
time = 6.30, size = 162, normalized size = 1.76

$$\frac{3\,(dx+c)a^3+18\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\,\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\,\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-16\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*a^3 + 18*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/
2*d*x + 1/2*c) - 3*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)
- 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*a^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^3*
tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - 16*a^3)/(tan(1/2*d*x
+ 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 6.77, size = 264, normalized size = 2.87

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{a^3\,\mathrm{atan}\left(\frac{a^6}{6\,a^6-a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{6\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^6-a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{-7\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+24\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{32\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-a^3}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d + (a^3*atan(a^6/(6*a^6 - a^6*tan(c/2 + (d
*x)/2)) + (6*a^6*tan(c/2 + (d*x)/2))/(6*a^6 - a^6*tan(c/2 + (d*x)/2))))/d +
 (a^3*tan(c/2 + (d*x)/2))/(2*d) + (3*a^3*tan(c/2 + (d*x)/2)^2 + 24*a^3*tan(
c/2 + (d*x)/2)^3 - 3*a^3*tan(c/2 + (d*x)/2)^4 + 8*a^3*tan(c/2 + (d*x)/2)^5
- 7*a^3*tan(c/2 + (d*x)/2)^6 - a^3 + (32*a^3*tan(c/2 + (d*x)/2))/3)/(d*(2*t
an(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3 + 6*tan(c/2 + (d*x)/2)^5 + 2*tan
(c/2 + (d*x)/2)^7))
```

3.34 $\int (a+a\sin(c+dx))^4\tan^5(c+dx)\,dx$

Optimal. Leaf size=129

$$-\frac{25a^4\log(1-\sin(c+dx))}{d}-\frac{16a^4\sin(c+dx)}{d}-\frac{9a^4\sin^2(c+dx)}{2d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{a^4\sin^4(c+dx)}{4d}+\frac{}{d(a-a}$$

```
[Out] -25*a^4*ln(1-sin(d*x+c))/d-16*a^4*sin(d*x+c)/d-9/2*a^4*sin(d*x+c)^2/d-4/3*a
^4*sin(d*x+c)^3/d-1/4*a^4*sin(d*x+c)^4/d+a^6/d/(a-a*sin(d*x+c))^2-11*a^5/d/
(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2786, 78}

$$\frac{a^6}{d(a-a\sin(c+dx))^2}-\frac{11a^5}{d(a-a\sin(c+dx))}-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{9a^4\sin^2(c+dx)}{2d}-\frac{16a^4\sin(c+dx)}{d}-\frac{25a^4\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^5,x]
```

```
[Out] (-25*a^4*Log[1 - Sin[c + d*x]])/d - (16*a^4*Sin[c + d*x])/d - (9*a^4*Sin[c
+ d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^4*Sin[c + d*x]^4)/(4*d)
 + a^6/(d*(a - a*Sin[c + d*x])^2) - (11*a^5)/(d*(a - a*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-16a^3+\frac{2a^6}{(a-x)^3}-\frac{11a^5}{(a-x)^2}+\frac{25a^4}{a-x}-9a^2x-4ax^2-x^3\right)\,dx\right.}{d}$$

$$= -\frac{25a^4\log(1-\sin(c+dx))}{d}-\frac{16a^4\sin(c+dx)}{d}-\frac{9a^4\sin^2(c+dx)}{2d}$$

Mathematica [**A**]
time = 0.30, size = 83, normalized size = 0.64

$$-\frac{a^4\left(300\log(1-\sin(c+dx))+\frac{120-132\sin(c+dx)}{(-1+\sin(c+dx))^2}+192\sin(c+dx)+54\sin^2(c+dx)+16\sin^3(c+dx)+3\sin^4(c+dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^5,x]
```

```
[Out] -1/12*(a^4*(300*Log[1 - Sin[c + d*x]] + (120 - 132*Sin[c + d*x])/(-1 + Sin[
c + d*x])^2 + 192*Sin[c + d*x] + 54*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3 + 3*
Sin[c + d*x]^4))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(123) = 246$.
time = 0.21, size = 393, normalized size = 3.05

method	result
risch	$25ia^4x-\frac{ia^4e^{3i(dx+c)}}{6d}+\frac{19a^4e^{2i(dx+c)}}{16d}+\frac{17ia^4e^{i(dx+c)}}{2d}-\frac{17ia^4e^{-i(dx+c)}}{2d}+\frac{19a^4e^{-2i(dx+c)}}{16d}+\frac{ia^4e^{-3i(dx}}{6d}$
derivativedivides	$\frac{a^4\left(\frac{\sin^{10}(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^{10}(dx+c)\right)}{4\cos(dx+c)^2}-\frac{3\left(\sin^8(dx+c)\right)}{4}-\left(\sin^6(dx+c)\right)-\frac{3\left(\sin^4(dx+c)\right)}{2}-3\left(\sin^2(dx+c)\right)-6\ln(\cos(dx+c))\right)+4}{}$
default	$\frac{a^4\left(\frac{\sin^{10}(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^{10}(dx+c)\right)}{4\cos(dx+c)^2}-\frac{3\left(\sin^8(dx+c)\right)}{4}-\left(\sin^6(dx+c)\right)-\frac{3\left(\sin^4(dx+c)\right)}{2}-3\left(\sin^2(dx+c)\right)-6\ln(\cos(dx+c))\right)+4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/4*sin(d*x+c)^10/cos(d*x+c)^4-3/4*sin(d*x+c)^10/cos(d*x+c)^2-3/4
*sin(d*x+c)^8-sin(d*x+c)^6-3/2*sin(d*x+c)^4-3*sin(d*x+c)^2-6*ln(cos(d*x+c))
)+4*a^4*(1/4*sin(d*x+c)^9/cos(d*x+c)^4-5/8*sin(d*x+c)^9/cos(d*x+c)^2-5/8*si
n(d*x+c)^7-7/8*sin(d*x+c)^5-35/24*sin(d*x+c)^3-35/8*sin(d*x+c)+35/8*ln(sec(
```

```
d*x+c)+tan(d*x+c)))+6*a^4*(1/4*sin(d*x+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/c
os(d*x+c)^2-1/2*sin(d*x+c)^6-3/4*sin(d*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x
+c)))+4*a^4*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/
8*sin(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+
c)))+a^4*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.29, size = 109, normalized size = 0.84

$$-\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+54\,a^4\sin\left(dx+c\right)^2+300\,a^4\log\left(\sin\left(dx+c\right)-1\right)+192\,a^4\sin\left(dx+c\right)-\frac{12\,\left(11\,a^4\sin(dx+c)-10\,a^4\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 54*a^4*sin(d*x + c)^2
 + 300*a^4*log(sin(d*x + c) - 1) + 192*a^4*sin(d*x + c) - 12*(11*a^4*sin(d*
x + c) - 10*a^4)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.37, size = 154, normalized size = 1.19

$$-\frac{24\,a^4\cos\left(dx+c\right)^6-272\,a^4\cos\left(dx+c\right)^4-2393\,a^4\cos\left(dx+c\right)^2+1906\,a^4+2400\,\left(a^4\cos\left(dx+c\right)^2+2\,a^4\sin\left(dx+c\right)-2\,a^4\right)\log\left(-\sin\left(dx+c\right)+1\right)-10\,\left(8\,a^4\cos\left(dx+c\right)^4-96\,a^4\cos\left(dx+c\right)^2+181\,a^4\right)\sin\left(dx+c\right)}{96\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/96*(24*a^4*cos(d*x + c)^6 - 272*a^4*cos(d*x + c)^4 - 2393*a^4*cos(d*x +
c)^2 + 1906*a^4 + 2400*(a^4*cos(d*x + c)^2 + 2*a^4*sin(d*x + c) - 2*a^4)*lo
g(-sin(d*x + c) + 1) - 10*(8*a^4*cos(d*x + c)^4 - 96*a^4*cos(d*x + c)^2 + 1
81*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin\left(c+dx\right)\tan^5\left(c+dx\right)dx+\int 6\sin^2\left(c+dx\right)\tan^5\left(c+dx\right)dx+\int 4\sin^3\left(c+dx\right)\tan^5\left(c+dx\right)dx+\int \sin^4\left(c+dx\right)\tan^5\left(c+dx\right)dx+\int \tan^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**5,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*tan(c + d*x)**5, x) + Integral(6*sin(c + d*x)
**2*tan(c + d*x)**5, x) + Integral(4*sin(c + d*x)**3*tan(c + d*x)**5, x) +
Integral(sin(c + d*x)**4*tan(c + d*x)**5, x) + Integral(tan(c + d*x)**5, x)
)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 7.88, size = 379, normalized size = 2.94

$$\frac{25\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{50\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}-\frac{50\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-150\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{950\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{3}-\frac{1700\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+\frac{2180\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{2452\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{2180\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{1700\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{950\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-150\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+50\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+31\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-40\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+44\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-40\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+31\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (25*a^4*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (50*a^4*log(tan(c/2 + (d*x)/2) -
 1))/d - ((950*a^4*tan(c/2 + (d*x)/2)^3)/3 - 150*a^4*tan(c/2 + (d*x)/2)^2 -
 (1700*a^4*tan(c/2 + (d*x)/2)^4)/3 + (2180*a^4*tan(c/2 + (d*x)/2)^5)/3 - (2
452*a^4*tan(c/2 + (d*x)/2)^6)/3 + (2180*a^4*tan(c/2 + (d*x)/2)^7)/3 - (1700
*a^4*tan(c/2 + (d*x)/2)^8)/3 + (950*a^4*tan(c/2 + (d*x)/2)^9)/3 - 150*a^4*t
an(c/2 + (d*x)/2)^10 + 50*a^4*tan(c/2 + (d*x)/2)^11 + 50*a^4*tan(c/2 + (d*x
)/2))/(d*(10*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 20*tan(c/2 + (d*
x)/2)^3 + 31*tan(c/2 + (d*x)/2)^4 - 40*tan(c/2 + (d*x)/2)^5 + 44*tan(c/2 +
(d*x)/2)^6 - 40*tan(c/2 + (d*x)/2)^7 + 31*tan(c/2 + (d*x)/2)^8 - 20*tan(c/2
 + (d*x)/2)^9 + 10*tan(c/2 + (d*x)/2)^10 - 4*tan(c/2 + (d*x)/2)^11 + tan(c/
2 + (d*x)/2)^12 + 1))
```

3.35 $\int (a + a\sin(c + dx))^4 \tan^3(c + dx)\, dx$

Optimal. Leaf size=107

$$\frac{16a^4\log(1-\sin(c+dx))}{d}+\frac{12a^4\sin(c+dx)}{d}+\frac{4a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin^4(c+dx)}{4d}+\frac{4\ldots}{d(a-a\,\mathrm{s}\ldots}$$

```
[Out] 16*a^4*ln(1-sin(d*x+c))/d+12*a^4*sin(d*x+c)/d+4*a^4*sin(d*x+c)^2/d+4/3*a^4*
sin(d*x+c)^3/d+1/4*a^4*sin(d*x+c)^4/d+4*a^5/d/(a-a*sin(d*x+c))
```

Rubi [A]

time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$\frac{4a^5}{d(a-a\sin(c+dx))}+\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{4a^4\sin^2(c+dx)}{d}+\frac{12a^4\sin(c+dx)}{d}+\frac{16a^4\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^3,x]
```

```
[Out] (16*a^4*Log[1 - Sin[c + d*x]])/d + (12*a^4*Sin[c + d*x])/d + (4*a^4*Sin[c +
 d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d) + (4
*a^5)/(d*(a - a*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^2}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(12a^3+\frac{4a^5}{(a-x)^2}-\frac{16a^4}{a-x}+8a^2x+4ax^2+x^3\right)dx, x, a\sin(c\right.}{d}$$
$$= \frac{16a^4\log(1-\sin(c+dx))}{d}+\frac{12a^4\sin(c+dx)}{d}+\frac{4a^4\sin^2(c+dx)}{d}$$

Mathematica [A]
time = 0.11, size = 76, normalized size = 0.71

$$\frac{a^4\left(192\log(1-\sin(c+dx))+\frac{48}{1-\sin(c+dx)}+144\sin(c+dx)+48\sin^2(c+dx)+16\sin^3(c+dx)+3\sin^4(c+dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^3,x]
```

```
[Out] (a^4*(192*Log[1 - Sin[c + d*x]] + 48/(1 - Sin[c + d*x]) + 144*Sin[c + d*x]
+ 48*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4))/(12*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(103) = 206$.
time = 0.17, size = 267, normalized size = 2.50

method	result
risch	$-16ia^4x-\frac{13ia^4e^{i(dx+c)}}{2d}+\frac{13ia^4e^{-i(dx+c)}}{2d}-\frac{32ia^4c}{d}-\frac{8ia^4e^{i(dx+c)}}{\left(e^{i(dx+c)}-i\right)^2 d}+\frac{32a^4\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a^4\cos(4}{32}$
derivativedivides	$a^4\left(\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^6(dx+c)\right)}{2}+\frac{3\left(\sin^4(dx+c)\right)}{4}+\frac{3\left(\sin^2(dx+c)\right)}{2}+3\ln(\cos(dx+c))\right)+4a^4\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\right.$
default	$a^4\left(\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^6(dx+c)\right)}{2}+\frac{3\left(\sin^4(dx+c)\right)}{4}+\frac{3\left(\sin^2(dx+c)\right)}{2}+3\ln(\cos(dx+c))\right)+4a^4\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/2*sin(d*x+c)^8/cos(d*x+c)^2+1/2*sin(d*x+c)^6+3/4*sin(d*x+c)^4+3
/2*sin(d*x+c)^2+3*ln(cos(d*x+c)))+4*a^4*(1/2*sin(d*x+c)^7/cos(d*x+c)^2+1/2*
sin(d*x+c)^5+5/6*sin(d*x+c)^3+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(d*x+c)))
+6*a^4*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(co
s(d*x+c)))+4*a^4*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*
x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c)))+a^4*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 85, normalized size = 0.79

$$\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+48\,a^4\sin\left(dx+c\right)^2+192\,a^4\log\left(\sin\left(dx+c\right)-1\right)+144\,a^4\sin\left(dx+c\right)-\frac{48\,a^4}{\sin(dx+c)-1}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 48*a^4*sin(d*x + c)^2
+ 192*a^4*log(sin(d*x + c) - 1) + 144*a^4*sin(d*x + c) - 48*a^4/(sin(d*x +
c) - 1))/d
```

Fricas [A]
time = 0.35, size = 116, normalized size = 1.08

$$\frac{104\,a^4\cos\left(dx+c\right)^4-976\,a^4\cos\left(dx+c\right)^2+689\,a^4+1536\left(a^4\sin\left(dx+c\right)-a^4\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(24\,a^4\cos\left(dx+c\right)^4-304\,a^4\cos\left(dx+c\right)^2-1073\,a^4\right)\sin\left(dx+c\right)}{96\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(104*a^4*cos(d*x + c)^4 - 976*a^4*cos(d*x + c)^2 + 689*a^4 + 1536*(a^4
*sin(d*x + c) - a^4)*log(-sin(d*x + c) + 1) + (24*a^4*cos(d*x + c)^4 - 304*
a^4*cos(d*x + c)^2 - 1073*a^4)*sin(d*x + c))/(d*sin(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin\left(c+dx\right)\tan^3\left(c+dx\right)dx+\int 6\sin^2\left(c+dx\right)\tan^3\left(c+dx\right)dx+\int 4\sin^3\left(c+dx\right)\tan^3\left(c+dx\right)dx+\int\sin^4\left(c+dx\right)\tan^3\left(c+dx\right)dx+\int\tan^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**3,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*tan(c + d*x)**3, x) + Integral(6*sin(c + d*x)
**2*tan(c + d*x)**3, x) + Integral(4*sin(c + d*x)**3*tan(c + d*x)**3, x) +
Integral(sin(c + d*x)**4*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x)
)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 7.56, size = 320, normalized size = 2.99

$$\frac{32\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}+\frac{32\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-32\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{320\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{340\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{424\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{340\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{320\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-32\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+32\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}-\frac{16\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (32*a^4*log(tan(c/2 + (d*x)/2) - 1))/d + ((320*a^4*tan(c/2 + (d*x)/2)^3)/3
- 32*a^4*tan(c/2 + (d*x)/2)^2 - (340*a^4*tan(c/2 + (d*x)/2)^4)/3 + (424*a^4
*tan(c/2 + (d*x)/2)^5)/3 - (340*a^4*tan(c/2 + (d*x)/2)^6)/3 + (320*a^4*tan(
c/2 + (d*x)/2)^7)/3 - 32*a^4*tan(c/2 + (d*x)/2)^8 + 32*a^4*tan(c/2 + (d*x)/
2)^9 + 32*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 +
(d*x)/2) - 8*tan(c/2 + (d*x)/2)^3 + 10*tan(c/2 + (d*x)/2)^4 - 12*tan(c/2 +
(d*x)/2)^5 + 10*tan(c/2 + (d*x)/2)^6 - 8*tan(c/2 + (d*x)/2)^7 + 5*tan(c/2 +
 (d*x)/2)^8 - 2*tan(c/2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^10 + 1)) - (16*a^
4*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.36 $\int (a + a\sin(c + dx))^4 \tan(c + dx)\,dx$

Optimal. Leaf size=88

$$-\frac{8a^4\log(1-\sin(c+dx))}{d}-\frac{8a^4\sin(c+dx)}{d}-\frac{7a^4\sin^2(c+dx)}{2d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{a^4\sin^4(c+dx)}{4d}$$

```
[Out] -8*a^4*ln(1-sin(d*x+c))/d-8*a^4*sin(d*x+c)/d-7/2*a^4*sin(d*x+c)^2/d-4/3*a^4
*sin(d*x+c)^3/d-1/4*a^4*sin(d*x+c)^4/d
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 78}

$$-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{7a^4\sin^2(c+dx)}{2d}-\frac{8a^4\sin(c+dx)}{d}-\frac{8a^4\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x],x]
```

```
[Out] (-8*a^4*Log[1 - Sin[c + d*x]])/d - (8*a^4*Sin[c + d*x])/d - (7*a^4*Sin[c +
d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^4*Sin[c + d*x]^4)/(4*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^3}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-8a^3+\frac{8a^4}{a-x}-7a^2x-4ax^2-x^3\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{8a^4\log(1-\sin(c+dx))}{d}-\frac{8a^4\sin(c+dx)}{d}-\frac{7a^4\sin^2(c+dx)}{2d}-$$

Mathematica [A]
time = 0.05, size = 62, normalized size = 0.70

$$-\frac{a^4(96\log(1-\sin(c+dx))+96\sin(c+dx)+42\sin^2(c+dx)+16\sin^3(c+dx)+3\sin^4(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x],x]
[Out] -1/12*(a^4*(96*Log[1 - Sin[c + d*x]] + 96*Sin[c + d*x] + 42*Sin[c + d*x]^2
+ 16*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4))/d
```

Maple [A]
time = 0.18, size = 143, normalized size = 1.62

method	result
risch	$8ia^4x+\frac{9ia^4e^{i(dx+c)}}{2d}-\frac{9ia^4e^{-i(dx+c)}}{2d}+\frac{16ia^4c}{d}-\frac{16a^4\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{a^4\cos(4dx+4c)}{32d}+\frac{a^4\sin(3dx+3c}{3d}$
derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+4a^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+6}{d}$
default	$\frac{a^4\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+4a^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+6}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(a^4*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+4*a^4*(-1/3*si
n(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+6*a^4*(-1/2*sin(d*x+c)^2-l
n(cos(d*x+c)))+4*a^4*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-a^4*ln(cos(d*x
+c)))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.80

$$-\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+42\,a^4\sin\left(dx+c\right)^2+96\,a^4\log\left(\sin\left(dx+c\right)-1\right)+96\,a^4\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 42*a^4*sin(d*x + c)^2
 + 96*a^4*log(sin(d*x + c) - 1) + 96*a^4*sin(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 74, normalized size = 0.84

$$-\frac{3\,a^4\cos\left(dx+c\right)^4-48\,a^4\cos\left(dx+c\right)^2+96\,a^4\log\left(-\sin\left(dx+c\right)+1\right)-16\,\left(a^4\cos\left(dx+c\right)^2-7\,a^4\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*cos(d*x + c)^4 - 48*a^4*cos(d*x + c)^2 + 96*a^4*log(-sin(d*x +
 c) + 1) - 16*(a^4*cos(d*x + c)^2 - 7*a^4)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin\left(c+dx\right)\tan\left(c+dx\right)dx+\int 6\sin^2\left(c+dx\right)\tan\left(c+dx\right)dx+\int 4\sin^3\left(c+dx\right)\tan\left(c+dx\right)dx+\int \sin^4\left(c+dx\right)\tan\left(c+dx\right)dx+\int \tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c),x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*tan(c + d*x), x) + Integral(6*sin(c + d*x)**2
*tan(c + d*x), x) + Integral(4*sin(c + d*x)**3*tan(c + d*x), x) + Integral(
sin(c + d*x)**4*tan(c + d*x), x) + Integral(tan(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 67058 vs. $2(82) = 164$.
time = 19.83, size = 67058, normalized size = 762.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/96*(384*a^4*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/
2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 - 384*a^4*log(2*(tan(1/2*d
*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2
*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
```

2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*
d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(
1/2*c)^6*tan(c)^4 + 384*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 - 177*a^4*tan(d*x)^4*tan
(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 768*a^4*log(2*(tan(1/2*d*x)^4*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2
*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)
^2 - 768*a^4*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*
c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*t
an(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 768*a^4*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(
c)^2 + 1152*a^4*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1
/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)
)*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^4 - 1152*a^4*log(2*(tan(1/2
*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^4*tan(1/2*d*x)^6*ta
n(1/2*c)^4*tan(c)^4 + 1152*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
+ 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^4 - 1536*a^4*tan(d*x)^4
*tan(1/2*d*x)^6*tan(1/2*c)^5*tan(c)^4 + 1152*a^4*log(2*(tan(1/2*d*x)^4*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + ta
n(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*t
an(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^4*tan(1/2*d*x)^4*tan(1/2*c)^6*t
an(c)^4 - 1152*a^4*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*ta
n(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 +
 1))*tan(d*x)^4*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^4 + 1152*a^4*log(4*(tan(
d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(1/2*d*x)^4*tan(1/2*c)
^6*tan(c)^4 - 1536*a^4*tan(d*x)^4*tan(1/2*d*x)^5*tan(1/2*c)^6*tan(c)^4 + 76
8*a^4*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*
tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan

```
(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)
^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 - 768*a^4*log(2*(tan(1/2*d*x)^4*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + t
an(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*
tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*
tan(c)^4 + 768*a^4*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*
x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 - 18*a^4*tan(d*x)^4*tan(1/2*d*x)^
6*tan(1/2*c)^6*tan(c)^2 + 672*a^4*tan(d*x)^3*tan(1/2*d*x)^6*tan(1/2*c)^6*ta
n(c)^3 - 531*a^4*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^4 - 531*a^4*
tan(d*x)^4*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^4 - 18*a^4*tan(d*x)^2*tan(1/2
*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 384*a^4*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x
)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2...
```

Mupad [B]
time = 6.63, size = 131, normalized size = 1.49

$$\frac{8\,a^4\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}-\frac{28\,a^4\,\sin\left(c+d\,x\right)}{3\,d}-\frac{16\,a^4\,\ln\left(\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,a^4\,{\cos\left(c+d\,x\right)}^2}{d}-\frac{a^4\,{\cos\left(c+d\,x\right)}^4}{4\,d}+\frac{4\,a^4\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (8*a^4*log(1/cos(c/2 + (d*x)/2)^2))/d - (28*a^4*sin(c + d*x))/(3*d) - (16*a
^4*log((cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))/cos(c/2 + (d*x)/2)))/d + (
4*a^4*cos(c + d*x)^2)/d - (a^4*cos(c + d*x)^4)/(4*d) + (4*a^4*cos(c + d*x)^
2*sin(c + d*x))/(3*d)
```

3.37 $\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=102

$$-\frac{4a^4\csc(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}+\frac{5a^4\log(\sin(c+dx))}{d}-\frac{5a^4\sin^2(c+dx)}{2d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{a^4\sin^4(c+$$

```
[Out] -4*a^4*csc(d*x+c)/d-1/2*a^4*csc(d*x+c)^2/d+5*a^4*ln(sin(d*x+c))/d-5/2*a^4*s
in(d*x+c)^2/d-4/3*a^4*sin(d*x+c)^3/d-1/4*a^4*sin(d*x+c)^4/d
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$,
Rules used = {2786, 76}

$$-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{5a^4\sin^2(c+dx)}{2d}-\frac{a^4\csc^2(c+dx)}{2d}-\frac{4a^4\csc(c+dx)}{d}+\frac{5a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a^4*Csc[c + d*x])/d - (a^4*Csc[c + d*x]^2)/(2*d) + (5*a^4*Log[Sin[c + d
*x]])/d - (5*a^4*Sin[c + d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^
4*Sin[c + d*x]^4)/(4*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^5}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^6}{x^3}+\frac{4a^5}{x^2}+\frac{5a^4}{x}-5a^2x-4ax^2-x^3\right)dx, x, a\sin(c+dx\right.}{d}\\ &= -\frac{4a^4\csc(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}+\frac{5a^4\log(\sin(c+dx))}{d}-\frac{5a^4}{}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 78, normalized size = 0.76

$$-\frac{a^4(3+16\csc(c+dx)+30\csc^2(c+dx)+48\csc^5(c+dx)+6\csc^6(c+dx)+\csc^4(c+dx)(90-60\log(\sin(c+dx))))\sin^4(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/12*(a^4*(3 + 16*Csc[c + d*x] + 30*Csc[c + d*x]^2 + 48*Csc[c + d*x]^5 + 6
*Csc[c + d*x]^6 + Csc[c + d*x]^4*(90 - 60*Log[Sin[c + d*x]]))*Sin[c + d*x]^
4)/d
```

Maple [A]
time = 0.21, size = 129, normalized size = 1.26

method	result
derivativedivides	$\frac{-\frac{a^4\left(\cos^4(dx+c)\right)}{4}+\frac{4a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^4\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+4a^4\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+\right.}{d}$
default	$\frac{-\frac{a^4\left(\cos^4(dx+c)\right)}{4}+\frac{4a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^4\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+4a^4\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+\right.}{d}$
risch	$-5ia^4x-\frac{ia^4e^{3i(dx+c)}}{6d}+\frac{11a^4e^{2i(dx+c)}}{16d}+\frac{ia^4e^{i(dx+c)}}{2d}-\frac{ia^4e^{-i(dx+c)}}{2d}+\frac{11a^4e^{-2i(dx+c)}}{16d}+\frac{ia^4e^{-3i(dx+c)}}{6d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a^4*cos(d*x+c)^4+4/3*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^4*(1/2*c
os(d*x+c)^2+ln(sin(d*x+c)))+4*a^4*(-1/sin(d*x+c)*cos(d*x+c)^4-(2+cos(d*x+c)
^2)*sin(d*x+c))+a^4*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 82, normalized size = 0.80

$$-\frac{3a^4\sin(dx+c)^4+16a^4\sin(dx+c)^3+30a^4\sin(dx+c)^2-60a^4\log(\sin(dx+c))+\frac{6(8a^4\sin(dx+c)+a^4)}{\sin(dx+c)^2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2
 - 60*a^4*log(sin(d*x + c)) + 6*(8*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^2)/
d
```

Fricas [A]
time = 0.38, size = 131, normalized size = 1.28

$$-\frac{24\,a^{4}\cos\left(dx+c\right)^{6}-312\,a^{4}\cos\left(dx+c\right)^{4}+423\,a^{4}\cos\left(dx+c\right)^{2}-183\,a^{4}-480\,\left(a^{4}\cos\left(dx+c\right)^{2}-a^{4}\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-128\,\left(a^{4}\cos\left(dx+c\right)^{4}-2\,a^{4}\cos\left(dx+c\right)^{2}+4\,a^{4}\right)\sin\left(dx+c\right)}{96\,\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/96*(24*a^4*cos(d*x + c)^6 - 312*a^4*cos(d*x + c)^4 + 423*a^4*cos(d*x + c
)^2 - 183*a^4 - 480*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*sin(d*x + c)) - 128*
(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*x + c)^2 + 4*a^4)*sin(d*x + c))/(d*cos(d*
x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{4}\left(\int 4\sin\left(c+dx\right)\cot^{3}\left(c+dx\right)\,dx+\int 6\sin^{2}\left(c+dx\right)\cot^{3}\left(c+dx\right)\,dx+\int 4\sin^{3}\left(c+dx\right)\cot^{3}\left(c+dx\right)\,dx+\int\sin^{4}\left(c+dx\right)\cot^{3}\left(c+dx\right)\,dx+\int\cot^{3}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(6*sin(c + d*x)
**2*cot(c + d*x)**3, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**3, x) +
Integral(sin(c + d*x)**4*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x)
)
```

Giac [A]
time = 15.68, size = 96, normalized size = 0.94

$$-\frac{3\,a^{4}\sin\left(dx+c\right)^{4}+16\,a^{4}\sin\left(dx+c\right)^{3}+30\,a^{4}\sin\left(dx+c\right)^{2}-60\,a^{4}\log\left(\left|\sin\left(dx+c\right)\right|\right)+\frac{6\left(15\,a^{4}\sin(dx+c)^{2}+8\,a^{4}\sin(dx+c)+a^{4}\right)}{\sin(dx+c)^{2}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2
 - 60*a^4*log(abs(sin(d*x + c))) + 6*(15*a^4*sin(d*x + c)^2 + 8*a^4*sin(d*x
 + c) + a^4)/sin(d*x + c)^2)/d
```

Mupad [B]
time = 6.40, size = 298, normalized size = 2.92

$$\frac{5\,a^{4}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^{4}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8\,d}-\frac{8\,a^{4}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\frac{81\,a^{4}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{2}+\frac{224\,a^{4}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{3}+98\,a^{4}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\frac{272\,a^{4}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{3}+43\,a^{4}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+32\,a^{4}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+2\,a^{4}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+8\,a^{4}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^{4}}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\right)}-\frac{2\,a^{4}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}-\frac{5\,a^{4}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + a*sin(c + d*x))^4,x)

[Out] (5*a^4*log(tan(c/2 + (d*x)/2)))/d - (a^4*tan(c/2 + (d*x)/2)^2)/(8*d) - (2*a
^4*tan(c/2 + (d*x)/2)^2 + 32*a^4*tan(c/2 + (d*x)/2)^3 + 43*a^4*tan(c/2 + (d
*x)/2)^4 + (272*a^4*tan(c/2 + (d*x)/2)^5)/3 + 98*a^4*tan(c/2 + (d*x)/2)^6 +
 (224*a^4*tan(c/2 + (d*x)/2)^7)/3 + (81*a^4*tan(c/2 + (d*x)/2)^8)/2 + 8*a^4
*tan(c/2 + (d*x)/2)^9 + a^4/2 + 8*a^4*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (
d*x)/2)^2 + 16*tan(c/2 + (d*x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2
+ (d*x)/2)^8 + 4*tan(c/2 + (d*x)/2)^10)) - (2*a^4*tan(c/2 + (d*x)/2))/d - (
5*a^4*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.38 $\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx$

Optimal. Leaf size=143

$$\frac{163a^4x}{8}-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{56a^4\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{35a^4\cos(c+dx}{8d}$$

```
[Out] 163/8*a^4*x-16*a^4*cos(d*x+c)/d+4/3*a^4*cos(d*x+c)^3/d+4/3*a^4*cos(d*x+c)/d
/(1-sin(d*x+c))^2-56/3*a^4*cos(d*x+c)/d/(1-sin(d*x+c))-35/8*a^4*cos(d*x+c)*
sin(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2788, 2729, 2727, 2718, 2715, 8, 2713}

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{16a^4\cos(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{35a^4\sin(c+dx)\cos(c+dx)}{8d}-\frac{56a^4\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{163a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (163*a^4*x)/8 - (16*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) + (4
*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (56*a^4*Cos[c + d*x])/(3*d*
(1 - Sin[c + d*x])) - (35*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c
 + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx &= a^4\int\left(16+\frac{4}{(-1+\sin(c+dx))^2}+\frac{20}{-1+\sin(c+dx)}+12\sin(c+
\right.\\
&= 16a^4x+a^4\int\sin^4(c+dx)\,dx+\left(4a^4\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+\\
&= 16a^4x-\frac{12a^4\cos(c+dx)}{d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{20a^4\cos(c+}{d(1-\sin(c+}\\
&= 20a^4x-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx)}\\
&= \frac{163a^4x}{8}-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx}{3d(1-\sin(c+dx}
\end{aligned}$$

Mathematica [A]
time = 1.08, size = 252, normalized size = 1.76

$$\frac{a^4\left(24(209+489c+489dx)\cos\left(\frac{1}{2}(c+dx)\right)-24(453+163c+163dx)\cos\left(\frac{3}{2}(c+dx)\right)+885\cos\left(\frac{5}{2}(c+dx)\right)-129\cos\left(\frac{7}{2}(c+dx)\right)-23\cos\left(\frac{9}{2}(c+dx)\right)+3\cos\left(\frac{11}{2}(c+dx)\right)-16488\sin\left(\frac{1}{2}(c+dx)\right)-11736c\sin\left(\frac{1}{2}(c+dx)\right)-11736dx\sin\left(\frac{1}{2}(c+dx)\right)+3704\sin\left(\frac{3}{2}(c+dx)\right)-3912c\sin\left(\frac{3}{2}(c+dx)\right)-3912dx\sin\left(\frac{3}{2}(c+dx)\right)+885\sin\left(\frac{5}{2}(c+dx)\right)+129\sin\left(\frac{7}{2}(c+dx)\right)-23\sin\left(\frac{9}{2}(c+dx)\right)-3\sin\left(\frac{11}{2}(c+dx)\right)\right)}{384d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (a^4*(24*(209 + 489*c + 489*d*x)*Cos[(c + d*x)/2] - 24*(453 + 163*c + 163*d
*x)*Cos[(3*(c + d*x))/2] + 885*Cos[(5*(c + d*x))/2] - 129*Cos[(7*(c + d*x))
/2] - 23*Cos[(9*(c + d*x))/2] + 3*Cos[(11*(c + d*x))/2] - 16488*Sin[(c + d*
x)/2] - 11736*c*Sin[(c + d*x)/2] - 11736*d*x*Sin[(c + d*x)/2] + 3704*Sin[(3
*(c + d*x))/2] - 3912*c*Sin[(3*(c + d*x))/2] - 3912*d*x*Sin[(3*(c + d*x))/2
] + 885*Sin[(5*(c + d*x))/2] + 129*Sin[(7*(c + d*x))/2] - 23*Sin[(9*(c + d*
x))/2] - 3*Sin[(11*(c + d*x))/2]))/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(131) = 262$.
time = 0.31, size = 360, normalized size = 2.52

method	result
risch	$\frac{163a^4x}{8} + \frac{9ia^4e^{2i(dx+c)}}{8d} - \frac{15a^4e^{i(dx+c)}}{2d} - \frac{15a^4e^{-i(dx+c)}}{2d} - \frac{9ia^4e^{-2i(dx+c)}}{8d} - \frac{8(-27ia^4e^{i(dx+c)}+15a^4e^{2i(dx+}}{3(e^{i(dx+c)}-i)^3d}$
derivativedivides	$a^4\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3} - \frac{2(\sin^9(dx+c))}{\cos(dx+c)} - 2\left(\sin^7(dx+c) + \frac{7(\sin^5(dx+c))}{6} + \frac{35(\sin^3(dx+c))}{24} + \frac{35\sin(dx+c)}{16}\right)\cos(dx+c) + \frac{35dx}{8} + \frac{35c}{8}\right.$
default	$a^4\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3} - \frac{2(\sin^9(dx+c))}{\cos(dx+c)} - 2\left(\sin^7(dx+c) + \frac{7(\sin^5(dx+c))}{6} + \frac{35(\sin^3(dx+c))}{24} + \frac{35\sin(dx+c)}{16}\right)\cos(dx+c) + \frac{35dx}{8} + \frac{35c}{8}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/3*sin(d*x+c)^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*
x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/
8*d*x+35/8*c)+4*a^4*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x
+c)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+6
*a^4*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*
x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+4*a^4*(1
/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*
sin(d*x+c)^2)*cos(d*x+c))+a^4*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.51, size = 238, normalized size = 1.66

$$\frac{32\left(\cos(dx+c)^3 - \frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3} - 9\cos(dx+c)\right)a^4 + \left(8\tan(dx+c)^3 + 105\,dx + 105\,c - \frac{3\left(13\tan(dx+c)^3+11\tan(dx+c)\right)}{\tan(dx+c)^4+2\tan(dx+c)^2+1} - 72\tan(dx+c)\right)a^4 + 24\left(2\tan(dx+c)^3 + 15\,dx + 15\,c - \frac{3\tan(dx+c)}{\tan(dx+c)^2+1} - 12\tan(dx+c)\right)a^4 + 8\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\tan(dx+c)\right)a^4 - 32\,a^4\left(\frac{6\cos(dx+c)^2-1}{\cos(dx+c)^3} + 3\cos(dx+c)\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/24*(32*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*
x + c))*a^4 + (8*tan(d*x + c)^3 + 105*d*x + 105*c - 3*(13*tan(d*x + c)^3 +
11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 72*tan(d*x + c))
*a^4 + 24*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^
2 + 1) - 12*tan(d*x + c))*a^4 + 8*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^4 - 32*a^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)
))/d
```

Fricas [**A**]
time = 0.35, size = 247, normalized size = 1.73

$$-\frac{6\,a^4\cos\left(dx+c\right)^6-20\,a^4\cos\left(dx+c\right)^5-85\,a^4\cos\left(dx+c\right)^4+214\,a^4\cos\left(dx+c\right)^3+978\,a^4dx+32\,a^4-\left(489\,a^4dx+721\,a^4\right)\cos\left(dx+c\right)^2+\left(489\,a^4dx-962\,a^4\right)\cos\left(dx+c\right)-\left(6\,a^4\cos\left(dx+c\right)^5+26\,a^4\cos\left(dx+c\right)^4-59\,a^4\cos\left(dx+c\right)^3+978\,a^4dx-273\,a^4\cos\left(dx+c\right)^2-32\,a^4+\left(489\,a^4dx-994\,a^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^6 - 20*a^4*cos(d*x + c)^5 - 85*a^4*cos(d*x + c)^4
 + 214*a^4*cos(d*x + c)^3 + 978*a^4*d*x + 32*a^4 - (489*a^4*d*x + 721*a^4)*
cos(d*x + c)^2 + (489*a^4*d*x - 962*a^4)*cos(d*x + c) - (6*a^4*cos(d*x + c)
^5 + 26*a^4*cos(d*x + c)^4 - 59*a^4*cos(d*x + c)^3 + 978*a^4*d*x - 273*a^4*
cos(d*x + c)^2 - 32*a^4 + (489*a^4*d*x - 994*a^4)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c)
 - 2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int 6\sin^2\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int 4\sin^3\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int\sin^4\left(c+dx\right)\tan^4\left(c+dx\right)dx+\int\tan^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**4,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*tan(c + d*x)**4, x) + Integral(6*sin(c + d*x)
**2*tan(c + d*x)**4, x) + Integral(4*sin(c + d*x)**3*tan(c + d*x)**4, x) +
Integral(sin(c + d*x)**4*tan(c + d*x)**4, x) + Integral(tan(c + d*x)**4, x)
)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 11.05, size = 437, normalized size = 3.06

$$\frac{163\,a^4\,x}{8}+\frac{\frac{163\,a^4\,\left(c+d\,x\right)}{8}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{489\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(1467\,c+1467\,d\,x-3630\right)}{24}\right)-\frac{a^4\,\left(489\,c+489\,d\,x-1536\right)}{24}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(\frac{489\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(1467\,c+1467\,d\,x-978\right)}{24}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(\frac{1141\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(3423\,c+3423\,d\,x-2934\right)}{24}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{1141\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(3423\,c+3423\,d\,x-7818\right)}{24}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{2119\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(6357\,c+6357\,d\,x-6520\right)}{24}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{2119\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(6357\,c+6357\,d\,x-13448\right)}{24}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(\frac{1467\,a^4\,\left(c+d\,x\right)}{4}-\frac{a^4\,\left(8802\,c+8802\,d\,x-11736\right)}{24}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{1467\,a^4\,\left(c+d\,x\right)}{4}-\frac{a^4\,\left(8802\,c+8802\,d\,x-15912\right)}{24}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{1793\,a^4\,\left(c+d\,x\right)}{4}-\frac{a^4\,\left(10758\,c+10758\,d\,x-15364\right)}{24}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{1793\,a^4\,\left(c+d\,x\right)}{4}-\frac{a^4\,\left(10758\,c+10758\,d\,x-18428\right)}{24}\right)}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (163*a^4*x)/8 + ((163*a^4*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((489*a^4*(c +
d*x))/8 - (a^4*(1467*c + 1467*d*x - 3630))/24) - (a^4*(489*c + 489*d*x - 15
36))/24 + tan(c/2 + (d*x)/2)^10*((489*a^4*(c + d*x))/8 - (a^4*(1467*c + 146
7*d*x - 978))/24) - tan(c/2 + (d*x)/2)^9*((1141*a^4*(c + d*x))/8 - (a^4*(34
23*c + 3423*d*x - 2934))/24) + tan(c/2 + (d*x)/2)^2*((1141*a^4*(c + d*x))/8
 - (a^4*(3423*c + 3423*d*x - 7818))/24) + tan(c/2 + (d*x)/2)^8*((2119*a^4*(
c + d*x))/8 - (a^4*(6357*c + 6357*d*x - 6520))/24) - tan(c/2 + (d*x)/2)^3*(
(2119*a^4*(c + d*x))/8 - (a^4*(6357*c + 6357*d*x - 13448))/24) - tan(c/2 +
(d*x)/2)^7*((1467*a^4*(c + d*x))/4 - (a^4*(8802*c + 8802*d*x - 11736))/24)
+ tan(c/2 + (d*x)/2)^4*((1467*a^4*(c + d*x))/4 - (a^4*(8802*c + 8802*d*x -
15912))/24) + tan(c/2 + (d*x)/2)^6*((1793*a^4*(c + d*x))/4 - (a^4*(10758*c
+ 10758*d*x - 15364))/24) - tan(c/2 + (d*x)/2)^5*((1793*a^4*(c + d*x))/4 -
(a^4*(10758*c + 10758*d*x - 18428))/24))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan
(c/2 + (d*x)/2)^2 + 1)^4)
```

3.39 $\int (a+a\sin(c+dx))^4\tan^2(c+dx)\,dx$

Optimal. Leaf size=113

$$-\frac{95a^4x}{8}+\frac{12a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{31a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos(c+$$

```
[Out] -95/8*a^4*x+12*a^4*cos(d*x+c)/d-4/3*a^4*cos(d*x+c)^3/d+8*a^4*cos(d*x+c)/d/(
1-sin(d*x+c))+31/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(d*x+c)*sin(d*x+c
)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {2788, 2727, 2718, 2715, 8, 2713}

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{12a^4\cos(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{31a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{95a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] (-95*a^4*x)/8 + (12*a^4*Cos[c + d*x])/d - (4*a^4*Cos[c + d*x]^3)/(3*d) + (8
*a^4*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (31*a^4*Cos[c + d*x]*Sin[c + d*
x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sin(c+dx))^4\tan^2(c+dx)\,dx &= a^2\int\left(-8a^2-\frac{8a^2}{-1+\sin(c+dx)}-8a^2\sin(c+dx)-7a^2\sin^2(c+\right.\\
&= -8a^4x-a^4\int\sin^4(c+dx)\,dx-\left(4a^4\right)\int\sin^3(c+dx)\,dx-\left(7a^4\right)\\
&= -8a^4x+\frac{8a^4\cos(c+dx)}{d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{7a^4\cos(c+dx)}{2d}\\
&= -\frac{23a^4x}{2}+\frac{12a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{8a^4\cos(c+d}{d(1-\sin(c+d}\\
&= -\frac{95a^4x}{8}+\frac{12a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{8a^4\cos(c+d}{d(1-\sin(c+d}
\end{aligned}
$$

Mathematica [A]
time = 0.71, size = 125, normalized size = 1.11

$$\frac{(a+a\sin(c+dx))^4\left(-1140(c+dx)+1056\cos(c+dx)-32\cos(3(c+dx))+\frac{1536\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+192\sin(2(c+dx))-3\sin(4(c+dx))\right)}{96d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^4*(-1140*(c + d*x) + 1056*Cos[c + d*x] - 32*Cos[3*(c
+ d*x)] + (1536*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 1
92*Sin[2*(c + d*x)] - 3*Sin[4*(c + d*x)]))/(96*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(105) = 210$.
time = 0.27, size = 231, normalized size = 2.04

method	result
risch	$-\frac{95a^4x}{8}+\frac{11a^4e^{i(dx+c)}}{2d}+\frac{11a^4e^{-i(dx+c)}}{2d}+\frac{16a^4}{d\left(e^{i(dx+c)}-i\right)}-\frac{a^4\sin(4dx+4c)}{32d}-\frac{a^4\cos(3dx+3c)}{3d}+\frac{2a^4\sin(2dx}{d}$
derivativedivides	$\frac{a^4\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+4a^4\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c\right.\right.}{}$
default	$\frac{a^4\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+4a^4\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d
*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+4*a^4*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(
d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+6*a^4*(sin(d*x+c)^5/cos(d*x+c)+(sin(
d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+4*a^4*(sin(d*x+c)^4/cos(
d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a^4*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.53, size = 181, normalized size = 1.60

$$-\frac{32\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\cos(dx+c)\right)a^4+3\left(15\,dx+15\,c-\frac{9\tan(dx+c)^3+7\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}-8\tan(dx+c)\right)a^4+72\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan(dx+c)\right)a^4+24\,(dx+c-\tan(dx+c))a^4-96\,a^4\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(32*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^4 + 3*(15*d*
x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x
+ c)^2 + 1) - 8*tan(d*x + c))*a^4 + 72*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x
 + c)^2 + 1) - 2*tan(d*x + c))*a^4 + 24*(d*x + c - tan(d*x + c))*a^4 - 96*a
^4*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.34, size = 179, normalized size = 1.58

$$-\frac{6\,a^4\cos(dx+c)^5+32\,a^4\cos(dx+c)^4-73\,a^4\cos(dx+c)^3+285\,a^4dx-288\,a^4\cos(dx+c)^2-192\,a^4+3\,(95\,a^4dx-127\,a^4)\cos(dx+c)+(6\,a^4\cos(dx+c)^4-26\,a^4\cos(dx+c)^3-285\,a^4dx-99\,a^4\cos(dx+c)^2+189\,a^4\cos(dx+c)-192\,a^4)\sin(dx+c)}{24\,(d\cos(dx+c)-d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^5 + 32*a^4*cos(d*x + c)^4 - 73*a^4*cos(d*x + c)^3
 + 285*a^4*d*x - 288*a^4*cos(d*x + c)^2 - 192*a^4 + 3*(95*a^4*d*x - 127*a^4
)*cos(d*x + c) + (6*a^4*cos(d*x + c)^4 - 26*a^4*cos(d*x + c)^3 - 285*a^4*d*
x - 99*a^4*cos(d*x + c)^2 + 189*a^4*cos(d*x + c) - 192*a^4)*sin(d*x + c))/(
d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin(c+dx)\tan^2(c+dx)\,dx+\int 6\sin^2(c+dx)\tan^2(c+dx)\,dx+\int 4\sin^3(c+dx)\tan^2(c+dx)\,dx+\int \sin^4(c+dx)\tan^2(c+dx)\,dx+\int \tan^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**2,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*tan(c + d*x)**2, x) + Integral(6*sin(c + d*x)
**2*tan(c + d*x)**2, x) + Integral(4*sin(c + d*x)**3*tan(c + d*x)**2, x) +
Integral(sin(c + d*x)**4*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x)
)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 10.26, size = 363, normalized size = 3.21

$$-\frac{95a^4x}{8}-\frac{\frac{95a^4(c+dx)}{8}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{95a^4(c+dx)}{8}-\frac{a^4(285c+285dx-326)}{24}\right)-\frac{a^4(285c+285dx-896)}{24}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\frac{95a^4(c+dx)}{8}-\frac{a^4(285c+285dx-570)}{24}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{95a^4(c+dx)}{2}-\frac{a^4(1140c+1140dx-570)}{24}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{95a^4(c+dx)}{2}-\frac{a^4(1140c+1140dx-1430)}{24}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{95a^4(c+dx)}{2}-\frac{a^4(1140c+1140dx-2154)}{24}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{95a^4(c+dx)}{2}-\frac{a^4(1140c+1140dx-3014)}{24}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{285a^4(c+dx)}{4}-\frac{a^4(1710c+1710dx-1770)}{24}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{285a^4(c+dx)}{4}-\frac{a^4(1710c+1710dx-3606)}{24}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + a*sin(c + d*x))^4,x)
```

```
[Out] - (95*a^4*x)/8 - ((95*a^4*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((95*a^4*(c + d
*x))/8 - (a^4*(285*c + 285*d*x - 326))/24) - (a^4*(285*c + 285*d*x - 896))/
24 + tan(c/2 + (d*x)/2)^8*((95*a^4*(c + d*x))/8 - (a^4*(285*c + 285*d*x - 5
70))/24) - tan(c/2 + (d*x)/2)^7*((95*a^4*(c + d*x))/2 - (a^4*(1140*c + 1140
*d*x - 570))/24) - tan(c/2 + (d*x)/2)^3*((95*a^4*(c + d*x))/2 - (a^4*(1140*
c + 1140*d*x - 1430))/24) + tan(c/2 + (d*x)/2)^6*((95*a^4*(c + d*x))/2 - (a
^4*(1140*c + 1140*d*x - 2154))/24) + tan(c/2 + (d*x)/2)^2*((95*a^4*(c + d*x
))/2 - (a^4*(1140*c + 1140*d*x - 3014))/24) - tan(c/2 + (d*x)/2)^5*((285*a^
4*(c + d*x))/4 - (a^4*(1710*c + 1710*d*x - 1770))/24) + tan(c/2 + (d*x)/2)^
4*((285*a^4*(c + d*x))/4 - (a^4*(1710*c + 1710*d*x - 3606))/24))/(d*(tan(c/
2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.40 $\int (a + a\sin(c + dx))^4 \, dx$

Optimal. Leaf size=87

$$\frac{35a^4x}{8} - \frac{8a^4\cos(c+dx)}{d} + \frac{4a^4\cos^3(c+dx)}{3d} - \frac{27a^4\cos(c+dx)\sin(c+dx)}{8d} - \frac{a^4\cos(c+dx)\sin^3(c+dx)}{4d}$$

```
[Out] 35/8*a^4*x-8*a^4*cos(d*x+c)/d+4/3*a^4*cos(d*x+c)^3/d-27/8*a^4*cos(d*x+c)*si
n(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2724, 2718, 2715, 8, 2713}

$$\frac{4a^4\cos^3(c+dx)}{3d} - \frac{8a^4\cos(c+dx)}{d} - \frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{27a^4\sin(c+dx)\cos(c+dx)}{8d} + \frac{35a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (35*a^4*x)/8 - (8*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) - (27*
a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\,dx = \int \left(a^4+4a^4\sin(c+dx)+6a^4\sin^2(c+dx)+4a^4\sin^3(c+dx)+a^4\sin^4(c+dx)\right.$$
$$= a^4x+a^4\int \sin^4(c+dx)\,dx+\left(4a^4\right)\int \sin(c+dx)\,dx+\left(4a^4\right)\int \sin^3(c+dx)\,dx$$
$$= a^4x-\frac{4a^4\cos(c+dx)}{d}-\frac{3a^4\cos(c+dx)\sin(c+dx)}{d}-\frac{a^4\cos(c+dx)\sin^3(c+}{4d}$$
$$= 4a^4x-\frac{8a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}-$$
$$= \frac{35a^4x}{8}-\frac{8a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}-$$

Mathematica [A]
time = 0.27, size = 57, normalized size = 0.66

$$\frac{a^4(-672\cos(c+dx)+32\cos(3(c+dx))+3(140c+140dx-56\sin(2(c+dx))+\sin(4(c+dx))))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4,x]
[Out] (a^4*(-672*Cos[c + d*x] + 32*Cos[3*(c + d*x)] + 3*(140*c + 140*d*x - 56*Sin
[2*(c + d*x)] + Sin[4*(c + d*x)])))/(96*d)
```

Maple [A]
time = 0.21, size = 111, normalized size = 1.28

method	result
risch	$\frac{35a^4x}{8}-\frac{7a^4\cos(dx+c)}{d}+\frac{a^4\sin(4dx+4c)}{32d}+\frac{a^4\cos(3dx+3c)}{3d}-\frac{7a^4\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{4a^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+6a^4\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$
default	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{4a^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+6a^4\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$

norman	$\frac{\frac{35a^4x}{8}-\frac{40a^4}{3d}-\frac{27a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{35a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{35a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{27a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{35a^4x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+}{(1+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-4/3*
a^4*(2+sin(d*x+c)^2)*cos(d*x+c)+6*a^4*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1
/2*c)-4*a^4*cos(d*x+c)+a^4*(d*x+c))
```

Maxima [A]
time = 0.28, size = 108, normalized size = 1.24

$$a^4x+\frac{4\left(\cos\left(dx+c\right)^3-3\cos\left(dx+c\right)\right)a^4}{3\,d}+\frac{\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\sin\left(2\,dx+2\,c\right)\right)a^4}{32\,d}+\frac{3\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)a^4}{2\,d}-\frac{4\,a^4\cos\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 4/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^4/d + 1/32*(12*d*x + 12*c +
 sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^4/d + 3/2*(2*d*x + 2*c - sin(2*d*
x + 2*c))*a^4/d - 4*a^4*cos(d*x + c)/d
```

Fricas [A]
time = 0.35, size = 70, normalized size = 0.80

$$\frac{32\,a^4\cos\left(dx+c\right)^3+105\,a^4dx-192\,a^4\cos\left(dx+c\right)+3\left(2\,a^4\cos\left(dx+c\right)^3-29\,a^4\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(32*a^4*cos(d*x + c)^3 + 105*a^4*d*x - 192*a^4*cos(d*x + c) + 3*(2*a^4
*cos(d*x + c)^3 - 29*a^4*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(82) = 164$.
time = 0.23, size = 224, normalized size = 2.57

$$\begin{cases}\frac{3a^4x\sin^4\left(c+dx\right)}{8}+\frac{3a^4x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+3a^4x\sin^2\left(c+dx\right)+\frac{3a^4x\cos^4\left(c+dx\right)}{8}+3a^4x\cos^2\left(c+dx\right)+a^4x-\frac{5a^4\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{4a^4\sin^2\left(c+dx\right)\cos\left(c+dx\right)}{d}-\frac{3a^4\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d}-\frac{3a^4\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}-\frac{8a^4\cos^3\left(c+dx\right)}{3d}-\frac{4a^4\cos\left(c+dx\right)}{d} & \text{for}\: d\neq 0\\x\left(a\sin\left(c\right)+a\right)^4 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*x*
```

```
cos(c + d*x)**2 + a**4*x - 5*a**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 4*a*
*4*sin(c + d*x)**2*cos(c + d*x)/d - 3*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*
d) - 3*a**4*sin(c + d*x)*cos(c + d*x)/d - 8*a**4*cos(c + d*x)**3/(3*d) - 4*
a**4*cos(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**4, True))
```

Giac [A]
time = 3.68, size = 72, normalized size = 0.83

$$\frac{35}{8}\,a^4x+\frac{a^4\cos\left(3\,dx+3\,c\right)}{3\,d}-\frac{7\,a^4\cos\left(dx+c\right)}{d}+\frac{a^4\sin\left(4\,dx+4\,c\right)}{32\,d}-\frac{7\,a^4\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 35/8*a^4*x + 1/3*a^4*cos(3*d*x + 3*c)/d - 7*a^4*cos(d*x + c)/d + 1/32*a^4*s
in(4*d*x + 4*c)/d - 7/4*a^4*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 8.59, size = 237, normalized size = 2.72

$$\frac{35\,a^4\,x}{8}-\frac{\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{27\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{a^4\,\left(105\,c+105\,d\,x\right)}{24}-\frac{a^4\,\left(105\,c+105\,d\,x-320\right)}{24}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{a^4\,\left(105\,c+105\,d\,x\right)}{6}-\frac{a^4\,\left(420\,c+420\,d\,x-192\right)}{24}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^4\,\left(105\,c+105\,d\,x\right)}{6}-\frac{a^4\,\left(420\,c+420\,d\,x-1088\right)}{24}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^4\,\left(105\,c+105\,d\,x\right)}{4}-\frac{a^4\,\left(630\,c+630\,d\,x-960\right)}{24}\right)+\frac{27\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4,x)
```

```
[Out] (35*a^4*x)/8 - ((35*a^4*tan(c/2 + (d*x)/2)^3)/4 - (35*a^4*tan(c/2 + (d*x)/2
)^5)/4 - (27*a^4*tan(c/2 + (d*x)/2)^7)/4 + (a^4*(105*c + 105*d*x))/24 - (a^
4*(105*c + 105*d*x - 320))/24 + tan(c/2 + (d*x)/2)^6*((a^4*(105*c + 105*d*x
))/6 - (a^4*(420*c + 420*d*x - 192))/24) + tan(c/2 + (d*x)/2)^2*((a^4*(105*
c + 105*d*x))/6 - (a^4*(420*c + 420*d*x - 1088))/24) + tan(c/2 + (d*x)/2)^4
*((a^4*(105*c + 105*d*x))/4 - (a^4*(630*c + 630*d*x - 960))/24) + (27*a^4*t
an(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.41 $\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=116

$$\frac{17a^4x}{8}-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot(c+dx)}{d}+\frac{23a^4\cos(c+dx)\sin(c}{8d}$$

```
[Out] 17/8*a^4*x-4*a^4*arctanh(cos(d*x+c))/d+4*a^4*cos(d*x+c)/d-4/3*a^4*cos(d*x+c
)^3/d-a^4*cot(d*x+c)/d+23/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(d*x+c)*
sin(d*x+c)^3/d
```

Rubi [A]

time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2788, 3855, 3852, 8, 2715, 2713}

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{23a^4\sin(c+dx)\cos(c+dx)}{8d}-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{17a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (17*a^4*x)/8 - (4*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x])/d - (
4*a^4*Cos[c + d*x]^3)/(3*d) - (a^4*Cot[c + d*x])/d + (23*a^4*Cos[c + d*x]*S
in[c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
```

```
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\int (5a^6+4a^6\csc(c+dx)+a^6\csc^2(c+dx)-5a^6\sin^2(c+dx)-4a}{a^2} \\
&= 5a^4x+a^4\int \csc^2(c+dx)\,dx-a^4\int \sin^4(c+dx)\,dx+\left(4a^4\right)\int \csc \\
&= 5a^4x-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{5a^4\cos(c+dx)\sin(c+dx)}{2d}+ \\
&= \frac{5a^4x}{2}-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+}{3d} \\
&= \frac{17a^4x}{8}-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c}{3d}
\end{aligned}
$$

Mathematica [A]
time = 1.02, size = 136, normalized size = 1.17

$$\frac{a^4\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-48\cos(c+dx)-147\cos(3(c+dx))+3\cos(5(c+dx))+408c\sin(c+dx)+408dx\sin(c+dx)-768\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+768\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+320\sin(2(c+dx))-32\sin(4(c+dx))\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-48*Cos[c + d*x] - 147*Cos[3*(c + d
*x)] + 3*Cos[5*(c + d*x)] + 408*c*Sin[c + d*x] + 408*d*x*Sin[c + d*x] - 768
*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 768*Log[Sin[(c + d*x)/2]]*Sin[c + d*x
] + 320*Sin[2*(c + d*x)] - 32*Sin[4*(c + d*x)]))/(384*d)
```

Maple [A]
time = 0.13, size = 136, normalized size = 1.17

method	result
derivativedivides	$\frac{a^4\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{4a^4\left(\cos^3(dx+c)\right)}{3}+6a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^4(\mathrm{co}}{d}$
default	$\frac{a^4\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{4a^4\left(\cos^3(dx+c)\right)}{3}+6a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^4(\mathrm{co}}{d}$
risch	$\frac{17a^4x}{8}-\frac{3ia^4\mathrm{e}^{2i(dx+c)}}{4d}+\frac{3a^4\mathrm{e}^{i(dx+c)}}{2d}+\frac{3a^4\mathrm{e}^{-i(dx+c)}}{2d}+\frac{3ia^4\mathrm{e}^{-2i(dx+c)}}{4d}-\frac{2ia^4}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{4a^4\ln\left(\mathrm{e}^{i(dx+c)}+\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-4/3*a^4*cos(d*x+c)^3+6*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4
*a^4*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a^4*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.51, size = 117, normalized size = 1.01

$$-\frac{128\,a^4\cos\left(dx+c\right)^3-3\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^4-144\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^4+96\left(dx+c+\frac{1}{\tan\left(dx+c\right)}\right)a^4-192\,a^4\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/96*(128*a^4*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 14
4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 96*(d*x + c + 1/tan(d*x + c))*a^4
- 192*a^4*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))
/d
```

Fricas [A]
time = 0.37, size = 135, normalized size = 1.16

$$\frac{6\,a^4\cos\left(dx+c\right)^5-81\,a^4\cos\left(dx+c\right)^3-48\,a^4\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+48\,a^4\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+51\,a^4\cos\left(dx+c\right)-\left(32\,a^4\cos\left(dx+c\right)^3-51\,a^4dx-96\,a^4\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^4*cos(d*x + c)^5 - 81*a^4*cos(d*x + c)^3 - 48*a^4*log(1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) + 48*a^4*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 51*a^4*cos(d*x + c) - (32*a^4*cos(d*x + c)^3 - 51*a^4*d*x - 96*a^4*cos(
d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int 6\sin^2\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int 4\sin^3\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int \sin^4\left(c+dx\right)\cot^2\left(c+dx\right)dx+\int \cot^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(6*sin(c + d*x)
**2*cot(c + d*x)**2, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**2, x) +
Integral(sin(c + d*x)**4*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x)
)
```

Giac [A]
time = 8.12, size = 194, normalized size = 1.67

$$\frac{51\,(dx+c)a^4+96\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{12\,\left(8\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\,\left(69\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+93\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-192\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-93\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-256\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-69\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-64\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(51*(d*x + c)*a^4 + 96*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^4*tan
(1/2*d*x + 1/2*c) - 12*(8*a^4*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2
*c) - 2*(69*a^4*tan(1/2*d*x + 1/2*c)^7 + 93*a^4*tan(1/2*d*x + 1/2*c)^5 - 19
2*a^4*tan(1/2*d*x + 1/2*c)^4 - 93*a^4*tan(1/2*d*x + 1/2*c)^3 - 256*a^4*tan(
1/2*d*x + 1/2*c)^2 - 69*a^4*tan(1/2*d*x + 1/2*c) - 64*a^4)/(tan(1/2*d*x + 1
/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 6.78, size = 295, normalized size = 2.54

$$\frac{4\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{17\,a^4\,\mathrm{atan}\left(\frac{289\,a^8}{16\,\left(34\,a^8-\frac{289\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{34\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{34\,a^8-\frac{289\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}}\right)}{4\,d}+\frac{-\frac{25\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}-\frac{39\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+32\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{19\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\frac{128\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{15\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{32\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-a^4}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (4*a^4*log(tan(c/2 + (d*x)/2)))/d + (17*a^4*atan((289*a^8)/(16*(34*a^8 - (2
89*a^8*tan(c/2 + (d*x)/2))/16)) + (34*a^8*tan(c/2 + (d*x)/2))/(34*a^8 - (28
9*a^8*tan(c/2 + (d*x)/2))/16)))/(4*d) + ((15*a^4*tan(c/2 + (d*x)/2)^2)/2 +
(128*a^4*tan(c/2 + (d*x)/2)^3)/3 + (19*a^4*tan(c/2 + (d*x)/2)^4)/2 + 32*a^4
*tan(c/2 + (d*x)/2)^5 - (39*a^4*tan(c/2 + (d*x)/2)^6)/2 - (25*a^4*tan(c/2 +
 (d*x)/2)^8)/2 - a^4 + (32*a^4*tan(c/2 + (d*x)/2))/3)/(d*(2*tan(c/2 + (d*x)
/2) + 8*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/
2)^7 + 2*tan(c/2 + (d*x)/2)^9)) + (a^4*tan(c/2 + (d*x)/2))/(2*d)
```

3.42 $\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=140

$$-\frac{61a^4x}{8}+\frac{2a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{2a^4\cot(c+dx)\csc$$

```
[Out] -61/8*a^4*x+2*a^4*arctanh(cos(d*x+c))/d+4/3*a^4*cos(d*x+c)^3/d-5*a^4*cot(d*
x+c)/d-1/3*a^4*cot(d*x+c)^3/d-2*a^4*cot(d*x+c)*csc(d*x+c)/d-19/8*a^4*cos(d*
x+c)*sin(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2788, 3855, 3852, 8, 3853, 2718, 2715, 2713}

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{19a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{2a^4\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^4\cot(c+dx)\csc(c+dx)}{d}-\frac{61a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-61*a^4*x)/8 + (2*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x]^3)/(3
*d) - (5*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]^3)/(3*d) - (2*a^4*Cot[c +
d*x]*Csc[c + d*x])/d - (19*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int\left(-10a^8-4a^8\csc(c+dx)+4a^8\csc^2(c+dx)+4a^8\csc^3(c+dx)\right.}{}$$

$$= -10a^4x+a^4\int\csc^4(c+dx)\,dx+a^4\int\sin^4(c+dx)\,dx-\left(4a^4\right)\int$$

$$= -10a^4x+\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos(c+dx)}{d}-\frac{2a^4\cot(c}{}$$

$$= -8a^4x+\frac{2a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{5a^4\cot(c}{d}$$

$$= -\frac{61a^4x}{8}+\frac{2a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{5a^4\cot(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 685 vs. 2(140) = 280.
time = 6.23, size = 685, normalized size = 4.89

$$-\frac{61(c+dx)(a+a\sin(c+dx))^4}{8d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{\cos(c+dx)(a+a\sin(c+dx))^4}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{\cos(3(c+dx))(a+a\sin(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}-\frac{7\cot\left(\frac{1}{2}(c+dx)\right)(a+a\sin(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)(a+a\sin(c+dx))^4}{2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+a\sin(c+dx))^4}{24d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+a\sin(c+dx))^4}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}-\frac{2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+a\sin(c+dx))^4}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sin(c+dx))^4}{2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}-\frac{5(a+a\sin(c+dx))^4\sin(2(c+dx))}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{(a+a\sin(c+dx))^4\sin(4(c+dx))}{32d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{7(a+a\sin(c+dx))^4\tan\left(\frac{1}{2}(c+dx)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sin(c+dx))^4\tan\left(\frac{1}{2}(c+dx)\right)}{24d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-61*(c + d*x)*(a + a*Sin[c + d*x])^4)/(8*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^8) + (Cos[c + d*x]*(a + a*Sin[c + d*x])^4)/(d*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^8) + (Cos[3*(c + d*x)]*(a + a*Sin[c + d*x])^4)/(3*d*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^8) - (7*Cot[(c + d*x)/2]*(a + a*Sin[c + d*x]
)^4)/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) - (Csc[(c + d*x)/2]^2*(a
 + a*Sin[c + d*x])^4)/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) - (Cot[
(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + a*Sin[c + d*x])^4)/(24*d*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^8) + (2*Log[Cos[(c + d*x)/2]]*(a + a*Sin[c + d*x])
^4)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) - (2*Log[Sin[(c + d*x)/2]]*
(a + a*Sin[c + d*x])^4)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) + (Sec[
(c + d*x)/2]^2*(a + a*Sin[c + d*x])^4)/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^8) - (5*(a + a*Sin[c + d*x])^4*Sin[2*(c + d*x)])/(4*d*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^8) + ((a + a*Sin[c + d*x])^4*Sin[4*(c + d*x)])/(32*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) + (7*(a + a*Sin[c + d*x])^4*Tan[
(c + d*x)/2])/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) + (Sec[(c + d*x
)/2]^2*(a + a*Sin[c + d*x])^4*Tan[(c + d*x)/2])/(24*d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^8)
```

Maple [A]
time = 0.23, size = 222, normalized size = 1.59

method	result
derivativedivides	$\frac{a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4a^4\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+6a^4\left(\right.}{}$
default	$\frac{a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4a^4\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+6a^4\left(\right.}{}$
risch	$-\frac{61a^4x}{8}-\frac{ia^4e^{4i(dx+c)}}{64d}+\frac{5ia^4e^{2i(dx+c)}}{8d}+\frac{a^4e^{i(dx+c)}}{2d}+\frac{a^4e^{-i(dx+c)}}{2d}-\frac{5ia^4e^{-2i(dx+c)}}{8d}+\frac{ia^4e^{-4i(dx+c)}}{64d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*a^4
*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+6*a^4*(-1/sin(d*x+
c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)-3/2*d*x-3/2*c)+4*a
^4*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(c
sc(d*x+c)-cot(d*x+c)))+a^4*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.50, size = 218, normalized size = 1.56

$$\frac{64\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a^4+3\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)a^4-288\left(3dx+3c+\frac{3\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^4+32\left(3dx+3c+\frac{3\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^4+96a^4\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-4\cos(dx+c)+3\log(\cos(dx+c)+1)-3\log(\cos(dx+c)-1)\right)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/96*(64*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*a^4 - 288*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3
+ tan(d*x + c)))*a^4 + 32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c
)^3)*a^4 + 96*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3
*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.39, size = 219, normalized size = 1.56

$$-\frac{6\,a^4\cos\left(dx+c\right)^7-75\,a^4\cos\left(dx+c\right)^5+244\,a^4\cos\left(dx+c\right)^3-183\,a^4\cos\left(dx+c\right)-24\,\left(a^4\cos\left(dx+c\right)^2-a^4\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+24\,\left(a^4\cos\left(dx+c\right)^2-a^4\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-\left(32\,a^4\cos\left(dx+c\right)^5-183\,a^4dx\cos\left(dx+c\right)^2-32\,a^4\cos\left(dx+c\right)^3+183\,a^4dx+48\,a^4\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^7 - 75*a^4*cos(d*x + c)^5 + 244*a^4*cos(d*x + c)^
3 - 183*a^4*cos(d*x + c) - 24*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) + 24*(a^4*cos(d*x + c)^2 - a^4)*log(-1/2*cos(d*x + c
) + 1/2)*sin(d*x + c) - (32*a^4*cos(d*x + c)^5 - 183*a^4*d*x*cos(d*x + c)^2
 - 32*a^4*cos(d*x + c)^3 + 183*a^4*d*x + 48*a^4*cos(d*x + c))*sin(d*x + c))
/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int 4\sin\left(c+dx\right)\cot^4\left(c+dx\right)\,dx+\int 6\sin^2\left(c+dx\right)\cot^4\left(c+dx\right)\,dx+\int 4\sin^3\left(c+dx\right)\cot^4\left(c+dx\right)\,dx+\int \sin^4\left(c+dx\right)\cot^4\left(c+dx\right)\,dx+\int \cot^4\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**4, x) + Integral(6*sin(c + d*x)
**2*cot(c + d*x)**4, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**4, x) +
Integral(sin(c + d*x)**4*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4, x)
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(130) = 260$.
time = 12.88, size = 274, normalized size = 1.96

$$\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-183\,(dx+c)a^4-48\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+57\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{88\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-57\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{2\left(57\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+96\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+81\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+96\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-81\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+32\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-57\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+32\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*tan(1/2*d*x + 1/2*c)^2 - 183*(d*x
 + c)*a^4 - 48*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 57*a^4*tan(1/2*d*x + 1/
2*c) + (88*a^4*tan(1/2*d*x + 1/2*c)^3 - 57*a^4*tan(1/2*d*x + 1/2*c)^2 - 12*
a^4*tan(1/2*d*x + 1/2*c) - a^4)/tan(1/2*d*x + 1/2*c)^3 + 2*(57*a^4*tan(1/2*
d*x + 1/2*c)^7 + 96*a^4*tan(1/2*d*x + 1/2*c)^6 + 81*a^4*tan(1/2*d*x + 1/2*c
)^5 + 96*a^4*tan(1/2*d*x + 1/2*c)^4 - 81*a^4*tan(1/2*d*x + 1/2*c)^3 + 32*a^
4*tan(1/2*d*x + 1/2*c)^2 - 57*a^4*tan(1/2*d*x + 1/2*c) + 32*a^4)/(tan(1/2*d
*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 6.71, size = 384, normalized size = 2.74

$$\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2\,d}+\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d}-\frac{2\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{61\,a^4\,\mathrm{atan}\left(\frac{3721\,a^8}{16\left(61\,a^8-\frac{3721\,a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}\right)}+\frac{61\,a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{61\,a^8-\frac{3721\,a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}}\right)}{4\,d}+\frac{19\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,d}-\frac{-19\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-60\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\frac{67\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{3}-48\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{508\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{8\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3}+116\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\frac{16\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{61\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+4\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a^4}{3}}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^2)/(2*d) + (a^4*tan(c/2 + (d*x)/2)^3)/(24*d) - (2*a
^4*log(tan(c/2 + (d*x)/2)))/d - (61*a^4*atan((3721*a^8)/(16*(61*a^8 - (3721
*a^8*tan(c/2 + (d*x)/2))/16)) + (61*a^8*tan(c/2 + (d*x)/2))/(61*a^8 - (3721
*a^8*tan(c/2 + (d*x)/2))/16)))/(4*d) + (19*a^4*tan(c/2 + (d*x)/2))/(8*d) -
((61*a^4*tan(c/2 + (d*x)/2)^2)/3 - (16*a^4*tan(c/2 + (d*x)/2)^3)/3 + 116*a^
4*tan(c/2 + (d*x)/2)^4 + (8*a^4*tan(c/2 + (d*x)/2)^5)/3 + (508*a^4*tan(c/2
+ (d*x)/2)^6)/3 - 48*a^4*tan(c/2 + (d*x)/2)^7 + (67*a^4*tan(c/2 + (d*x)/2)^
8)/3 - 60*a^4*tan(c/2 + (d*x)/2)^9 - 19*a^4*tan(c/2 + (d*x)/2)^10 + a^4/3 +
 4*a^4*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 32*tan(c/2 + (d*x)/
2)^5 + 48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9 + 8*tan(c/2 + (d*x
)/2)^11))
```

3.43 $\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=198

$$\frac{97a^4x}{8}+\frac{5a^4\tanh^{-1}(\cos(c+dx))}{2d}-\frac{4a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{10a^4\cot(c+dx)}{d}-\frac{5a^4\cot^3(c+dx)}{3d}-$$

```
[Out] 97/8*a^4*x+5/2*a^4*arctanh(cos(d*x+c))/d-4*a^4*cos(d*x+c)/d-4/3*a^4*cos(d*x
+c)^3/d+10*a^4*cot(d*x+c)/d-5/3*a^4*cot(d*x+c)^3/d-1/5*a^4*cot(d*x+c)^5/d+5
/2*a^4*cot(d*x+c)*csc(d*x+c)/d-a^4*cot(d*x+c)*csc(d*x+c)^3/d+15/8*a^4*cos(d
*x+c)*sin(d*x+c)/d+1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.29, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2788, 3852, 8, 3853, 3855, 2718, 2715, 2713}

$$-\frac{4a^4\cos^3(c+dx)}{3d}-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot^5(c+dx)}{5d}-\frac{5a^4\cot^3(c+dx)}{3d}+\frac{10a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{15a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{5a^4\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^4\cot(c+dx)\csc^3(c+dx)}{d}+\frac{5a^4\cot(c+dx)\csc(c+dx)}{2d}+\frac{97a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (97*a^4*x)/8 + (5*a^4*ArcTanh[Cos[c + d*x]])/(2*d) - (4*a^4*Cos[c + d*x])/d
 - (4*a^4*Cos[c + d*x]^3)/(3*d) + (10*a^4*Cot[c + d*x])/d - (5*a^4*Cot[c +
d*x]^3)/(3*d) - (a^4*Cot[c + d*x]^5)/(5*d) + (5*a^4*Cot[c + d*x]*Csc[c + d*
x])/(2*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/d + (15*a^4*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(14a^{10} - 14a^{10}\csc^2(c+dx) - 8a^{10}\csc^3(c+dx) + 3a^{10}\csc^4(c+dx\right.}{}$$

$$= 14a^4x + a^4\int \csc^6(c+dx)\,dx - a^4\int \sin^4(c+dx)\,dx + \left(3a^4\right)\int \csc$$

$$= 14a^4x - \frac{8a^4\cos(c+dx)}{d} + \frac{4a^4\cot(c+dx)\csc(c+dx)}{d} - \frac{a^4\cot(c+}{}$$

$$= \frac{25a^4x}{2} + \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+}{3d}$$

$$= \frac{97a^4x}{8} + \frac{5a^4\tanh^{-1}(\cos(c+dx))}{2d} - \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+}{3d}$$

Mathematica [A]
time = 1.07, size = 283, normalized size = 1.43

$$\frac{a^4(1+\sin(c+dx))^4\left(5820(c+dx)-2400\cos(c+dx)-160\cos(3(c+dx))+2752\cot\left(\frac{1}{2}(c+dx)\right)+300\csc^2\left(\frac{1}{2}(c+dx)\right)-30\csc^4\left(\frac{1}{2}(c+dx)\right)+1200\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-1200\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-300\sec^2\left(\frac{1}{2}(c+dx)\right)+30\sec^4\left(\frac{1}{2}(c+dx)\right)+632\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+96\csc^5(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)-\frac{79}{2}\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-\frac{3}{2}\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)+480\sin(2(c+dx))-15\sin(4(c+dx))-2752\tan\left(\frac{1}{2}(c+dx)\right)\right)}{480d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(5820*(c + d*x) - 2400*Cos[c + d*x] - 160*Cos[3*(
c + d*x)] + 2752*Cot[(c + d*x)/2] + 300*Csc[(c + d*x)/2]^2 - 30*Csc[(c + d*
x)/2]^4 + 1200*Log[Cos[(c + d*x)/2]] - 1200*Log[Sin[(c + d*x)/2]] - 300*Sec
[(c + d*x)/2]^2 + 30*Sec[(c + d*x)/2]^4 + 632*Csc[c + d*x]^3*Sin[(c + d*x)/
2]^4 + 96*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - (79*Csc[(c + d*x)/2]^4*Sin[c
+ d*x])/2 - (3*Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 + 480*Sin[2*(c + d*x)] -
15*Sin[4*(c + d*x)] - 2752*Tan[(c + d*x)/2]))/(480*d*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^8)
```

Maple [A]
time = 0.26, size = 353, normalized size = 1.78

method	result
risch	$\frac{97a^4x}{8}+\frac{ia^4e^{4i(dx+c)}}{64d}-\frac{a^4e^{3i(dx+c)}}{6d}-\frac{ia^4e^{2i(dx+c)}}{2d}-\frac{5a^4e^{i(dx+c)}}{2d}-\frac{5a^4e^{-i(dx+c)}}{2d}+\frac{ia^4e^{-2i(dx+c)}}{2d}-\frac{a^4e}{}$
derivativedivides	$a^4\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+4a^4\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^5(d\right.}{2}\right.$
default	$a^4\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+4a^4\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^5(d\right.}{2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/sin(d*x+c)*cos(d*x+c)^7-(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*co
s(d*x+c))*sin(d*x+c)-15/8*d*x-15/8*c)+4*a^4*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7
-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*cos(d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x
+c)))+6*a^4*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3
*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+
4*a^4*(-1/4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos
(d*x+c)^5+5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))+
a^4*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.53, size = 313, normalized size = 1.58

$$-\frac{40\left(4\cos\left(dx+c\right)^{3}-\frac{6\cos(dx+c)}{\cos(dx+c)^{2}-1}+24\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^{4}+15\left(15\,dx+15\,c+\frac{15\tan(dx+c)^{4}+25\tan(dx+c)^{2}+8}{\tan(dx+c)^{5}+2\tan(dx+c)^{3}+\tan(dx+c)}\right)a^{4}-120\left(15\,dx+15\,c+\frac{15\tan(dx+c)^{4}+10\tan(dx+c)^{2}-2}{\tan(dx+c)^{5}+\tan(dx+c)^{3}}\right)a^{4}+8\left(15\,dx+15\,c+\frac{15\tan(dx+c)^{4}-5\tan(dx+c)^{2}+3}{\tan(dx+c)^{5}}\right)a^{4}+30\,a^{4}\left(\frac{2\left(9\cos(dx+c)^{3}-7\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\cos(dx+c)^{2}+1}-16\cos\left(dx+c\right)+15\log\left(\cos\left(dx+c\right)+1\right)-15\log\left(\cos\left(dx+c\right)-1\right)\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/120*(40*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos
(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^4 + 15*(
15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5
 + 2*tan(d*x + c)^3 + tan(d*x + c)))*a^4 - 120*(15*d*x + 15*c + (15*tan(d*x
 + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^4 + 8
*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5
)*a^4 + 30*a^4*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*c
os(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(co
s(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.39, size = 291, normalized size = 1.47

$$\frac{30\,a^{4}\cos\left(dx+c\right)^{9}-345\,a^{4}\cos\left(dx+c\right)^{7}+2231\,a^{4}\cos\left(dx+c\right)^{5}-3395\,a^{4}\cos\left(dx+c\right)^{3}+1455\,a^{4}\cos\left(dx+c\right)+150\left(a^{4}\cos\left(dx+c\right)^{4}-2\,a^{4}\cos\left(dx+c\right)^{2}+a^{4}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-150\left(a^{4}\cos\left(dx+c\right)^{4}-2\,a^{4}\cos\left(dx+c\right)^{2}+a^{4}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-5\left(32\,a^{4}\cos\left(dx+c\right)^{7}-291\,a^{4}dx\cos\left(dx+c\right)^{4}+32\,a^{4}\cos\left(dx+c\right)^{5}+582\,a^{4}dx\cos\left(dx+c\right)^{2}-100\,a^{4}\cos\left(dx+c\right)^{3}-291\,a^{4}dx+60\,a^{4}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/120*(30*a^4*cos(d*x + c)^9 - 345*a^4*cos(d*x + c)^7 + 2231*a^4*cos(d*x +
c)^5 - 3395*a^4*cos(d*x + c)^3 + 1455*a^4*cos(d*x + c) + 150*(a^4*cos(d*x +
 c)^4 - 2*a^4*cos(d*x + c)^2 + a^4)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) - 150*(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*x + c)^2 + a^4)*log(-1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) - 5*(32*a^4*cos(d*x + c)^7 - 291*a^4*d*x*cos(d*x +
 c)^4 + 32*a^4*cos(d*x + c)^5 + 582*a^4*d*x*cos(d*x + c)^2 - 100*a^4*cos(d*
x + c)^3 - 291*a^4*d*x + 60*a^4*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{4}\left(\int 4\sin\left(c+dx\right)\cot^{6}\left(c+dx\right)\,dx+\int 6\sin^{2}\left(c+dx\right)\cot^{6}\left(c+dx\right)\,dx+\int 4\sin^{3}\left(c+dx\right)\cot^{6}\left(c+dx\right)\,dx+\int \sin^{4}\left(c+dx\right)\cot^{6}\left(c+dx\right)\,dx+\int \cot^{6}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**6, x) + Integral(6*sin(c + d*x)
**2*cot(c + d*x)**6, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**6, x) +
```

```
Integral(sin(c + d*x)**4*cot(c + d*x)**6, x) + Integral(cot(c + d*x)**6, x)
)
```

Giac [A]
time = 14.06, size = 339, normalized size = 1.71

$$\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+85\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5820\,(dx+c)a^4-1200\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-2670\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{40\left(45\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+192\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+69\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+384\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-69\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+320\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-45\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+128\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}+\frac{2740\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2670\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+240\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-85\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^4*tan(1/2*d*x + 1/2*c)^5 + 30*a^4*tan(1/2*d*x + 1/2*c)^4 + 85*a^
4*tan(1/2*d*x + 1/2*c)^3 - 240*a^4*tan(1/2*d*x + 1/2*c)^2 + 5820*(d*x + c)*
a^4 - 1200*a^4*log(abs(tan(1/2*d*x + 1/2*c))) - 2670*a^4*tan(1/2*d*x + 1/2*
c) - 40*(45*a^4*tan(1/2*d*x + 1/2*c)^7 + 192*a^4*tan(1/2*d*x + 1/2*c)^6 + 6
9*a^4*tan(1/2*d*x + 1/2*c)^5 + 384*a^4*tan(1/2*d*x + 1/2*c)^4 - 69*a^4*tan(
1/2*d*x + 1/2*c)^3 + 320*a^4*tan(1/2*d*x + 1/2*c)^2 - 45*a^4*tan(1/2*d*x +
1/2*c) + 128*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4 + (2740*a^4*tan(1/2*d*x +
1/2*c)^5 + 2670*a^4*tan(1/2*d*x + 1/2*c)^4 + 240*a^4*tan(1/2*d*x + 1/2*c)^3
 - 85*a^4*tan(1/2*d*x + 1/2*c)^2 - 30*a^4*tan(1/2*d*x + 1/2*c) - 3*a^4)/tan
(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 6.85, size = 454, normalized size = 2.29

$$\frac{17\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d}+\frac{a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16\,d}+\frac{a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{5\,a^4\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{97\,a^4\,\mathrm{atan}\left(\frac{9409\,a^8}{16\,\left(\frac{485\,a^8}{4}+\frac{9409\,a^8\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}-\frac{485\,a^8\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(\frac{485\,a^8}{4}+\frac{9409\,a^8\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}-\frac{-58\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+496\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-\frac{1567\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{3}+962\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-\frac{18437\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{15}+\frac{2296\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{3986\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{5}+\frac{868\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{2312\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}-8\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{97\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+2\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^4}{5}}{d\,\left(32\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+128\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+192\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+128\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}-\frac{89\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (17*a^4*tan(c/2 + (d*x)/2)^3)/(96*d) - (a^4*tan(c/2 + (d*x)/2)^2)/(2*d) + (
a^4*tan(c/2 + (d*x)/2)^4)/(16*d) + (a^4*tan(c/2 + (d*x)/2)^5)/(160*d) - (5*
a^4*log(tan(c/2 + (d*x)/2)))/(2*d) - (97*a^4*atan((9409*a^8)/(16*((485*a^8)
/4 + (9409*a^8*tan(c/2 + (d*x)/2))/16)) - (485*a^8*tan(c/2 + (d*x)/2))/(4*(
(485*a^8)/4 + (9409*a^8*tan(c/2 + (d*x)/2))/16))))/(4*d) - ((97*a^4*tan(c/2
 + (d*x)/2)^2)/15 - 8*a^4*tan(c/2 + (d*x)/2)^3 - (2312*a^4*tan(c/2 + (d*x)/
2)^4)/15 + (868*a^4*tan(c/2 + (d*x)/2)^5)/3 - (3986*a^4*tan(c/2 + (d*x)/2)^
6)/5 + (2296*a^4*tan(c/2 + (d*x)/2)^7)/3 - (18437*a^4*tan(c/2 + (d*x)/2)^8)
/15 + 962*a^4*tan(c/2 + (d*x)/2)^9 - (1567*a^4*tan(c/2 + (d*x)/2)^10)/3 + 4
96*a^4*tan(c/2 + (d*x)/2)^11 - 58*a^4*tan(c/2 + (d*x)/2)^12 + a^4/5 + 2*a^4
*tan(c/2 + (d*x)/2))/(d*(32*tan(c/2 + (d*x)/2)^5 + 128*tan(c/2 + (d*x)/2)^7
 + 192*tan(c/2 + (d*x)/2)^9 + 128*tan(c/2 + (d*x)/2)^11 + 32*tan(c/2 + (d*x
)/2)^13)) - (89*a^4*tan(c/2 + (d*x)/2))/(16*d)
```

3.44 $\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=130

$$-\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{35\sec(c+dx)\tan(c+dx)}{128ad}-\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad}+\frac{7\sec(c+dx)\tan^5(c+d}{48ad}$$

```
[Out] -35/128*arctanh(sin(d*x+c))/a/d+35/128*sec(d*x+c)*tan(d*x+c)/a/d-35/192*sec
(d*x+c)*tan(d*x+c)^3/a/d+7/48*sec(d*x+c)*tan(d*x+c)^5/a/d-1/8*sec(d*x+c)*ta
n(d*x+c)^7/a/d+1/8*tan(d*x+c)^8/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2691, 3855}

$$\frac{\tan^8(c+dx)}{8ad}-\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^7(c+dx)\sec(c+dx)}{8ad}+\frac{7\tan^5(c+dx)\sec(c+dx)}{48ad}-\frac{35\tan^3(c+dx)\sec(c+dx)}{192ad}+\frac{35\tan(c+dx)\sec(c+dx)}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-35*ArcTanh[Sin[c + d*x]])/(128*a*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(128
*a*d) - (35*Sec[c + d*x]*Tan[c + d*x]^3)/(192*a*d) + (7*Sec[c + d*x]*Tan[c
+ d*x]^5)/(48*a*d) - (Sec[c + d*x]*Tan[c + d*x]^7)/(8*a*d) + Tan[c + d*x]^8
/(8*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^2(c+dx)\tan^7(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^8(c+dx)\,dx}{a}$$
$$= -\frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \frac{7\int \sec(c+dx)\tan^6(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^7\,dx, x\right.}{ad}$$
$$= \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \frac{\tan^8(c+dx)}{8ad} - \frac{35\int \sec}{}$$
$$= -\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \frac{\sec(c+dx)\tan^7(c}{8ad}$$
$$= \frac{35\sec(c+dx)\tan(c+dx)}{128ad} - \frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \frac{7\sec(c+dx)\tan^5(c}{48ad}$$
$$= -\frac{35\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{35\sec(c+dx)\tan(c+dx)}{128ad} - \frac{35\sec(c+dx)\tan^3(c+}{192ad}$$

Mathematica [A]
time = 0.65, size = 101, normalized size = 0.78

$$-\frac{105\tanh^{-1}(\sin(c+dx)) + \frac{-48+57\sin(c+dx)+249\sin^2(c+dx)-136\sin^3(c+dx)-424\sin^4(c+dx)+87\sin^5(c+dx)+279\sin^6(c+dx)}{(-1+\sin(c+dx))^3(1+\sin(c+dx))^4}}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/384*(105*ArcTanh[Sin[c + d*x]] + (-48 + 57*Sin[c + d*x] + 249*Sin[c + d*
x]^2 - 136*Sin[c + d*x]^3 - 424*Sin[c + d*x]^4 + 87*Sin[c + d*x]^5 + 279*Si
n[c + d*x]^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(a*d)
```

Maple [A]
time = 0.24, size = 115, normalized size = 0.88

method	result
derivativedivides	$\frac{-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{9}{128(\sin(dx+c)-1)^2}-\frac{29}{128(\sin(dx+c)-1)}+\frac{35\ln(\sin(dx+c)-1)}{256}+\frac{1}{64(1+\sin(dx+c))^4}-\frac{5}{48(1+\sin(dx+c))^3}+\frac{}{64(1+}}{da}$
default	$\frac{-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{9}{128(\sin(dx+c)-1)^2}-\frac{29}{128(\sin(dx+c)-1)}+\frac{35\ln(\sin(dx+c)-1)}{256}+\frac{1}{64(1+\sin(dx+c))^4}-\frac{5}{48(1+\sin(dx+c))^3}+\frac{}{64(1+}}{da}$
risch	$-\frac{i(279\,e^{i(dx+c)}-174ie^{2i(dx+c)}+174ie^{12i(dx+c)}+279\,e^{13i(dx+c)}+22\,e^{11i(dx+c)}+300ie^{8i(dx+c)}-300ie^{6i(dx+c)}-218ie^{4i(d}}{192(e^{i(dx+c)}+i)^8(e^{i(dx+c)}-i)^6da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/96/(sin(d*x+c)-1)^3-9/128/(sin(d*x+c)-1)^2-29/128/(sin(d*x+c)-1)+
35/256*ln(sin(d*x+c)-1)+1/64/(1+sin(d*x+c))^4-5/48/(1+sin(d*x+c))^3+19/64/(
1+sin(d*x+c))^2-1/2/(1+sin(d*x+c))-35/256*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 175, normalized size = 1.35

$$-\frac{\frac{2\left(279\sin(dx+c)^6+87\sin(dx+c)^5-424\sin(dx+c)^4-136\sin(dx+c)^3+249\sin(dx+c)^2+57\sin(dx+c)-48\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}+\frac{105\log(\sin(dx+c)+1)}{a}-\frac{105\log(\sin(dx+c)-1)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(279*sin(d*x + c)^6 + 87*sin(d*x + c)^5 - 424*sin(d*x + c)^4 - 13
6*sin(d*x + c)^3 + 249*sin(d*x + c)^2 + 57*sin(d*x + c) - 48)/(a*sin(d*x +
c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin
(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 105*log(sin(d*x +
c) + 1)/a - 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.37, size = 167, normalized size = 1.28

$$-\frac{558\cos(dx+c)^6-826\cos(dx+c)^4+476\cos(dx+c)^2+105\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)-105\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)-2\left(87\cos(dx+c)^4-38\cos(dx+c)^2+8\right)\sin(dx+c)-112}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(558*cos(d*x + c)^6 - 826*cos(d*x + c)^4 + 476*cos(d*x + c)^2 + 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(
87*cos(d*x + c)^4 - 38*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x
 + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^7{(c+dx)}}{\sin{(c+dx)}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**7/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 16.36, size = 136, normalized size = 1.05

$$-\frac{\frac{420\log\left(|\sin(dx+c)+1|\right)}{a}-\frac{420\log\left(|\sin(dx+c)-1|\right)}{a}+\frac{2\left(385\sin(dx+c)^3-807\sin(dx+c)^2+567\sin(dx+c)-129\right)}{a(\sin(dx+c)-1)^3}-\frac{875\sin(dx+c)^4+1964\sin(dx+c)^3+1554\sin(dx+c)^2+396\sin(dx+c)-21}{a(\sin(dx+c)+1)^4}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 2*(385*sin(d*x + c)^3 - 807*sin(d*x + c)^2 + 567*sin(d*x + c) - 129)/(a
*(sin(d*x + c) - 1)^3) - (875*sin(d*x + c)^4 + 1964*sin(d*x + c)^3 + 1554*s
in(d*x + c)^2 + 396*sin(d*x + c) - 21)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 10.74, size = 388, normalized size = 2.98

$$\frac{\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{32}-\frac{245\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{96}-\frac{595\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{96}+\frac{791\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}+\frac{231\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{16}-\frac{25\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{16}+\frac{231\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{16}+\frac{791\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{192}-\frac{595\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{96}-\frac{245\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{96}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{32}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+9a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+30a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-40a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+30a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+9a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{35\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + a*sin(c + d*x)),x)
```

```
[Out] ((35*tan(c/2 + (d*x)/2))/64 + (35*tan(c/2 + (d*x)/2)^2)/32 - (245*tan(c/2 +
 (d*x)/2)^3)/96 - (595*tan(c/2 + (d*x)/2)^4)/96 + (791*tan(c/2 + (d*x)/2)^5
)/192 + (231*tan(c/2 + (d*x)/2)^6)/16 - (25*tan(c/2 + (d*x)/2)^7)/16 + (231
*tan(c/2 + (d*x)/2)^8)/16 + (791*tan(c/2 + (d*x)/2)^9)/192 - (595*tan(c/2 +
 (d*x)/2)^10)/96 - (245*tan(c/2 + (d*x)/2)^11)/96 + (35*tan(c/2 + (d*x)/2)^
12)/32 + (35*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*
a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)
^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 +
(d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan
(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12
 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) - (35*atanh(tan(c/
2 + (d*x)/2)))/(64*a*d)
```

3.45 $\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{5\sec(c+dx)\tan(c+dx)}{16ad}+\frac{5\sec(c+dx)\tan^3(c+dx)}{24ad}-\frac{\sec(c+dx)\tan^5(c+dx)}{6ad}+\text{ta}$$

```
[Out] 5/16*arctanh(sin(d*x+c))/a/d-5/16*sec(d*x+c)*tan(d*x+c)/a/d+5/24*sec(d*x+c)
*tan(d*x+c)^3/a/d-1/6*sec(d*x+c)*tan(d*x+c)^5/a/d+1/6*tan(d*x+c)^6/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2691, 3855}

$$\frac{\tan^6(c+dx)}{6ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{\tan^5(c+dx)\sec(c+dx)}{6ad}+\frac{5\tan^3(c+dx)\sec(c+dx)}{24ad}-\frac{5\tan(c+dx)\sec(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(16*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(16*a*d)
 + (5*Sec[c + d*x]*Tan[c + d*x]^3)/(24*a*d) - (Sec[c + d*x]*Tan[c + d*x]^5)
/(6*a*d) + Tan[c + d*x]^6/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^5(c+dx)\,dx}{a}-\frac{\int \sec(c+dx)\tan^6(c+dx)\,dx}{a}\\ &= -\frac{\sec(c+dx)\tan^5(c+dx)}{6ad}+\frac{5\int \sec(c+dx)\tan^4(c+dx)\,dx}{6a}+\frac{\operatorname{Subst}\left(\int x^5\,dx,x\right.}{ad}\\ &= \frac{5\sec(c+dx)\tan^3(c+dx)}{24ad}-\frac{\sec(c+dx)\tan^5(c+dx)}{6ad}+\frac{\tan^6(c+dx)}{6ad}-\frac{5\int \sec(}{}\\ &= -\frac{5\sec(c+dx)\tan(c+dx)}{16ad}+\frac{5\sec(c+dx)\tan^3(c+dx)}{24ad}-\frac{\sec(c+dx)\tan^5(c+}{6ad}\\ &= \frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{5\sec(c+dx)\tan(c+dx)}{16ad}+\frac{5\sec(c+dx)\tan^3(c+dx)}{24ad}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 84, normalized size = 0.79

$$\frac{30\tanh^{-1}(\sin(c+dx))+\frac{3}{(1-\sin(c+dx))^2}-\frac{18}{1-\sin(c+dx)}+\frac{4}{(1+\sin(c+dx))^3}-\frac{21}{(1+\sin(c+dx))^2}+\frac{48}{1+\sin(c+dx)}}{96ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x]),x]
[Out] (30*ArcTanh[Sin[c + d*x]] + 3/(1 - Sin[c + d*x])^2 - 18/(1 - Sin[c + d*x])
+ 4/(1 + Sin[c + d*x])^3 - 21/(1 + Sin[c + d*x])^2 + 48/(1 + Sin[c + d*x]))
/(96*a*d)
```

Maple [A]
time = 0.21, size = 91, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{1}{24(1+\sin(dx+c))^3}-\frac{7}{32(1+\sin(dx+c))^2}+\frac{1}{2+2\sin(dx+c)}+\frac{5\ln(1+\sin(dx+c))}{32}+\frac{1}{32(\sin(dx+c)-1)^2}+\frac{3}{16(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+}{32}}{da}$

default	$\frac{\frac{1}{24(1+\sin(dx+c))^3}-\frac{7}{32(1+\sin(dx+c))^2}+\frac{1}{2+2\sin(dx+c)}+\frac{5\ln(1+\sin(dx+c))}{32}+\frac{1}{32(\sin(dx+c)-1)^2}+\frac{3}{16(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)\cdots}{32}}{da}$
risch	$\frac{i\left(-8\,e^{7i(dx+c)}+2ie^{6i(dx+c)}+78\,e^{5i(dx+c)}+18ie^{8i(dx+c)}+33\,e^{9i(dx+c)}-2ie^{4i(dx+c)}-8\,e^{3i(dx+c)}-18ie^{2i(dx+c)}+33\,e^{i(dx+c)}\cdots\right.}{24\left(e^{i(dx+c)}+i\right)^6\left(e^{i(dx+c)}-i\right)^4da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/24/(1+sin(d*x+c))^3-7/32/(1+sin(d*x+c))^2+1/2/(1+sin(d*x+c))+5/32*
ln(1+sin(d*x+c))+1/32/(sin(d*x+c)-1)^2+3/16/(sin(d*x+c)-1)-5/32*ln(sin(d*x+
c)-1))
```

Maxima [A]
time = 0.30, size = 130, normalized size = 1.23

$$\frac{\frac{2\left(33\sin(dx+c)^4+9\sin(dx+c)^3-31\sin(dx+c)^2-7\sin(dx+c)+8\right)}{a\sin(dx+c)^5+a\sin(dx+c)^4-2a\sin(dx+c)^3-2a\sin(dx+c)^2+a\sin(dx+c)+a}+\frac{15\log(\sin(dx+c)+1)}{a}-\frac{15\log(\sin(dx+c)-1)}{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(33*sin(d*x + c)^4 + 9*sin(d*x + c)^3 - 31*sin(d*x + c)^2 - 7*sin(d
*x + c) + 8)/(a*sin(d*x + c)^5 + a*sin(d*x + c)^4 - 2*a*sin(d*x + c)^3 - 2*
a*sin(d*x + c)^2 + a*sin(d*x + c) + a) + 15*log(sin(d*x + c) + 1)/a - 15*lo
g(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.38, size = 147, normalized size = 1.39

$$\frac{66\cos(dx+c)^4-70\cos(dx+c)^2+15\left(\cos(dx+c)^4\sin(dx+c)+\cos(dx+c)^4\right)\log(\sin(dx+c)+1)-15\left(\cos(dx+c)^4\sin(dx+c)+\cos(dx+c)^4\right)\log(-\sin(dx+c)+1)-2\left(9\cos(dx+c)^2-2\right)\sin(dx+c)+20}{96\left(ad\cos(dx+c)^4\sin(dx+c)+ad\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*cos(d*x + c)^4 - 70*cos(d*x + c)^2 + 15*(cos(d*x + c)^4*sin(d*x +
c) + cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4*sin(d*x + c
) + cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 2*(9*cos(d*x + c)^2 - 2)*sin(d
*x + c) + 20)/(a*d*cos(d*x + c)^4*sin(d*x + c) + a*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^5(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 12.55, size = 116, normalized size = 1.09

$$\frac{\frac{30\log(|\sin(dx+c)+1|)}{a}-\frac{30\log(|\sin(dx+c)-1|)}{a}+\frac{3\left(15\sin(dx+c)^2-18\sin(dx+c)+5\right)}{a(\sin(dx+c)-1)^2}-\frac{55\sin(dx+c)^3+69\sin(dx+c)^2+15\sin(dx+c)-7}{a(\sin(dx+c)+1)^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(30*log(abs(sin(d*x + c) + 1))/a - 30*log(abs(sin(d*x + c) - 1))/a +
3*(15*sin(d*x + c)^2 - 18*sin(d*x + c) + 5)/(a*(sin(d*x + c) - 1)^2) - (55*
sin(d*x + c)^3 + 69*sin(d*x + c)^2 + 15*sin(d*x + c) - 7)/(a*(sin(d*x + c)
+ 1)^3))/d
```

Mupad [B]
time = 10.43, size = 281, normalized size = 2.65

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}-\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{8}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{55\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{12}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{55\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{12}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*sin(c + d*x)),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(8*a*d) - ((5*tan(c/2 + (d*x)/2))/8 + (5*tan(
c/2 + (d*x)/2)^2)/4 - (5*tan(c/2 + (d*x)/2)^3)/3 - (55*tan(c/2 + (d*x)/2)^4
)/12 + (3*tan(c/2 + (d*x)/2)^5)/4 - (55*tan(c/2 + (d*x)/2)^6)/12 - (5*tan(c
/2 + (d*x)/2)^7)/3 + (5*tan(c/2 + (d*x)/2)^8)/4 + (5*tan(c/2 + (d*x)/2)^9)/
8)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 3*a*tan(c/2 + (d*x)/2)^2 - 8*a*tan(c/2
+ (d*x)/2)^3 + 2*a*tan(c/2 + (d*x)/2)^4 + 12*a*tan(c/2 + (d*x)/2)^5 + 2*a*t
an(c/2 + (d*x)/2)^6 - 8*a*tan(c/2 + (d*x)/2)^7 - 3*a*tan(c/2 + (d*x)/2)^8 +
 2*a*tan(c/2 + (d*x)/2)^9 + a*tan(c/2 + (d*x)/2)^10))
```

3.46 $\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}-\frac{\sec(c+dx)\tan^3(c+dx)}{4ad}+\frac{\tan^4(c+dx)}{4ad}$$

```
[Out] -3/8*arctanh(sin(d*x+c))/a/d+3/8*sec(d*x+c)*tan(d*x+c)/a/d-1/4*sec(d*x+c)*t
an(d*x+c)^3/a/d+1/4*tan(d*x+c)^4/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$,
Rules used = {2785, 2687, 30, 2691, 3855}

$$\frac{\tan^4(c+dx)}{4ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{\tan^3(c+dx)\sec(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(8*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(8*a*d)
- (Sec[c + d*x]*Tan[c + d*x]^3)/(4*a*d) + Tan[c + d*x]^4/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^4(c+dx)\,dx}{a}\\ &= -\frac{\sec(c+dx)\tan^3(c+dx)}{4ad} + \frac{3\int \sec(c+dx)\tan^2(c+dx)\,dx}{4a} + \frac{\operatorname{Subst}\left(\int x^3\,dx, x\right.}{ad}\\ &= \frac{3\sec(c+dx)\tan(c+dx)}{8ad} - \frac{\sec(c+dx)\tan^3(c+dx)}{4ad} + \frac{\tan^4(c+dx)}{4ad} - \frac{3\int \sec(c}{8}\\ &= -\frac{3\tanh^{-1}(\sin(c+dx))}{8ad} + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} - \frac{\sec(c+dx)\tan^3(c+dx)}{4ad}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 54, normalized size = 0.66

$$-\frac{3\tanh^{-1}(\sin(c+dx)) + \frac{1}{-1+\sin(c+dx)} - \frac{1}{(1+\sin(c+dx))^2} + \frac{4}{1+\sin(c+dx)}}{8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x]),x]
[Out] -1/8*(3*ArcTanh[Sin[c + d*x]] + (-1 + Sin[c + d*x])^(-1) - (1 + Sin[c + d*x
])^(-2) + 4/(1 + Sin[c + d*x]))/(a*d)
```

Maple [A]
time = 0.21, size = 67, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{2(1+\sin(dx+c))}-\frac{3\ln(1+\sin(dx+c))}{16}-\frac{1}{8(\sin(dx+c)-1)}+\frac{3\ln(\sin(dx+c)-1)}{16}}{da}$	67
default	$\frac{\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{2(1+\sin(dx+c))}-\frac{3\ln(1+\sin(dx+c))}{16}-\frac{1}{8(\sin(dx+c)-1)}+\frac{3\ln(\sin(dx+c)-1)}{16}}{da}$	67

risch	$-\frac{i\left(2i e^{4i(dx+c)}-2\,e^{3i(dx+c)}-2i e^{2i(dx+c)}+5\,e^{5i(dx+c)}+5\,e^{i(dx+c)}\right)}{4\left(e^{i(dx+c)}+i\right)^{4}\left(e^{i(dx+c)}-i\right)^{2}da}+\frac{3\ln\left(e^{i(dx+c)}-i\right)}{8ad}-\frac{3\ln\left(e^{i(dx+c)}+i\right)}{8ad}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/8/(1+sin(d*x+c))^2-1/2/(1+sin(d*x+c))-3/16*ln(1+sin(d*x+c))-1/8/(s
in(d*x+c)-1)+3/16*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 89, normalized size = 1.09

$$-\frac{\frac{2\left(5\,\sin(dx+c)^{2}+\sin(dx+c)-2\right)}{a\sin(dx+c)^{3}+a\sin(dx+c)^{2}-a\sin(dx+c)-a}+\frac{3\,\log(\sin(dx+c)+1)}{a}-\frac{3\,\log(\sin(dx+c)-1)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(5*sin(d*x + c)^2 + sin(d*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin(d*
x + c)^2 - a*sin(d*x + c) - a) + 3*log(sin(d*x + c) + 1)/a - 3*log(sin(d*x
+ c) - 1)/a)/d
```

Fricas [A]
time = 0.36, size = 125, normalized size = 1.52

$$-\frac{10\,\cos\left(dx+c\right)^{2}+3\,\left(\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,\left(\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\sin\left(dx+c\right)-6}{16\,\left(ad\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)
*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*l
og(-sin(d*x + c) + 1) - 2*sin(d*x + c) - 6)/(a*d*cos(d*x + c)^2*sin(d*x + c
) + a*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{3}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 5.72, size = 96, normalized size = 1.17

$$-\frac{\frac{6\,\log\left(|\sin\left(dx+c\right)+1|\right)}{a}-\frac{6\,\log\left(|\sin\left(dx+c\right)-1|\right)}{a}+\frac{2\,\left(3\,\sin\left(dx+c\right)-1\right)}{a\left(\sin\left(dx+c\right)-1\right)}-\frac{9\,\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)-3}{a\left(\sin\left(dx+c\right)+1\right)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c) - 1))/a + 2*
(3*sin(d*x + c) - 1)/(a*(sin(d*x + c) - 1)) - (9*sin(d*x + c)^2 + 2*sin(d*x
 + c) - 3)/(a*(sin(d*x + c) + 1)^2))/d
```

Mupad [B]
time = 9.11, size = 172, normalized size = 2.10

$$\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2))/4 + (3*tan(c/2 + (d*x)/2)^2)/2 - tan(c/2 + (d*x)/2)
^3/2 + (3*tan(c/2 + (d*x)/2)^4)/2 + (3*tan(c/2 + (d*x)/2)^5)/4)/(d*(a + 2*a
*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2 - 4*a*tan(c/2 + (d*x)/2)^3 - a
*tan(c/2 + (d*x)/2)^4 + 2*a*tan(c/2 + (d*x)/2)^5 + a*tan(c/2 + (d*x)/2)^6))
 - (3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d)
```

3.47 $\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{1}{2d(a+a\sin(c+dx))}$$

```
[Out] 1/2*arctanh(sin(d*x+c))/a/d+1/2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.57, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2785, 2686, 30, 2691, 3855}

$$\frac{\sec^2(c+dx)}{2ad}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) + Sec[c + d*x]^2/(2*a*d) - (Sec[c + d*x]*Tan[
c + d*x])/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^2(c+dx)\,dx}{a}\\ &= -\frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \sec(c+dx)\,dx}{2a} + \frac{\text{Subst}(\int x\,dx, x, \sec(c+dx))}{ad}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{\sec^2(c+dx)}{2ad} - \frac{\sec(c+dx)\tan(c+dx)}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 0.76

$$\frac{\tanh^{-1}(\sin(c+dx)) + \frac{1}{1+\sin(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x]),x]
[Out] (ArcTanh[Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(2*a*d)
```

Maple [A]
time = 0.16, size = 43, normalized size = 1.16

method	result	size
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{4}+\frac{1}{2+2\sin(dx+c)}+\frac{\ln(1+\sin(dx+c))}{4}}{da}$	43
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{4}+\frac{1}{2+2\sin(dx+c)}+\frac{\ln(1+\sin(dx+c))}{4}}{da}$	43
risch	$\frac{i e^{i(dx+c)}}{da\left(e^{i(dx+c)}+i\right)^2} - \frac{\ln\left(e^{i(dx+c)}-i\right)}{2ad} + \frac{\ln\left(e^{i(dx+c)}+i\right)}{2ad}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-1/4*ln(sin(d*x+c)-1)+1/2/(1+sin(d*x+c))+1/4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.31, size = 47, normalized size = 1.27

$$\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c)-1)}{a} + \frac{2}{a\sin(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c) - 1)/a + 2/(a*sin(d*x + c)
+ a))/d
```

Fricas [A]
time = 0.37, size = 58, normalized size = 1.57

$$\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1) - (\sin(dx+c)+1)\log(-\sin(dx+c)+1) + 2}{4\,(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (sin(d*x + c) + 1)*log(-sin
(d*x + c) + 1) + 2)/(a*d*sin(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 9.67, size = 58, normalized size = 1.57

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)-1|)}{a} - \frac{\sin(dx+c)-1}{a(\sin(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c) - 1))/a - (sin(d*x
 + c) - 1)/(a*(sin(d*x + c) + 1)))/d
```

Mupad [B]
time = 6.66, size = 61, normalized size = 1.65

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*sin(c + d*x)),x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(a*d) - tan(c/2 + (d*x)/2)/(d*(a + 2*a*tan(c/2 +
(d*x)/2) + a*tan(c/2 + (d*x)/2)^2))
```

3.48 $\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(1+\sin(c+dx))}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2786, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a\sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(1+\sin(c+dx))}{ad}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 1.00

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(1+\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Maple [A]
time = 0.11, size = 27, normalized size = 0.84

method	result	size
derivativedivides	$\frac{\ln(\sin(dx+c))-\ln(1+\sin(dx+c))}{ad}$	27
default	$\frac{\ln(\sin(dx+c))-\ln(1+\sin(dx+c))}{ad}$	27
risch	$-\frac{2\ln\left(e^{i(dx+c)}+i\right)}{ad}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{da}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/a/d*(ln(sin(d*x+c))-ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 31, normalized size = 0.97

$$-\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c) + 1)/a - log(sin(d*x + c))/a)/d
```

Fricas [A]
time = 0.35, size = 28, normalized size = 0.88

$$\frac{\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-\log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] (log(1/2*sin(d*x + c)) - log(sin(d*x + c) + 1))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] Integral(cot(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 9.18, size = 33, normalized size = 1.03

$$-\frac{\frac{\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c)))/a)/d
```

Mupad [B]
time = 6.53, size = 32, normalized size = 1.00

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*sin(c + d*x)),x)
[Out] (log(tan(c/2 + (d*x)/2)) - 2*log(tan(c/2 + (d*x)/2) + 1))/(a*d)
```

3.49 $\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2785, 2686, 30, 8}

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot(c+dx)\csc(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}(\int 1\,dx, x, \csc(c+dx))}{ad} - \frac{\text{Subst}(\int x\,dx, x, \csc(c+dx))}{ad}$$
$$= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.75

$$-\frac{(-2+\csc(c+dx))\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*((-2 + Csc[c + d*x])*Csc[c + d*x])/(a*d)
```

Maple [A]
time = 0.12, size = 30, normalized size = 0.94

method	result	size
derivativedivides	$-\frac{\frac{1}{2\sin(dx+c)^2}-\frac{1}{\sin(dx+c)}}{ad}$	30
default	$-\frac{\frac{1}{2\sin(dx+c)^2}-\frac{1}{\sin(dx+c)}}{ad}$	30
risch	$\frac{2i\left(-i e^{2i(dx+c)}+e^{3i(dx+c)}-e^{i(dx+c)}\right)}{da\left(e^{2i(dx+c)}-1\right)^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/d*(1/2/sin(d*x+c)^2-1/sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.81

$$\frac{2\sin(dx+c)-1}{2\,ad\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

Fricas [A]
time = 0.35, size = 30, normalized size = 0.94

$$-\frac{2\,\sin\left(dx+c\right)-1}{2\,\left(ad\cos\left(dx+c\right)^{2}-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^{3}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c)),x)
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 6.35, size = 26, normalized size = 0.81

$$\frac{2\,\sin\left(dx+c\right)-1}{2\,ad\sin\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

Mupad [B]
time = 6.58, size = 23, normalized size = 0.72

$$\frac{\sin\left(c+d\,x\right)-\frac{1}{2}}{a\,d\,\sin\left(c+d\,x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*sin(c + d*x)),x)
[Out] (sin(c + d*x) - 1/2)/(a*d*sin(c + d*x)^2)
```

3.50 $\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{\cot^4(c+dx)}{4ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] -1/4*cot(d*x+c)^4/a/d-csc(d*x+c)/a/d+1/3*csc(d*x+c)^3/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2785, 2687, 30, 2686}

$$-\frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*Cot[c + d*x]^4/(a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^3(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^3(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^3\,dx,x,-\cot(c+dx)\right)}{ad}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^4(c+dx)}{4ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 0.59

$$-\frac{(-1+\csc(c+dx))^3(5+3\csc(c+dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
[Out] -1/12*((-1 + Csc[c + d*x])^3*(5 + 3*Csc[c + d*x]))/(a*d)
```

Maple [A]
time = 0.24, size = 49, normalized size = 0.96

method	result	size
derivativedivides	$\frac{-\frac{1}{\sin(dx+c)}-\frac{1}{4\sin(dx+c)^4}+\frac{1}{3\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^2}}{da}$	49
default	$\frac{-\frac{1}{\sin(dx+c)}-\frac{1}{4\sin(dx+c)^4}+\frac{1}{3\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^2}}{da}$	49
risch	$-\frac{2i\left(-3i\,e^{6i(dx+c)}+3\,e^{7i(dx+c)}-5\,e^{5i(dx+c)}-3i\,e^{2i(dx+c)}+5\,e^{3i(dx+c)}-3\,e^{i(dx+c)}\right)}{3da\left(e^{2i(dx+c)}-1\right)^4}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-1/sin(d*x+c)-1/4/sin(d*x+c)^4+1/3/sin(d*x+c)^3+1/2/sin(d*x+c)^2)
```

Maxima [A]
time = 0.29, size = 46, normalized size = 0.90

$$-\frac{12\,\sin\left(dx+c\right)^3-6\,\sin\left(dx+c\right)^2-4\,\sin\left(dx+c\right)+3}{12\,ad\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

Fricas [A]
time = 0.35, size = 63, normalized size = 1.24

$$-\frac{6\,\cos\left(dx+c\right)^{2}-4\left(3\,\cos\left(dx+c\right)^{2}-2\right)\sin\left(dx+c\right)-3}{12\left(ad\cos\left(dx+c\right)^{4}-2\,ad\cos\left(dx+c\right)^{2}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^2 - 2)*sin(d*x + c) - 3)/(a*d*c
os(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{5}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 5.30, size = 46, normalized size = 0.90

$$-\frac{12\,\sin\left(dx+c\right)^{3}-6\,\sin\left(dx+c\right)^{2}-4\,\sin\left(dx+c\right)+3}{12\,ad\sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

Mupad [B]
time = 6.56, size = 45, normalized size = 0.88

$$\frac{-\sin\left(c+d\,x\right)^{3}+\frac{\sin\left(c+d\,x\right)^{2}}{2}+\frac{\sin\left(c+d\,x\right)}{3}-\frac{1}{4}}{a\,d\sin\left(c+d\,x\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)/3 + sin(c + d*x)^2/2 - sin(c + d*x)^3 - 1/4)/(a*d*sin(c + d*x
)^4)
```

3.51 $\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc(c+dx)}{ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^5(c+dx)}{5ad}$$

```
[Out] -1/6*cot(d*x+c)^6/a/d+csc(d*x+c)/a/d-2/3*csc(d*x+c)^3/a/d+1/5*csc(d*x+c)^5/
a/d
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2686, 200}

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*Cot[c + d*x]^6/(a*d) + Csc[c + d*x]/(a*d) - (2*Csc[c + d*x]^3)/(3*a*d)
 + Csc[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^2(c+dx)\,dx}{a}\\
&= -\frac{\operatorname{Subst}\left(\int x^5\,dx,x,-\cot(c+dx)\right)}{ad}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{ad}\\
&= -\frac{\cot^6(c+dx)}{6ad}+\frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\csc(c+dx)\right)}{ad}\\
&= -\frac{\cot^6(c+dx)}{6ad}+\frac{\csc(c+dx)}{ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^5(c+dx)}{5ad}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 61, normalized size = 0.90

$$\frac{\csc^6(c+dx)(-15\cos(4(c+dx))+78\sin(c+dx)-5(5+7\sin(3(c+dx))-3\sin(5(c+dx))))}{240ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
[Out] (Csc[c + d*x]^6*(-15*Cos[4*(c + d*x)] + 78*Sin[c + d*x] - 5*(5 + 7*Sin[3*(c
 + d*x)] - 3*Sin[5*(c + d*x)])))/(240*a*d)
```

Maple [A]
time = 0.24, size = 67, normalized size = 0.99

method	result
derivativedivides	$\frac{\frac{1}{2\sin(dx+c)^4}+\frac{1}{\sin(dx+c)}-\frac{2}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}+\frac{1}{5\sin(dx+c)^5}-\frac{1}{6\sin(dx+c)^6}}{da}$
default	$\frac{\frac{1}{2\sin(dx+c)^4}+\frac{1}{\sin(dx+c)}-\frac{2}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}+\frac{1}{5\sin(dx+c)^5}-\frac{1}{6\sin(dx+c)^6}}{da}$
risch	$\frac{2i\left(-15i e^{10i(dx+c)}+15\,e^{11i(dx+c)}-35\,e^{9i(dx+c)}-50i e^{6i(dx+c)}+78\,e^{7i(dx+c)}-78\,e^{5i(dx+c)}-15i e^{2i(dx+c)}+35\,e^{3i(dx+c)}-\right.}{15da\left(e^{2i(dx+c)}-1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/2/sin(d*x+c)^4+1/sin(d*x+c)-2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2+1/5/
sin(d*x+c)^5-1/6/sin(d*x+c)^6)
```

Maxima [A]
time = 0.30, size = 66, normalized size = 0.97

$$\frac{30 \sin(dx+c)^5 - 15 \sin(dx+c)^4 - 20 \sin(dx+c)^3 + 15 \sin(dx+c)^2 + 6 \sin(dx+c) - 5}{30\, ad \sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*
x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)
```

Fricas [A]
time = 0.37, size = 96, normalized size = 1.41

$$\frac{15 \cos(dx+c)^4 - 15 \cos(dx+c)^2 - 2\left(15 \cos(dx+c)^4 - 20 \cos(dx+c)^2 + 8\right)\sin(dx+c) + 5}{30\left(ad\cos(dx+c)^6 - 3\,ad\cos(dx+c)^4 + 3\,ad\cos(dx+c)^2 - ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*cos(d*x + c)^4 - 15*cos(d*x + c)^2 - 2*(15*cos(d*x + c)^4 - 20*cos
(d*x + c)^2 + 8)*sin(d*x + c) + 5)/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)
^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^7(c+dx)}{\sin(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**7/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 4.26, size = 66, normalized size = 0.97

$$\frac{30\,\sin\left(dx+c\right)^{5}-15\,\sin\left(dx+c\right)^{4}-20\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)^{2}+6\,\sin\left(dx+c\right)-5}{30\,ad\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*
x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)
```

Mupad [B]
time = 6.79, size = 63, normalized size = 0.93

$$\frac{\sin\left(c+d\,x\right)^5-\frac{\sin\left(c+d\,x\right)^4}{2}-\frac{2\,\sin\left(c+d\,x\right)^3}{3}+\frac{\sin\left(c+d\,x\right)^2}{2}+\frac{\sin\left(c+d\,x\right)}{5}-\frac{1}{6}}{a\,d\,\sin\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)/5 + sin(c + d*x)^2/2 - (2*sin(c + d*x)^3)/3 - sin(c + d*x)^4/
2 + sin(c + d*x)^5 - 1/6)/(a*d*sin(c + d*x)^6)
```

3.52 $\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\cot^8(c+dx)}{8ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{ad}-\frac{3\csc^5(c+dx)}{5ad}+\frac{\csc^7(c+dx)}{7ad}$$

```
[Out] -1/8*cot(d*x+c)^8/a/d-csc(d*x+c)/a/d+csc(d*x+c)^3/a/d-3/5*csc(d*x+c)^5/a/d+
1/7*csc(d*x+c)^7/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2686, 200}

$$-\frac{\cot^8(c+dx)}{8ad}+\frac{\csc^7(c+dx)}{7ad}-\frac{3\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*Cot[c + d*x]^8/(a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(a*d) - (3*
Csc[c + d*x]^5)/(5*a*d) + Csc[c + d*x]^7/(7*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^7(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^7(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^7\,dx,x,-\cot(c+dx)\right)}{ad}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^8(c+dx)}{8ad}+\frac{\operatorname{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^8(c+dx)}{8ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{ad}-\frac{3\csc^5(c+dx)}{5ad}+\frac{\csc^7(c+dx)}{7ad}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 77, normalized size = 0.92

$$\frac{\csc^8(c+dx)(-245\cos(2(c+dx))-35\cos(6(c+dx))-513\sin(c+dx)+371\sin(3(c+dx))-105\sin(5(c+dx))+35\sin(7(c+dx)))}{2240ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^8*(-245*Cos[2*(c + d*x)] - 35*Cos[6*(c + d*x)] - 513*Sin[c +
d*x] + 371*Sin[3*(c + d*x)] - 105*Sin[5*(c + d*x)] + 35*Sin[7*(c + d*x)]))/
(2240*a*d)
```

Maple [A]
time = 0.32, size = 87, normalized size = 1.04

method	result
derivativedivides	$\frac{\frac{1}{7\sin(dx+c)^7}-\frac{1}{\sin(dx+c)}+\frac{1}{2\sin(dx+c)^6}-\frac{3}{5\sin(dx+c)^5}-\frac{1}{8\sin(dx+c)^8}-\frac{3}{4\sin(dx+c)^4}+\frac{1}{\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^2}}{da}$
default	$\frac{\frac{1}{7\sin(dx+c)^7}-\frac{1}{\sin(dx+c)}+\frac{1}{2\sin(dx+c)^6}-\frac{3}{5\sin(dx+c)^5}-\frac{1}{8\sin(dx+c)^8}-\frac{3}{4\sin(dx+c)^4}+\frac{1}{\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^2}}{da}$

risch	$-\frac{2i\left(-35i\mathrm{e}^{14i(dx+c)}+35\,\mathrm{e}^{15i(dx+c)}-105\,\mathrm{e}^{13i(dx+c)}-245i\mathrm{e}^{10i(dx+c)}+371\,\mathrm{e}^{11i(dx+c)}-513\,\mathrm{e}^{9i(dx+c)}-245i\mathrm{e}^{6i(dx+c)}+513\,\mathrm{e}\right.}{35da\left(\mathrm{e}^{2i(dx+c)}-1\right)^{8}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/7/sin(d*x+c)^7-1/sin(d*x+c)+1/2/sin(d*x+c)^6-3/5/sin(d*x+c)^5-1/8/
sin(d*x+c)^8-3/4/sin(d*x+c)^4+1/sin(d*x+c)^3+1/2/sin(d*x+c)^2)
```

Maxima [A]
time = 0.30, size = 86, normalized size = 1.02

$$-\frac{280\,\sin\left(dx+c\right)^{7}-140\,\sin\left(dx+c\right)^{6}-280\,\sin\left(dx+c\right)^{5}+210\,\sin\left(dx+c\right)^{4}+168\,\sin\left(dx+c\right)^{3}-140\,\sin\left(dx+c\right)^{2}-40\,\sin\left(dx+c\right)+35}{280\,ad\sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/280*(280*sin(d*x + c)^7 - 140*sin(d*x + c)^6 - 280*sin(d*x + c)^5 + 210*
sin(d*x + c)^4 + 168*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 40*sin(d*x + c)
+ 35)/(a*d*sin(d*x + c)^8)
```

Fricas [A]
time = 0.35, size = 127, normalized size = 1.51

$$-\frac{140\,\cos\left(dx+c\right)^{6}-210\,\cos\left(dx+c\right)^{4}+140\,\cos\left(dx+c\right)^{2}-8\left(35\,\cos\left(dx+c\right)^{6}-70\,\cos\left(dx+c\right)^{4}+56\,\cos\left(dx+c\right)^{2}-16\right)\sin\left(dx+c\right)-35}{280\left(ad\cos\left(dx+c\right)^{8}-4\,ad\cos\left(dx+c\right)^{6}+6\,ad\cos\left(dx+c\right)^{4}-4\,ad\cos\left(dx+c\right)^{2}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/280*(140*cos(d*x + c)^6 - 210*cos(d*x + c)^4 + 140*cos(d*x + c)^2 - 8*(3
5*cos(d*x + c)^6 - 70*cos(d*x + c)^4 + 56*cos(d*x + c)^2 - 16)*sin(d*x + c)
 - 35)/(a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^6 + 6*a*d*cos(d*x + c)^4 -
4*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^{9}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**9/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 4.22, size = 86, normalized size = 1.02

$$-\frac{280 \sin\left(dx+c\right)^{7}-140 \sin\left(dx+c\right)^{6}-280 \sin\left(dx+c\right)^{5}+210 \sin\left(dx+c\right)^{4}+168 \sin\left(dx+c\right)^{3}-140 \sin\left(dx+c\right)^{2}-40 \sin\left(dx+c\right)+35}{280\, ad \sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/280*(280*sin(d*x + c)^7 - 140*sin(d*x + c)^6 - 280*sin(d*x + c)^5 + 210*
sin(d*x + c)^4 + 168*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 40*sin(d*x + c)
+ 35)/(a*d*sin(d*x + c)^8)
```

Mupad [B]
time = 6.77, size = 83, normalized size = 0.99

$$\frac{-\sin\left(c+d\,x\right)^7+\frac{\sin(c+d\,x)^6}{2}+\sin\left(c+d\,x\right)^5-\frac{3\sin(c+d\,x)^4}{4}-\frac{3\sin(c+d\,x)^3}{5}+\frac{\sin(c+d\,x)^2}{2}+\frac{\sin(c+d\,x)}{7}-\frac{1}{8}}{a\,d\sin\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^9/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)/7 + sin(c + d*x)^2/2 - (3*sin(c + d*x)^3)/5 - (3*sin(c + d*x)
^4)/4 + sin(c + d*x)^5 + sin(c + d*x)^6/2 - sin(c + d*x)^7 - 1/8)/(a*d*sin(
c + d*x)^8)
```

3.53 $\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{ad}+\frac{3\sec^5(c+dx)}{5ad}-\frac{\sec^7(c+dx)}{7ad}+\frac{\tan^7(c+dx)}{7ad}$$

```
[Out] sec(d*x+c)/a/d-sec(d*x+c)^3/a/d+3/5*sec(d*x+c)^5/a/d-1/7*sec(d*x+c)^7/a/d+1
/7*tan(d*x+c)^7/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2686, 200}

$$\frac{\tan^7(c+dx)}{7ad}-\frac{\sec^7(c+dx)}{7ad}+\frac{3\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(a*d) + (3*Sec[c + d*x]^5)/(5*a*d) - Se
c[c + d*x]^7/(7*a*d) + Tan[c + d*x]^7/(7*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a} \\
&= \frac{\text{Subst}\left(\int x^6\,dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx, x, \sec(c+dx)\right)}{ad} \\
&= \frac{\tan^7(c+dx)}{7ad} - \frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx, x, \sec(c+dx)\right)}{ad} \\
&= \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{ad} + \frac{3\sec^5(c+dx)}{5ad} - \frac{\sec^7(c+dx)}{7ad} + \frac{\tan^7(c+dx)}{7ad}
\end{aligned}
$$

Mathematica [A]
time = 0.21, size = 146, normalized size = 1.74

$$\frac{\sec^5(c+dx)(2912-7620\cos(c+dx)+3760\cos(2(c+dx))-3810\cos(3(c+dx))+1440\cos(4(c+dx))-762\cos(5(c+dx))+80\cos(6(c+dx))+2432\sin(c+dx)-1905\sin(2(c+dx))+320\sin(3(c+dx))-1524\sin(4(c+dx))+960\sin(5(c+dx))-381\sin(6(c+dx)))}{17920ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^5*(2912 - 7620*Cos[c + d*x] + 3760*Cos[2*(c + d*x)] - 3810*Co
s[3*(c + d*x)] + 1440*Cos[4*(c + d*x)] - 762*Cos[5*(c + d*x)] + 80*Cos[6*(c
 + d*x)] + 2432*Sin[c + d*x] - 1905*Sin[2*(c + d*x)] + 320*Sin[3*(c + d*x)]
 - 1524*Sin[4*(c + d*x)] + 960*Sin[5*(c + d*x)] - 381*Sin[6*(c + d*x)]))/(1
7920*a*d*(1 + Sin[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(78) = 156$.
time = 0.20, size = 175, normalized size = 2.08

method	result
risch	$\frac{-\frac{10\,e^{i(dx+c)}}{7}+\frac{52i e^{6i(dx+c)}}{5}+\frac{52i e^{4i(dx+c)}}{7}+6i e^{8i(dx+c)}+\frac{22i e^{2i(dx+c)}}{7}-\frac{52\,e^{5i(dx+c)}}{35}+\frac{6\,e^{3i(dx+c)}}{7}+\frac{36\,e^{7i(dx+c)}}{5}+2\,e^{9i(dx}}{\left(e^{i(dx+c)}-i\right)^5\left(e^{i(dx+c)}+i\right)^7 da}$

derivativedivides	$\frac{-\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{da}$
default	$\frac{-\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{5}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 128/d/a*(-1/1280/(tan(1/2*d*x+1/2*c)-1)^5-1/512/(tan(1/2*d*x+1/2*c)-1)^4+1/
512/(tan(1/2*d*x+1/2*c)-1)^2-5/2048/(tan(1/2*d*x+1/2*c)-1)-1/448/(tan(1/2*d
*x+1/2*c)+1)^7+1/128/(tan(1/2*d*x+1/2*c)+1)^6-9/1280/(tan(1/2*d*x+1/2*c)+1)
^5-1/512/(tan(1/2*d*x+1/2*c)+1)^4+1/512/(tan(1/2*d*x+1/2*c)+1)^3+3/1024/(ta
n(1/2*d*x+1/2*c)+1)^2+5/2048/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(78) = 156$.
time = 0.31, size = 338, normalized size = 4.02

$$\frac{32\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+1\right)}{35\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{20a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{5a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{10a\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{4a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{2a\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 32/35*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 20*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a + 2*a*sin(d*x
 + c)/(cos(d*x + c) + 1) - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 10*a*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 20*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 20*a*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 5*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 10*a*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 + 4*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 2*a*si
n(d*x + c)^11/(cos(d*x + c) + 1)^11 - a*sin(d*x + c)^12/(cos(d*x + c) + 1)^
12)*d)
```

Fricas [A]
time = 0.35, size = 95, normalized size = 1.13

$$\frac{5\cos(dx+c)^6+15\cos(dx+c)^4-5\cos(dx+c)^2+2\left(15\cos(dx+c)^4-10\cos(dx+c)^2+3\right)\sin(dx+c)+1}{35\left(ad\cos(dx+c)^5\sin(dx+c)+ad\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(d*x + c)^6 + 15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 2*(15*cos(d
*x + c)^4 - 10*cos(d*x + c)^2 + 3)*sin(d*x + c) + 1)/(a*d*cos(d*x + c)^5*si
n(d*x + c) + a*d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^6(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sin(d*x+c)),x)
[Out] Integral(tan(c + d*x)**6/(sin(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(78) = 156.
time = 8.84, size = 172, normalized size = 2.05

$$-\frac{\frac{7\left(25\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+33\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^5}-\frac{175\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+1260\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3815\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+6020\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4641\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1792\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+281}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/560*(7*(25*tan(1/2*d*x + 1/2*c)^4 - 120*tan(1/2*d*x + 1/2*c)^3 + 210*tan
(1/2*d*x + 1/2*c)^2 - 140*tan(1/2*d*x + 1/2*c) + 33)/(a*(tan(1/2*d*x + 1/2*
c) - 1)^5) - (175*tan(1/2*d*x + 1/2*c)^6 + 1260*tan(1/2*d*x + 1/2*c)^5 + 38
15*tan(1/2*d*x + 1/2*c)^4 + 6020*tan(1/2*d*x + 1/2*c)^3 + 4641*tan(1/2*d*x
+ 1/2*c)^2 + 1792*tan(1/2*d*x + 1/2*c) + 281)/(a*(tan(1/2*d*x + 1/2*c) + 1)
^7))/d
```

Mupad [B]
time = 8.49, size = 99, normalized size = 1.18

$$-\frac{32\left(20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{35\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^5\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + a*sin(c + d*x)),x)
[Out] -(32*(2*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)
^3 + 5*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^5 + 1))/(35*a*d*(tan(c/
2 + (d*x)/2) - 1)^5*(tan(c/2 + (d*x)/2) + 1)^7)
```

3.54 $\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=69

$$-\frac{\sec(c+dx)}{ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\tan^5(c+dx)}{5ad}$$

```
[Out] -sec(d*x+c)/a/d+2/3*sec(d*x+c)^3/a/d-1/5*sec(d*x+c)^5/a/d+1/5*tan(d*x+c)^5/
a/d
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2686, 200}

$$\frac{\tan^5(c+dx)}{5ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]/(a*d)) + (2*Sec[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^5/(5*a*d)
 + Tan[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a}\\ &= \frac{\mathrm{Subst}\left(\int x^4\,dx, x, \tan(c+dx)\right)}{ad} - \frac{\mathrm{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \sec(c+dx)\right)}{ad}\\ &= \frac{\tan^5(c+dx)}{5ad} - \frac{\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \sec(c+dx)\right)}{ad}\\ &= -\frac{\sec(c+dx)}{ad} + \frac{2\sec^3(c+dx)}{3ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 106, normalized size = 1.54

$$-\frac{\sec^3(c+dx)(200-534\cos(c+dx)+288\cos(2(c+dx))-178\cos(3(c+dx))+24\cos(4(c+dx))-64\sin(c+dx)-178\sin(2(c+dx))+192\sin(3(c+dx))-89\sin(4(c+dx)))}{960ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(Sec[c + d*x]^3*(200 - 534*Cos[c + d*x] + 288*Cos[2*(c + d*x)] - 178
*Cos[3*(c + d*x)] + 24*Cos[4*(c + d*x)] - 64*Sin[c + d*x] - 178*Sin[2*(c +
d*x)] + 192*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.
time = 0.22, size = 130, normalized size = 1.88

method	result
risch	$-\frac{2\left(25i\,e^{4i(dx+c)}+5\,e^{5i(dx+c)}+21i\,e^{2i(dx+c)}+13\,e^{3i(dx+c)}+15i\,e^{6i(dx+c)}+15\,e^{7i(dx+c)}-9\,e^{i(dx+c)}+3i\right)}{15\left(e^{i(dx+c)}+i\right)^5\left(e^{i(dx+c)}-i\right)^3 da}$

derivativedivides	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}}{da}$
default	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2+3/25
6/(tan(1/2*d*x+1/2*c)-1)-1/80/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/
2*c)+1)^4-1/96/(tan(1/2*d*x+1/2*c)+1)^3-1/64/(tan(1/2*d*x+1/2*c)+1)^2-3/256
/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(63) = 126$.
time = 0.30, size = 214, normalized size = 3.10

$$-\frac{16\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+1\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{6a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{2a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -16/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 6*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a + 2*a*sin(d*x + c)/
(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A]
time = 0.35, size = 75, normalized size = 1.09

$$-\frac{3\cos(dx+c)^{4}+6\cos(dx+c)^{2}+4\left(3\cos(dx+c)^{2}-1\right)\sin(dx+c)-1}{15\left(ad\cos(dx+c)^{3}\sin(dx+c)+ad\cos(dx+c)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(d*x + c)^4 + 6*cos(d*x + c)^2 + 4*(3*cos(d*x + c)^2 - 1)*sin(d
*x + c) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^{4}(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 17.14, size = 120, normalized size = 1.74

$$\frac{\frac{5\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}-\frac{45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+490\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+73}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(9*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 11)/(a*(tan(
1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^4 + 240*tan(1/2*d*x + 1
/2*c)^3 + 490*tan(1/2*d*x + 1/2*c)^2 + 320*tan(1/2*d*x + 1/2*c) + 73)/(a*(t
an(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 6.73, size = 73, normalized size = 1.06

$$\frac{16\,\left(-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{15\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + a*sin(c + d*x)),x)
```

```
[Out] (16*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2)^3
 + 1))/(15*a*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.55 $\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\tan^3(c+dx)}{3ad}$$

```
[Out] sec(d*x+c)/a/d-1/3*sec(d*x+c)^3/a/d+1/3*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2785, 2687, 30, 2686}

$$\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a}-\frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^2\,dx,x,\tan(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\sec(c+dx)\right)}{ad}\\ &= \frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\tan^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(50) = 100.
time = 0.11, size = 106, normalized size = 2.12

$$\frac{6-10\cos(c+dx)+2\cos(2(c+dx))+8\sin(c+dx)-5\sin(2(c+dx))}{12ad\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6 - 10*Cos[c + d*x] + 2*Cos[2*(c + d*x)] + 8*Sin[c + d*x] - 5*Sin[2*(c + d
*x)])/(12*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.19, size = 70, normalized size = 1.40

method	result	size
derivativedivides	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{8}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16}}{da}$	70
default	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{8}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16}}{da}$	70
risch	$\frac{2i\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{3i(dx+c)}+\frac{2i}{3}-\frac{2\,\mathrm{e}^{i(dx+c)}}{3}}{\left(\mathrm{e}^{i(dx+c)}-i\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^{3}da}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a*(-1/16/(tan(1/2*d*x+1/2*c)-1)-1/12/(tan(1/2*d*x+1/2*c)+1)^3+1/8/(tan(
1/2*d*x+1/2*c)+1)^2+1/16/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.31, size = 90, normalized size = 1.80

$$\frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4/3*(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x
 + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4)*d)
```

Fricas [A]
time = 0.34, size = 47, normalized size = 0.94

$$\frac{\cos(dx+c)^2+2\sin(dx+c)+1}{3\left(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^2 + 2*sin(d*x + c) + 1)/(a*d*cos(d*x + c)*sin(d*x + c) +
a*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 5.99, size = 68, normalized size = 1.36

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) - (3*tan(1/2*d*x + 1/2*c)^2 + 12*tan
(1/2*d*x + 1/2*c) + 5)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 6.40, size = 47, normalized size = 0.94

$$-\frac{4\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

```
[Out] -(4*(2*tan(c/2 + (d*x)/2) + 1))/(3*a*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 +
(d*x)/2) + 1)^3)
```

3.56 $\int \frac{1}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}$$

```
[Out] -cos(d*x+c)/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2727}

$$-\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-1),x]
```

```
[Out] -(Cos[c + d*x]/(d*(a + a*Sin[c + d*x])))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\sin(c+dx)}\,dx = -\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(23) = 46.
time = 0.03, size = 48, normalized size = 2.09

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d(a+a\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*(a + a*Sin[c
+ d*x]))
```

Maple [A]
time = 0.09, size = 22, normalized size = 0.96

method	result	size
derivativedivides	$-\frac{2}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	22
default	$-\frac{2}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	22
risch	$-\frac{2}{da\left(e^{i(dx+c)}+i\right)}$	23
norman	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -2/d/a/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A]
time = 0.30, size = 27, normalized size = 1.17

$$-\frac{2}{\left(a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -2/((a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

Fricas [A]
time = 0.34, size = 42, normalized size = 1.83

$$-\frac{\cos\left(dx+c\right)-\sin\left(dx+c\right)+1}{ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -(cos(d*x + c) - sin(d*x + c) + 1)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a
*d)
```

Sympy [A]
time = 0.37, size = 27, normalized size = 1.17

$$\begin{cases} -\frac{2}{ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for}\ d\neq 0 \\ \frac{x}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-2/(a*d*tan(c/2 + d*x/2) + a*d), Ne(d, 0)), (x/(a*sin(c) + a), T
rue))
```

Giac [A]
time = 8.19, size = 21, normalized size = 0.91

$$-\frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/(a*d*(tan(1/2*d*x + 1/2*c) + 1))
```

Mupad [B]
time = 6.43, size = 21, normalized size = 0.91

$$-\frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sin(c + d*x)),x)
```

```
[Out] -2/(a*d*(tan(c/2 + (d*x)/2) + 1))
```

3.57 $\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

```
[Out] arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2785, 3852, 8, 3855}

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \csc(c+dx)\,dx}{a}+\frac{\int \csc^2(c+dx)\,dx}{a}\\
&= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{ad}\\
&= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 69 vs. $2(29) = 58$.
time = 0.18, size = 69, normalized size = 2.38

$$
-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)+\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)}{2ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x] + (-Log[Cos[(c + d*x)
/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]))/(a*d)
```

Maple [A]
time = 0.18, size = 44, normalized size = 1.52

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{2da}$	44
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{2da}$	44
risch	$-\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}-\frac{\ln\left(e^{i(dx+c)}-1\right)}{da}+\frac{\ln\left(e^{i(dx+c)}+1\right)}{da}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a*(tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+1/2*c))-1/tan(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(29) = 58$.
time = 0.28, size = 70, normalized size = 2.41

$$
-\frac{\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\cos(dx+c)+1}{a\sin(dx+c)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (cos(d*x + c) + 1)/(a*sin(
d*x + c)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(29) = 58$.
time = 0.35, size = 62, normalized size = 2.14

$$\frac{\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)}{2\,ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - log(-1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^2\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.
time = 10.36, size = 65, normalized size = 2.24

$$-\frac{\frac{2\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a - tan(1/2*d*x + 1/2*c)/a - (2*tan(
1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 6.64, size = 25, normalized size = 0.86

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\cot(c+d\,x)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

```
[Out] -(log(tan(c/2 + (d*x)/2)) + cot(c + d*x))/(a*d)
```

3.58 $\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d-1/3*cot(d*x+c)^3/a/d+1/2*cot(d*x+c)*csc(d*x+c)
/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2691, 3855}

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*C
sc[c + d*x])/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \csc(c+dx)\,dx}{2a}+\frac{\text{Subst}\left(\int x^2\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(58) = 116.
time = 0.36, size = 124, normalized size = 2.14

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\cos(3(c+dx))+\cos(c+dx)(3-6\sin(c+dx))+6\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^3(c+dx)\right)}{96ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/96*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/
2])^2*(Cos[3*(c + d*x)] + Cos[c + d*x]*(3 - 6*Sin[c + d*x]) + 6*(Log[Cos[(c
 + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3))/(a*d*(1 + Sin[c + d*x
]))
```

Maple [A]
time = 0.21, size = 94, normalized size = 1.62

method	result	siz
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{8da}$	94
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{8da}$	94

risch	$-\frac{-6i e^{4i(dx+c)}+3 e^{5i(dx+c)}-2i-3 e^{i(dx+c)}}{3da\left(e^{2i(dx+c)}-1\right)^3}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2da}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a*(1/3*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)-1
/3/tan(1/2*d*x+1/2*c)^3+4*ln(tan(1/2*d*x+1/2*c))+1/tan(1/2*d*x+1/2*c)+1/tan
(1/2*d*x+1/2*c)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(52) = 104$.
time = 0.29, size = 155, normalized size = 2.67

$$-\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - 12*log(sin(d*x + c)/(cos(
d*x + c) + 1))/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(52) = 104$.
time = 0.36, size = 111, normalized size = 1.91

$$\frac{4\cos(dx+c)^3-3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-6\cos(dx+c)\sin(dx+c)}{12\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 6*cos(d*x + c)*sin(d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x +
c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(52) = 104.
time = 8.76, size = 127, normalized size = 2.19

$$\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{22\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 3
*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c))/a^3 - (22*tan(1/2
*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/(a
*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 6.63, size = 115, normalized size = 1.98

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{3}\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a*sin(c + d*x)),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) + log(tan(c/2
+ (d*x)/2))/(2*a*d) - tan(c/2 + (d*x)/2)/(8*a*d) + (cot(c/2 + (d*x)/2)^3*(t
an(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2 - 1/3))/(8*a*d)
```

3.59 $\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}$$

```
[Out] 3/8*arctanh(cos(d*x+c))/a/d-1/5*cot(d*x+c)^5/a/d-3/8*cot(d*x+c)*csc(d*x+c)/
a/d+1/4*cot(d*x+c)^3*csc(d*x+c)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2691, 3855}

$$-\frac{\cot^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]^5/(5*a*d) - (3*Cot[c + d*x
]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot^3(c+dx)\csc(c+dx)}{4ad}+\frac{3\int \cot^2(c+dx)\csc(c+dx)\,dx}{4a}+\frac{\text{Subst}\left(\int x^4\,dx,x,-c\right.}{ad}\\ &= -\frac{\cot^5(c+dx)}{5ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\int \csc(c}{8}\\ &= \frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot^3(c+dx)}{4a}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(82) = 164.
time = 0.52, size = 189, normalized size = 2.30

$$-\frac{\csc^5(c+dx)\left(80\cos(c+dx)+40\cos(3(c+dx))+8\cos(5(c+dx))-150\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+150\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+20\sin(2(c+dx))+75\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-75\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-50\sin(4(c+dx))-15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))+15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{640ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/640*(Csc[c + d*x]^5*(80*Cos[c + d*x] + 40*Cos[3*(c + d*x)] + 8*Cos[5*(c
+ d*x)] - 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 150*Log[Sin[(c + d*x)/2]
]*Sin[c + d*x] + 20*Sin[2*(c + d*x)] + 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c +
d*x)] - 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 50*Sin[4*(c + d*x)] - 1
5*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*Log[Sin[(c + d*x)/2]]*Sin[5*(
c + d*x)]))/(a*d)
```

Maple [A]
time = 0.23, size = 148, normalized size = 1.80

method	result

risch	$\frac{-40i e^{8i(dx+c)}+25 e^{9i(dx+c)}-10 e^{7i(dx+c)}-80i e^{4i(dx+c)}+10 e^{3i(dx+c)}-8i-25 e^{i(dx+c)}}{20da\left(e^{2i(dx+c)}-1\right)^5}-\frac{3\ln\left(e^{i(dx+c)}-1\right)}{8da}+\frac{3\ln(e^{\cdot}$
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-}{32da}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-}{32da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a*(1/5*tan(1/2*d*x+1/2*c)^5-1/2*tan(1/2*d*x+1/2*c)^4-tan(1/2*d*x+1/2
*c)^3+4*tan(1/2*d*x+1/2*c)^2+2*tan(1/2*d*x+1/2*c)-4/tan(1/2*d*x+1/2*c)^2-1/
5/tan(1/2*d*x+1/2*c)^5+1/tan(1/2*d*x+1/2*c)^3-12*ln(tan(1/2*d*x+1/2*c))+1/2
/tan(1/2*d*x+1/2*c)^4-2/tan(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(74) = 148$.
time = 0.30, size = 234, normalized size = 2.85

$$\frac{\frac{\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{20\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-2\right)(\cos(dx+c)+1)^5}{a\sin(dx+c)^5}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/320*((20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 120*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a + (5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2
0*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2)*(cos(d*x + c) + 1)^5/(a*sin(d*x
+ c)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(74) = 148$.
time = 0.35, size = 155, normalized size = 1.89

$$-\frac{16\cos(dx+c)^5-15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-10\left(5\cos(dx+c)^3-3\cos(dx+c)\right)\sin(dx+c)}{80\left(ad\cos(dx+c)^4-2ad\cos(dx+c)^2+ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/80*(16*cos(d*x + c)^5 - 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2
```

```
 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(5*cos(d*x + c)^3 - 3*
cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a
*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{6}{\left(c+dx \right)}}{\sin{\left(c+dx \right)}+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**6/(sin(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(74) = 148.
time = 8.24, size = 187, normalized size = 2.28

$$-\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{2\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^5}-\frac{274\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/320*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (2*a^4*tan(1/2*d*x + 1/2*c)^
5 - 5*a^4*tan(1/2*d*x + 1/2*c)^4 - 10*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^4*t
an(1/2*d*x + 1/2*c)^2 + 20*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (274*tan(1/2*d*x
 + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x + 1/2*c)^3 + 10*ta
n(1/2*d*x + 1/2*c)^2 + 5*tan(1/2*d*x + 1/2*c) - 2)/(a*tan(1/2*d*x + 1/2*c)^
5))/d
```

Mupad [B]
time = 6.66, size = 183, normalized size = 2.23

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a\,d}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{1}{5}\right)}{32\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a*sin(c + d*x)),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(32*a*d) - tan(c/2 + (d
*x)/2)^4/(64*a*d) + tan(c/2 + (d*x)/2)^5/(160*a*d) - (3*log(tan(c/2 + (d*x)
/2)))/(8*a*d) + tan(c/2 + (d*x)/2)/(16*a*d) - (cot(c/2 + (d*x)/2)^5*(4*tan(
c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)/2 + 2*tan(c/2
+ (d*x)/2)^4 + 1/5))/(32*a*d)
```

3.60 $\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}$$

```
[Out] -5/16*arctanh(cos(d*x+c))/a/d-1/7*cot(d*x+c)^7/a/d+5/16*cot(d*x+c)*csc(d*x+
c)/a/d-5/24*cot(d*x+c)^3*csc(d*x+c)/a/d+1/6*cot(d*x+c)^5*csc(d*x+c)/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2691, 3855}

$$-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a*d) - Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(16*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) + (Co
t[c + d*x]^5*Csc[c + d*x])/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^6(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot^5(c+dx)\csc(c+dx)}{6ad}+\frac{5\int \cot^4(c+dx)\csc(c+dx)\,dx}{6a}+\frac{\text{Subst}\left(\int x^6\,dx,x,-c\right.}{ad}\\ &= -\frac{\cot^7(c+dx)}{7ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\int \cot^2}{}\\ &= -\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{\cot^5(c}{}\\ &= -\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}-\frac{5\cot^3(c+}{2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 284 vs. 2(106) = 212.
time = 0.63, size = 284, normalized size = 2.68

$$-\frac{\csc^5(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(1680\cos(c+dx)+1008\cos(3(c+dx))+336\cos(5(c+dx))+48\cos(7(c+dx))+3675\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-3675\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-1190\sin(2(c+dx))-2205\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+2205\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+392\sin(4(c+dx))+735\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-735\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-462\sin(6(c+dx))-105\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))+105\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))\right)}{86016ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/86016*(Csc[c + d*x]^5*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(1680*Cos[
c + d*x] + 1008*Cos[3*(c + d*x)] + 336*Cos[5*(c + d*x)] + 48*Cos[7*(c + d*x
)] + 3675*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 3675*Log[Sin[(c + d*x)/2]]*S
in[c + d*x] - 1190*Sin[2*(c + d*x)] - 2205*Log[Cos[(c + d*x)/2]]*Sin[3*(c +
 d*x)] + 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 392*Sin[4*(c + d*x)]
 + 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 735*Log[Sin[(c + d*x)/2]]*S
in[5*(c + d*x)] - 462*Sin[6*(c + d*x)] - 105*Log[Cos[(c + d*x)/2]]*Sin[7*(c
 + d*x)] + 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)]))/(a*d*(1 + Sin[c + d
*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(96) = 192$.
time = 0.25, size = 200, normalized size = 1.89

method	result
risch	$-\frac{-336i e^{12i(dx+c)}+231\,e^{13i(dx+c)}-196\,e^{11i(dx+c)}-1680i e^{8i(dx+c)}+595\,e^{9i(dx+c)}-1008i e^{4i(dx+c)}-595\,e^{5i(dx+c)}+196}{168da\left(e^{2i(dx+c)}-1\right)^7}$
derivativedivides	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-15\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\tan(}{}$
default	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-15\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\tan(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/d/a*(1/7*tan(1/2*d*x+1/2*c)^7-1/3*tan(1/2*d*x+1/2*c)^6-tan(1/2*d*x+1/
2*c)^5+3*tan(1/2*d*x+1/2*c)^4+3*tan(1/2*d*x+1/2*c)^3-15*tan(1/2*d*x+1/2*c)^
2-5*tan(1/2*d*x+1/2*c)+1/tan(1/2*d*x+1/2*c)^5+1/3/tan(1/2*d*x+1/2*c)^6-3/ta
n(1/2*d*x+1/2*c)^4-1/7/tan(1/2*d*x+1/2*c)^7-3/tan(1/2*d*x+1/2*c)^3+15/tan(1
/2*d*x+1/2*c)^2+5/tan(1/2*d*x+1/2*c)+40*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(96) = 192$.
time = 0.31, size = 315, normalized size = 2.97

$$-\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{315\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{63\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{63\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{7\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a}-\frac{840\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{63\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{63\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{315\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{105\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-3\right)(\cos(dx+c)+1)^7}{a\sin(dx+c)^7}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2688*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 315*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 63*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 7*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a - 840*
log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (7*sin(d*x + c)/(cos(d*x + c) + 1)
 + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 63*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 315*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 105*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3)*(cos(d*x + c
) + 1)^7/(a*sin(d*x + c)^7))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(96) = 192$.
time = 0.37, size = 198, normalized size = 1.87

$$\frac{96\cos(dx+c)^7-105\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+105\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-14\left(33\cos(dx+c)^5-40\cos(dx+c)^3+15\cos(dx+c)\right)\sin(dx+c)}{672\left(ad\cos(dx+c)^6-3ad\cos(dx+c)^4+3ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/672*(96*cos(d*x + c)^7 - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d
*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x + c)
^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 14*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))
*sin(d*x + c))/((a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x
+ c)^2 - a*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^8(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**8/(sin(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(96) = 192.
time = 12.65, size = 244, normalized size = 2.30

$$\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{3a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-7a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-21a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+63a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+63a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-315a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-105a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^7}-\frac{2178\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2688*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a + (3*a^6*tan(1/2*d*x + 1/2*c)^
7 - 7*a^6*tan(1/2*d*x + 1/2*c)^6 - 21*a^6*tan(1/2*d*x + 1/2*c)^5 + 63*a^6*t
an(1/2*d*x + 1/2*c)^4 + 63*a^6*tan(1/2*d*x + 1/2*c)^3 - 315*a^6*tan(1/2*d*x
 + 1/2*c)^2 - 105*a^6*tan(1/2*d*x + 1/2*c))/a^7 - (2178*tan(1/2*d*x + 1/2*c
)^7 - 105*tan(1/2*d*x + 1/2*c)^6 - 315*tan(1/2*d*x + 1/2*c)^5 + 63*tan(1/2*
d*x + 1/2*c)^4 + 63*tan(1/2*d*x + 1/2*c)^3 - 21*tan(1/2*d*x + 1/2*c)^2 - 7*
tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/2*c)^7))/d
```

Mupad [B]
time = 8.22, size = 387, normalized size = 3.65

$$\frac{3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9-105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5-63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+840\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{2688\,a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8/(a + a*sin(c + d*x)),x)
```

```
[Out] (3*sin(c/2 + (d*x)/2)^14 - 3*cos(c/2 + (d*x)/2)^14 - 7*cos(c/2 + (d*x)/2)*s
in(c/2 + (d*x)/2)^13 + 7*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) - 21*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 + 63*cos(c/2 + (d*x)/2)^3*sin(c/2 +
(d*x)/2)^11 + 63*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 315*cos(c/2 +
 (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 105*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)
/2)^8 + 105*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 315*cos(c/2 + (d*x)
/2)^9*sin(c/2 + (d*x)/2)^5 - 63*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4
- 63*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 21*cos(c/2 + (d*x)/2)^12*
sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c
/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/(2688*a*d*cos(c/2 + (d*x)/2)^7*sin(c/
2 + (d*x)/2)^7)
```

3.61 $\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$-\frac{7\tanh^{-1}(\sin(c+dx))}{128a^2d}+\frac{a}{192d(a-a\sin(c+dx))^3}-\frac{1}{32d(a-a\sin(c+dx))^2}+\frac{a^3}{80d(a+a\sin(c+dx))^5}-\frac{}{64d(}$$

```
[Out] -7/128*arctanh(sin(d*x+c))/a^2/d+1/192*a/d/(a-a*sin(d*x+c))^3-1/32/d/(a-a*s
in(d*x+c))^2+1/80*a^3/d/(a+a*sin(d*x+c))^5-5/64*a^2/d/(a+a*sin(d*x+c))^4+19
/96*a/d/(a+a*sin(d*x+c))^3-1/4/d/(a+a*sin(d*x+c))^2+21/256/d/(a^2-a^2*sin(d
*x+c))+35/256/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2786, 90, 212}

$$\frac{a^3}{80d(a\sin(c+dx)+a)^5}-\frac{5a^2}{64d(a\sin(c+dx)+a)^4}+\frac{21}{256d\,(a^2-a^2\sin(c+dx))}+\frac{35}{256d\,(a^2\sin(c+dx)+a^2)}-\frac{7\tanh^{-1}(\sin(c+dx))}{128a^2d}+\frac{a}{192d(a-a\sin(c+dx))^3}+\frac{19a}{96d(a\sin(c+dx)+a)^3}-\frac{1}{32d(a-a\sin(c+dx))^2}-\frac{1}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(128*a^2*d) + a/(192*d*(a - a*Sin[c + d*x])^3) -
 1/(32*d*(a - a*Sin[c + d*x])^2) + a^3/(80*d*(a + a*Sin[c + d*x])^5) - (5*a
^2)/(64*d*(a + a*Sin[c + d*x])^4) + (19*a)/(96*d*(a + a*Sin[c + d*x])^3) -
1/(4*d*(a + a*Sin[c + d*x])^2) + 21/(256*d*(a^2 - a^2*Sin[c + d*x])) + 35/(
256*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
```

```
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^7}{(a-x)^4(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{64(a-x)^4} - \frac{1}{16(a-x)^3} + \frac{21}{256a(a-x)^2} - \frac{a^3}{16(a+x)^6} + \frac{5a^2}{16(a+x)^5} - \frac{19a}{32(a+x)^4} + \frac{}{2(a}\right.\right.}{d}$$
$$= \frac{a}{192d(a-a\sin(c+dx))^3} - \frac{1}{32d(a-a\sin(c+dx))^2} + \frac{a^3}{80d(a+a\sin(c+dx))^5}$$
$$= -\frac{7\tanh^{-1}(\sin(c+dx))}{128a^2d} + \frac{a}{192d(a-a\sin(c+dx))^3} - \frac{1}{32d(a-a\sin(c+dx))^2}$$

Mathematica [A]
time = 1.06, size = 112, normalized size = 0.59

$$-\frac{210\tanh^{-1}(\sin(c+dx)) - \frac{2(-144-393\sin(c+dx)+78\sin^2(c+dx)+1039\sin^3(c+dx)+560\sin^4(c+dx)-815\sin^5(c+dx)-750\sin^6(c+dx)+105\sin^7(c+dx))}{(-1+\sin(c+dx))^3(1+\sin(c+dx))^5}}{3840a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
[Out] -1/3840*(210*ArcTanh[Sin[c + d*x]] - (2*(-144 - 393*Sin[c + d*x] + 78*Sin[c
 + d*x]^2 + 1039*Sin[c + d*x]^3 + 560*Sin[c + d*x]^4 - 815*Sin[c + d*x]^5 -
 750*Sin[c + d*x]^6 + 105*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^3*(1 + Sin[
c + d*x])^5))/(a^2*d)
```

Maple [A]
time = 0.27, size = 127, normalized size = 0.67

method	result
derivativedivides	$\frac{-\frac{1}{192(\sin(dx+c)-1)^3}-\frac{1}{32(\sin(dx+c)-1)^2}-\frac{21}{256(\sin(dx+c)-1)}+\frac{7\ln(\sin(dx+c)-1)}{256}+\frac{1}{80(1+\sin(dx+c))^5}-\frac{5}{64(1+\sin(dx+c))^4}+\frac{}{96(1}}{d\,a^2}$
default	$\frac{-\frac{1}{192(\sin(dx+c)-1)^3}-\frac{1}{32(\sin(dx+c)-1)^2}-\frac{21}{256(\sin(dx+c)-1)}+\frac{7\ln(\sin(dx+c)-1)}{256}+\frac{1}{80(1+\sin(dx+c))^5}-\frac{5}{64(1+\sin(dx+c))^4}+\frac{}{96(1}}{d\,a^2}$
risch	$\frac{i(-2525\,e^{3i(dx+c)}-2529\,e^{5i(dx+c)}+2529\,e^{11i(dx+c)}+2525\,e^{13i(dx+c)}-105\,e^{i(dx+c)}-2084i e^{6i(dx+c)}-4205\,e^{7i(dx+c)}+42}{960(e^{i(dx+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/192/(sin(d*x+c)-1)^3-1/32/(sin(d*x+c)-1)^2-21/256/(sin(d*x+c)-1
)+7/256*ln(sin(d*x+c)-1)+1/80/(1+sin(d*x+c))^5-5/64/(1+sin(d*x+c))^4+19/96/
(1+sin(d*x+c))^3-1/4/(1+sin(d*x+c))^2+35/256/(1+sin(d*x+c))-7/256*ln(1+sin(
d*x+c)))
```

Maxima [A]
time = 0.28, size = 202, normalized size = 1.07

$$\frac{\frac{2\left(105\,\sin\left(dx+c\right)^{7}-750\,\sin\left(dx+c\right)^{6}-815\,\sin\left(dx+c\right)^{5}+560\,\sin\left(dx+c\right)^{4}+1039\,\sin\left(dx+c\right)^{3}+78\,\sin\left(dx+c\right)^{2}-393\,\sin\left(dx+c\right)-144\right)}{a^{2}\sin\left(dx+c\right)^{8}+2\,a^{2}\sin\left(dx+c\right)^{7}-2\,a^{2}\sin\left(dx+c\right)^{6}-6\,a^{2}\sin\left(dx+c\right)^{5}+6\,a^{2}\sin\left(dx+c\right)^{3}+2\,a^{2}\sin\left(dx+c\right)^{2}-2\,a^{2}\sin\left(dx+c\right)-a^{2}}-\frac{105\,\log\left(\sin\left(dx+c\right)+1\right)}{a^{2}}+\frac{105\,\log\left(\sin\left(dx+c\right)-1\right)}{a^{2}}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3840*(2*(105*sin(d*x + c)^7 - 750*sin(d*x + c)^6 - 815*sin(d*x + c)^5 + 5
60*sin(d*x + c)^4 + 1039*sin(d*x + c)^3 + 78*sin(d*x + c)^2 - 393*sin(d*x +
 c) - 144)/(a^2*sin(d*x + c)^8 + 2*a^2*sin(d*x + c)^7 - 2*a^2*sin(d*x + c)^
6 - 6*a^2*sin(d*x + c)^5 + 6*a^2*sin(d*x + c)^3 + 2*a^2*sin(d*x + c)^2 - 2*
a^2*sin(d*x + c) - a^2) - 105*log(sin(d*x + c) + 1)/a^2 + 105*log(sin(d*x +
 c) - 1)/a^2)/d
```

Fricas [A]
time = 0.40, size = 218, normalized size = 1.15

$$\frac{1500\,\cos\left(dx+c\right)^{6}-3380\,\cos\left(dx+c\right)^{4}+2104\,\cos\left(dx+c\right)^{2}-105\,\left(\cos\left(dx+c\right)^{8}-2\,\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)^{6}\right)\log\left(\sin\left(dx+c\right)+1\right)+105\,\left(\cos\left(dx+c\right)^{8}-2\,\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)^{6}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(105\,\cos\left(dx+c\right)^{6}+500\,\cos\left(dx+c\right)^{4}-276\,\cos\left(dx+c\right)^{2}+64\right)\sin\left(dx+c\right)-512}{3840\,\left(a^{2}d\cos\left(dx+c\right)^{8}-2\,a^{2}d\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+c\right)^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3840*(1500*cos(d*x + c)^6 - 3380*cos(d*x + c)^4 + 2104*cos(d*x + c)^2 - 1
05*(cos(d*x + c)^8 - 2*cos(d*x + c)^6*sin(d*x + c) - 2*cos(d*x + c)^6)*log(
sin(d*x + c) + 1) + 105*(cos(d*x + c)^8 - 2*cos(d*x + c)^6*sin(d*x + c) - 2
*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(105*cos(d*x + c)^6 + 500*cos(d
*x + c)^4 - 276*cos(d*x + c)^2 + 64)*sin(d*x + c) - 512)/(a^2*d*cos(d*x + c
)^8 - 2*a^2*d*cos(d*x + c)^6*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{7}{\left(c+dx \right)}}{\sin^{2}{\left(c+dx \right)}+2\sin{\left(c+dx \right)}+1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**7/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 14.21, size = 146, normalized size = 0.77

$$-\frac{\frac{420 \log\left(|\sin\left(dx+c\right)+1|\right)}{a^2}-\frac{420 \log\left(|\sin\left(dx+c\right)-1|\right)}{a^2}+\frac{10 \left(77 \sin\left(dx+c\right)^3-105 \sin\left(dx+c\right)^2+27 \sin\left(dx+c\right)+9\right)}{a^2 \left(\sin\left(dx+c\right)-1\right)^3}-\frac{959 \sin\left(dx+c\right)^5+6895 \sin\left(dx+c\right)^4+14150 \sin\left(dx+c\right)^3+13710 \sin\left(dx+c\right)^2+6555 \sin\left(dx+c\right)+1251}{a^2 \left(\sin\left(dx+c\right)+1\right)^5}}{15360\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/15360*(420*log(abs(sin(d*x + c) + 1))/a^2 - 420*log(abs(sin(d*x + c) - 1
))/a^2 + 10*(77*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 27*sin(d*x + c) + 9)/
(a^2*(sin(d*x + c) - 1)^3) - (959*sin(d*x + c)^5 + 6895*sin(d*x + c)^4 + 14
150*sin(d*x + c)^3 + 13710*sin(d*x + c)^2 + 6555*sin(d*x + c) + 1251)/(a^2*
(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 10.46, size = 444, normalized size = 2.35

$$\frac{\frac{7 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}+\frac{7 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{16}+\frac{7 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192}-\frac{49 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{24}-\frac{693 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{320}+\frac{791 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{240}+\frac{1207 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}+\frac{123 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{4}+\frac{1207 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{192}+\frac{791 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{240}-\frac{693 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{320}-\frac{49 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{24}+\frac{7 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{192}+\frac{7 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{16}+\frac{7 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d \left(a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+4 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}-20 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-20 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+36 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+64 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-20 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-90 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-20 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+64 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+36 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}-20 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-20 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+4 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2\right)}-\frac{7 \mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7/(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((7*tan(c/2 + (d*x)/2))/64 + (7*tan(c/2 + (d*x)/2)^2)/16 + (7*tan(c/2 + (d*
x)/2)^3)/192 - (49*tan(c/2 + (d*x)/2)^4)/24 - (693*tan(c/2 + (d*x)/2)^5)/32
0 + (791*tan(c/2 + (d*x)/2)^6)/240 + (1207*tan(c/2 + (d*x)/2)^7)/192 + (123
*tan(c/2 + (d*x)/2)^8)/4 + (1207*tan(c/2 + (d*x)/2)^9)/192 + (791*tan(c/2 +
 (d*x)/2)^10)/240 - (693*tan(c/2 + (d*x)/2)^11)/320 - (49*tan(c/2 + (d*x)/2
)^12)/24 + (7*tan(c/2 + (d*x)/2)^13)/192 + (7*tan(c/2 + (d*x)/2)^14)/16 + (
7*tan(c/2 + (d*x)/2)^15)/64)/(d*(36*a^2*tan(c/2 + (d*x)/2)^5 - 20*a^2*tan(c
/2 + (d*x)/2)^4 - 20*a^2*tan(c/2 + (d*x)/2)^3 + 64*a^2*tan(c/2 + (d*x)/2)^6
 - 20*a^2*tan(c/2 + (d*x)/2)^7 - 90*a^2*tan(c/2 + (d*x)/2)^8 - 20*a^2*tan(c
/2 + (d*x)/2)^9 + 64*a^2*tan(c/2 + (d*x)/2)^10 + 36*a^2*tan(c/2 + (d*x)/2)^
11 - 20*a^2*tan(c/2 + (d*x)/2)^12 - 20*a^2*tan(c/2 + (d*x)/2)^13 + 4*a^2*ta
n(c/2 + (d*x)/2)^15 + a^2*tan(c/2 + (d*x)/2)^16 + a^2 + 4*a^2*tan(c/2 + (d*
x)/2))) - (7*atanh(tan(c/2 + (d*x)/2)))/(64*a^2*d)
```

3.62 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$\frac{5\tanh^{-1}(\sin(c+dx))}{64a^2d}+\frac{1}{64d(a-a\sin(c+dx))^2}+\frac{a^2}{32d(a+a\sin(c+dx))^4}-\frac{7a}{48d(a+a\sin(c+dx))^3}+\frac{}{4d(a+}$$

```
[Out] 5/64*arctanh(sin(d*x+c))/a^2/d+1/64/d/(a-a*sin(d*x+c))^2+1/32*a^2/d/(a+a*si
n(d*x+c))^4-7/48*a/d/(a+a*sin(d*x+c))^3+1/4/d/(a+a*sin(d*x+c))^2-5/64/d/(a^
2-a^2*sin(d*x+c))-5/32/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2786, 90, 212}

$$\frac{a^2}{32d(a\sin(c+dx)+a)^4}-\frac{5}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{7a}{48d(a\sin(c+dx)+a)^3}+\frac{1}{64d(a-a\sin(c+dx))^2}+\frac{1}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + 1/(64*d*(a - a*Sin[c + d*x])^2) + a^
2/(32*d*(a + a*Sin[c + d*x])^4) - (7*a)/(48*d*(a + a*Sin[c + d*x])^3) + 1/(
4*d*(a + a*Sin[c + d*x])^2) - 5/(64*d*(a^2 - a^2*Sin[c + d*x])) - 5/(32*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{32(a-x)^3} - \frac{5}{64a(a-x)^2} - \frac{a^2}{8(a+x)^5} + \frac{7a}{16(a+x)^4} - \frac{1}{2(a+x)^3} + \frac{5}{32a(a+x)^2} + \frac{}{64a(a\right.\right.}{d}$$

$$= \frac{1}{64d(a-a\sin(c+dx))^2} + \frac{a^2}{32d(a+a\sin(c+dx))^4} - \frac{7a}{48d(a+a\sin(c+dx))^3}$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{64a^2d} + \frac{1}{64d(a-a\sin(c+dx))^2} + \frac{a^2}{32d(a+a\sin(c+dx))^4} -$$

Mathematica [A]
time = 0.30, size = 91, normalized size = 0.62

$$\frac{15\tanh^{-1}(\sin(c+dx)) + \frac{-16-47\sin(c+dx)-14\sin^2(c+dx)+74\sin^3(c+dx)+66\sin^4(c+dx)-15\sin^5(c+dx)}{(-1+\sin(c+dx))^2(1+\sin(c+dx))^4}}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] + (-16 - 47*Sin[c + d*x] - 14*Sin[c + d*x]^2 + 74
*Sin[c + d*x]^3 + 66*Sin[c + d*x]^4 - 15*Sin[c + d*x]^5)/((-1 + Sin[c + d*x
])^2*(1 + Sin[c + d*x])^4))/(192*a^2*d)
```

Maple [A]
time = 0.27, size = 103, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{1}{64(\sin(dx+c)-1)^2}+\frac{5}{64(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{128}+\frac{1}{32(1+\sin(dx+c))^4}-\frac{7}{48(1+\sin(dx+c))^3}+\frac{1}{4(1+\sin(dx+c))^2}-\frac{5}{32(1+\sin(}}{d\,a^2}$
default	$\frac{\frac{1}{64(\sin(dx+c)-1)^2}+\frac{5}{64(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{128}+\frac{1}{32(1+\sin(dx+c))^4}-\frac{7}{48(1+\sin(dx+c))^3}+\frac{1}{4(1+\sin(dx+c))^2}-\frac{5}{32(1+\sin(}}{d\,a^2}$
risch	$-\frac{i\left(-56i e^{6i(dx+c)}-221\,e^{3i(dx+c)}+416i e^{4i(dx+c)}-14\,e^{5i(dx+c)}-15\,e^{i(dx+c)}-132i e^{2i(dx+c)}+14\,e^{7i(dx+c)}+416i e^{8i(dx+c}\right.}{96\left(e^{i(dx+c)}+i\right)^8\left(e^{i(dx+c)}-i\right)^4 d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/64/(sin(d*x+c)-1)^2+5/64/(sin(d*x+c)-1)-5/128*ln(sin(d*x+c)-1)+1
/32/(1+sin(d*x+c))^4-7/48/(1+sin(d*x+c))^3+1/4/(1+sin(d*x+c))^2-5/32/(1+sin
(d*x+c))+5/128*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 167, normalized size = 1.14

$$-\frac{\frac{2\left(15\sin(dx+c)^5-66\sin(dx+c)^4-74\sin(dx+c)^3+14\sin(dx+c)^2+47\sin(dx+c)+16\right)}{a^2\sin(dx+c)^6+2a^2\sin(dx+c)^5-a^2\sin(dx+c)^4-4a^2\sin(dx+c)^3-a^2\sin(dx+c)^2+2a^2\sin(dx+c)+a^2}-\frac{15\log(\sin(dx+c)+1)}{a^2}+\frac{15\log(\sin(dx+c)-1)}{a^2}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/384*(2*(15*sin(d*x + c)^5 - 66*sin(d*x + c)^4 - 74*sin(d*x + c)^3 + 14*s
in(d*x + c)^2 + 47*sin(d*x + c) + 16)/(a^2*sin(d*x + c)^6 + 2*a^2*sin(d*x +
 c)^5 - a^2*sin(d*x + c)^4 - 4*a^2*sin(d*x + c)^3 - a^2*sin(d*x + c)^2 + 2*
a^2*sin(d*x + c) + a^2) - 15*log(sin(d*x + c) + 1)/a^2 + 15*log(sin(d*x + c
) - 1)/a^2)/d
```

Fricas [A]
time = 0.37, size = 198, normalized size = 1.36

$$-\frac{132\cos(dx+c)^4-236\cos(dx+c)^2-15\left(\cos(dx+c)^6-2\cos(dx+c)^4\sin(dx+c)-2\cos(dx+c)^4\right)\log(\sin(dx+c)+1)+15\left(\cos(dx+c)^6-2\cos(dx+c)^4\sin(dx+c)-2\cos(dx+c)^4\right)\log(-\sin(dx+c)+1)-2\left(15\cos(dx+c)^4+44\cos(dx+c)^2-12\right)\sin(dx+c)+72}{384\left(a^2d\cos(dx+c)^6-2a^2d\cos(dx+c)^4\sin(dx+c)-2a^2d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(132*cos(d*x + c)^4 - 236*cos(d*x + c)^2 - 15*(cos(d*x + c)^6 - 2*co
s(d*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(sin(d*x + c) + 1) + 15*(c
os(d*x + c)^6 - 2*cos(d*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(-sin(
d*x + c) + 1) - 2*(15*cos(d*x + c)^4 + 44*cos(d*x + c)^2 - 12)*sin(d*x + c)
 + 72)/(a^2*d*cos(d*x + c)^6 - 2*a^2*d*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*
d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^5(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 8.19, size = 126, normalized size = 0.86

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^2}-\frac{60\log(|\sin(dx+c)-1|)}{a^2}+\frac{6\left(15\sin(dx+c)^2-10\sin(dx+c)-1\right)}{a^2(\sin(dx+c)-1)^2}-\frac{125\sin(dx+c)^4+740\sin(dx+c)^3+1086\sin(dx+c)^2+676\sin(dx+c)+157}{a^2(\sin(dx+c)+1)^4}}{1536d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(60*log(abs(sin(d*x + c) + 1))/a^2 - 60*log(abs(sin(d*x + c) - 1))/a
^2 + 6*(15*sin(d*x + c)^2 - 10*sin(d*x + c) - 1)/(a^2*(sin(d*x + c) - 1)^2)
 - (125*sin(d*x + c)^4 + 740*sin(d*x + c)^3 + 1086*sin(d*x + c)^2 + 676*sin
(d*x + c) + 157)/(a^2*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 10.50, size = 361, normalized size = 2.47

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{32\,a^2\,d}-\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{8}+\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{96}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}-\frac{121\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{48}-\frac{119\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{12}-\frac{121\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{48}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{3}+\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{96}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{32}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}-17\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+28\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-17\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(32*a^2*d) - ((5*tan(c/2 + (d*x)/2))/32 + (5*
tan(c/2 + (d*x)/2)^2)/8 + (35*tan(c/2 + (d*x)/2)^3)/96 - (5*tan(c/2 + (d*x)
/2)^4)/3 - (121*tan(c/2 + (d*x)/2)^5)/48 - (119*tan(c/2 + (d*x)/2)^6)/12 -
(121*tan(c/2 + (d*x)/2)^7)/48 - (5*tan(c/2 + (d*x)/2)^8)/3 + (35*tan(c/2 +
(d*x)/2)^9)/96 + (5*tan(c/2 + (d*x)/2)^10)/8 + (5*tan(c/2 + (d*x)/2)^11)/32
)/(d*(2*a^2*tan(c/2 + (d*x)/2)^2 - 12*a^2*tan(c/2 + (d*x)/2)^3 - 17*a^2*tan
(c/2 + (d*x)/2)^4 + 8*a^2*tan(c/2 + (d*x)/2)^5 + 28*a^2*tan(c/2 + (d*x)/2)^
6 + 8*a^2*tan(c/2 + (d*x)/2)^7 - 17*a^2*tan(c/2 + (d*x)/2)^8 - 12*a^2*tan(c
/2 + (d*x)/2)^9 + 2*a^2*tan(c/2 + (d*x)/2)^10 + 4*a^2*tan(c/2 + (d*x)/2)^11
 + a^2*tan(c/2 + (d*x)/2)^12 + a^2 + 4*a^2*tan(c/2 + (d*x)/2)))
```

3.63 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$-\frac{\tanh^{-1}(\sin(c+dx))}{8a^2d}+\frac{a}{12d(a+a\sin(c+dx))^3}-\frac{1}{4d(a+a\sin(c+dx))^2}+\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}+\frac{}{16d\left(a^2\right.}$$

```
[Out] -1/8*arctanh(sin(d*x+c))/a^2/d+1/12*a/d/(a+a*sin(d*x+c))^3-1/4/d/(a+a*sin(d
*x+c))^2+1/16/d/(a^2-a^2*sin(d*x+c))+3/16/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2786, 90, 212}

$$\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}+\frac{3}{16d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\tanh^{-1}(\sin(c+dx))}{8a^2d}+\frac{a}{12d(a\sin(c+dx)+a)^3}-\frac{1}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/8*ArcTanh[Sin[c + d*x]]/(a^2*d) + a/(12*d*(a + a*Sin[c + d*x])^3) - 1/(4
*d*(a + a*Sin[c + d*x])^2) + 1/(16*d*(a^2 - a^2*Sin[c + d*x])) + 3/(16*d*(a
^2 + a^2*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{16a(a-x)^2} - \frac{a}{4(a+x)^4} + \frac{1}{2(a+x)^3} - \frac{3}{16a(a+x)^2} - \frac{1}{8a(a^2-x^2)}\right) dx, x, a\sin(c+\right.}{d}$$
$$= \frac{a}{12d(a+a\sin(c+dx))^3} - \frac{1}{4d(a+a\sin(c+dx))^2} + \frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}$$
$$= -\frac{\tanh^{-1}(\sin(c+dx))}{8a^2d} + \frac{a}{12d(a+a\sin(c+dx))^3} - \frac{1}{4d(a+a\sin(c+dx))^2} + \cdot$$

Mathematica [A]
time = 0.22, size = 70, normalized size = 0.67

$$-\frac{6\tanh^{-1}(\sin(c+dx)) - \frac{3}{1-\sin(c+dx)} - \frac{4}{(1+\sin(c+dx))^3} + \frac{12}{(1+\sin(c+dx))^2} - \frac{9}{1+\sin(c+dx)}}{48a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
[Out] -1/48*(6*ArcTanh[Sin[c + d*x]] - 3/(1 - Sin[c + d*x]) - 4/(1 + Sin[c + d*x]
)^3 + 12/(1 + Sin[c + d*x])^2 - 9/(1 + Sin[c + d*x]))/(a^2*d)
```

Maple [A]
time = 0.24, size = 79, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{1}{12(1+\sin(dx+c))^3} - \frac{1}{4(1+\sin(dx+c))^2} + \frac{3}{16(1+\sin(dx+c))} - \frac{\ln(1+\sin(dx+c))}{16} - \frac{1}{16(\sin(dx+c)-1)} + \frac{\ln(\sin(dx+c)-1)}{16}}{d\,a^2}$
default	$\frac{\frac{1}{12(1+\sin(dx+c))^3} - \frac{1}{4(1+\sin(dx+c))^2} + \frac{3}{16(1+\sin(dx+c))} - \frac{\ln(1+\sin(dx+c))}{16} - \frac{1}{16(\sin(dx+c)-1)} + \frac{\ln(\sin(dx+c)-1)}{16}}{d\,a^2}$
risch	$\frac{i\left(-19\,e^{3i(dx+c)} - 12ie^{2i(dx+c)} - 3\,e^{i(dx+c)} + 19\,e^{5i(dx+c)} + 40ie^{4i(dx+c)} - 12ie^{6i(dx+c)} + 3\,e^{7i(dx+c)}\right)}{12\left(e^{i(dx+c)}+i\right)^6\left(e^{i(dx+c)}-i\right)^2 d\,a^2} + \frac{\ln\left(e^{i(dx+c)}-i\right)}{8a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/12/(1+sin(d*x+c))^3-1/4/(1+sin(d*x+c))^2+3/16/(1+sin(d*x+c))-1/1
6*ln(1+sin(d*x+c))-1/16/(sin(d*x+c)-1)+1/16*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.27, size = 110, normalized size = 1.06

$$\frac{\frac{2\left(3\,\sin(dx+c)^3 - 6\,\sin(dx+c)^2 - 7\,\sin(dx+c) - 2\right)}{a^2\sin(dx+c)^4 + 2\,a^2\sin(dx+c)^3 - 2\,a^2\sin(dx+c) - a^2} - \frac{3\,\log(\sin(dx+c)+1)}{a^2} + \frac{3\,\log(\sin(dx+c)-1)}{a^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(2*(3*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 7*sin(d*x + c) - 2)/(a^2*sin
(d*x + c)^4 + 2*a^2*sin(d*x + c)^3 - 2*a^2*sin(d*x + c) - a^2) - 3*log(sin(
d*x + c) + 1)/a^2 + 3*log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [A]
time = 0.36, size = 178, normalized size = 1.71

$$\frac{12 \cos \left(dx+c\right)^{2}-3 \left(\cos \left(dx+c\right)^{4}-2 \cos \left(dx+c\right)^{2} \sin \left(dx+c\right)-2 \cos \left(dx+c\right)^{2}\right) \log \left(\sin \left(dx+c\right)+1\right)+3 \left(\cos \left(dx+c\right)^{4}-2 \cos \left(dx+c\right)^{2} \sin \left(dx+c\right)-2 \cos \left(dx+c\right)^{2}\right) \log \left(-\sin \left(dx+c\right)+1\right)-2 \left(3 \cos \left(dx+c\right)^{2}+4\right) \sin \left(dx+c\right)-16}{48 \left(a^{2} d \cos \left(dx+c\right)^{4}-2 a^{2} d \cos \left(dx+c\right)^{2} \sin \left(dx+c\right)-2 a^{2} d \cos \left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(12*cos(d*x + c)^2 - 3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c)
 - 2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 3*(cos(d*x + c)^4 - 2*cos(d*x
+ c)^2*sin(d*x + c) - 2*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(3*cos(d
*x + c)^2 + 4)*sin(d*x + c) - 16)/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x +
 c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^{3}{\left(c+dx \right)}}{\sin^{2}{\left(c+dx \right)}+2 \sin{\left(c+dx \right)}+1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 6.29, size = 102, normalized size = 0.98

$$-\frac{\frac{6 \log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{2}}-\frac{6 \log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^{2}}+\frac{6 \sin\left(dx+c\right)}{a^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{11 \sin\left(dx+c\right)^{3}+51 \sin\left(dx+c\right)^{2}+45 \sin\left(dx+c\right)+13}{a^{2}\left(\sin\left(dx+c\right)+1\right)^{3}}}{96\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*(6*log(abs(sin(d*x + c) + 1))/a^2 - 6*log(abs(sin(d*x + c) - 1))/a^2
+ 6*sin(d*x + c)/(a^2*(sin(d*x + c) - 1)) - (11*sin(d*x + c)^3 + 51*sin(d*x
 + c)^2 + 45*sin(d*x + c) + 13)/(a^2*(sin(d*x + c) + 1)^3))/d
```

Mupad [B]
time = 10.05, size = 240, normalized size = 2.31

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-10\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^2\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)/4 + tan(c/2 + (d*x)/2)^2 + (13*tan(c/2 + (d*x)/2)^3)/12
 + (10*tan(c/2 + (d*x)/2)^4)/3 + (13*tan(c/2 + (d*x)/2)^5)/12 + tan(c/2 + (
d*x)/2)^6 + tan(c/2 + (d*x)/2)^7/4)/(d*(4*a^2*tan(c/2 + (d*x)/2)^2 - 4*a^2*
tan(c/2 + (d*x)/2)^3 - 10*a^2*tan(c/2 + (d*x)/2)^4 - 4*a^2*tan(c/2 + (d*x)/
2)^5 + 4*a^2*tan(c/2 + (d*x)/2)^6 + 4*a^2*tan(c/2 + (d*x)/2)^7 + a^2*tan(c/
2 + (d*x)/2)^8 + a^2 + 4*a^2*tan(c/2 + (d*x)/2))) - atanh(tan(c/2 + (d*x)/2
))/(4*a^2*d)
```

3.64 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2} \, dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}+\frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\left(a^2+a^2\sin(c+dx)\right)}$$

```
[Out] 1/4*arctanh(sin(d*x+c))/a^2/d+1/4/d/(a+a*sin(d*x+c))^2-1/4/d/(a^2+a^2*sin(d
*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2786, 78, 212}

$$-\frac{1}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}+\frac{1}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) + 1/(4*d*(a + a*Sin[c + d*x])^2) - 1/(4*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^3}+\frac{1}{4a(a+x)^2}+\frac{1}{4a(a^2-x^2)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\,(a^2+a^2\sin(c+dx))}+\frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin\right.}{4ad}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}+\frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\,(a^2+a^2\sin(c+dx))}$$

Mathematica [A]
time = 0.06, size = 36, normalized size = 0.60

$$\frac{\tanh^{-1}(\sin(c+dx))-\frac{\sin(c+dx)}{(1+\sin(c+dx))^2}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^2,x]
[Out] (ArcTanh[Sin[c + d*x]] - Sin[c + d*x]/(1 + Sin[c + d*x])^2)/(4*a^2*d)
```

Maple [A]
time = 0.22, size = 55, normalized size = 0.92

method	result	size
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{8}+\frac{1}{4(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{8}}{d\,a^2}$	55
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{8}+\frac{1}{4(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{8}}{d\,a^2}$	55
risch	$-\frac{i\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{2d\,a^2\left(e^{i(dx+c)}+i\right)^4}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{4a^2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{4a^2d}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(-1/8*ln(sin(d*x+c)-1)+1/4/(1+sin(d*x+c))^2-1/4/(1+sin(d*x+c))+1/8*
ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 1.17

$$-\frac{\frac{2\sin(dx+c)}{a^2\sin(dx+c)^2+2\,a^2\sin(dx+c)+a^2}-\frac{\log(\sin(dx+c)+1)}{a^2}+\frac{\log(\sin(dx+c)-1)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*sin(d*x + c)/(a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c) + a^2) - log(
sin(d*x + c) + 1)/a^2 + log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [A]
time = 0.35, size = 104, normalized size = 1.73

$$\frac{\left(\cos\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\cos\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)-2\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,\sin\left(dx+c\right)}{8\left(a^{2}d\cos\left(dx+c\right)^{2}-2\,a^{2}d\sin\left(dx+c\right)-2\,a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - (cos(d*x
 + c)^2 - 2*sin(d*x + c) - 2)*log(-sin(d*x + c) + 1) + 2*sin(d*x + c))/(a^2
*d*cos(d*x + c)^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan\left(c+dx\right)}{\sin^{2}\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 4.27, size = 90, normalized size = 1.50

$$\frac{\frac{\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right|\right)}{a^{2}}-\frac{\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)-2\right|\right)}{a^{2}}-\frac{\frac{1}{\sin(dx+c)}+\sin(dx+c)+6}{a^{2}\left(\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2))/a^2 - log(abs(1/sin(d*x +
 c) + sin(d*x + c) - 2))/a^2 - (1/sin(d*x + c) + sin(d*x + c) + 6)/(a^2*(1/
sin(d*x + c) + sin(d*x + c) + 2)))/d
```

Mupad [B]
time = 7.64, size = 116, normalized size = 1.93

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^{2}\,d}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+4\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+6\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+4\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(2*a^2*d) - (tan(c/2 + (d*x)/2)/2 + tan(c/2 + (d*
x)/2)^3/2)/(d*(6*a^2*tan(c/2 + (d*x)/2)^2 + 4*a^2*tan(c/2 + (d*x)/2)^3 + a^
2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a^2*tan(c/2 + (d*x)/2)))
```

3.65 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(1+\sin(c+dx))}{a^2d}+\frac{1}{d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] ln(sin(d*x+c))/a^2/d-ln(1+sin(d*x+c))/a^2/d+1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2786, 46}

$$\frac{1}{d\,(a^2\sin(c+dx)+a^2)}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{1}{a(a+x)^2}-\frac{1}{a^2(a+x)}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(1+\sin(c+dx))}{a^2d}+\frac{1}{d\,(a^2+a^2\sin(c+dx))}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 0.69

$$\frac{\log(\sin(c+dx))-\log(1+\sin(c+dx))+\frac{1}{1+\sin(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Log[Sin[c + d*x]] - Log[1 + Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(a^2*
d)
```

Maple [A]
time = 0.17, size = 37, normalized size = 0.71

method	result	size
derivativedivides	$\frac{\ln(\sin(dx+c))+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{a^2d}$	37
default	$\frac{\ln(\sin(dx+c))+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{a^2d}$	37
risch	$\frac{2i\mathrm{e}^{i(dx+c)}}{d\,a^2\left(\mathrm{e}^{i(dx+c)}+i\right)^2}-\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^2d}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{a^2d}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/d*(ln(sin(d*x+c))+1/(1+sin(d*x+c))-ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.88

$$\frac{\frac{1}{a^2\sin(dx+c)+a^2}-\frac{\log(\sin(dx+c)+1)}{a^2}+\frac{\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*sin(d*x + c) + a^2) - log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c))
/a^2)/d
```

Fricas [A]
time = 0.37, size = 59, normalized size = 1.13

$$\frac{(\sin(dx+c)+1)\log\left(\frac{1}{2}\sin(dx+c)\right)-(\sin(dx+c)+1)\log(\sin(dx+c)+1)+1}{a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((sin(d*x + c) + 1)*log(1/2*sin(d*x + c)) - (sin(d*x + c) + 1)*log(sin(d*x
+ c) + 1) + 1)/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot (c+dx)}{\sin^2 (c+dx)+2\sin (c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 3.60, size = 45, normalized size = 0.87

$$\frac{a\left(\frac{\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^3}+\frac{1}{(a\sin(dx+c)+a)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] a*(log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^3 + 1/((a*sin(d*x + c) + a)*a^2)
)/d
```

Mupad [B]
time = 6.59, size = 87, normalized size = 1.67

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^2\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^2*d) -
 (2*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 + a^2 + 2*a^2*tan(c/2
+ (d*x)/2)))
```

3.66 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{2\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\log(\sin(c+dx))}{a^2d}-\frac{2\log(1+\sin(c+dx))}{a^2d}$$

```
[Out] 2*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a^2/d+2*ln(sin(d*x+c))/a^2/d-2*ln(1+sin
(d*x+c))/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 78}

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{2\log(\sin(c+dx))}{a^2d}-\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + (2*Log[Sin[c + d*x]])
/(a^2*d) - (2*Log[1 + Sin[c + d*x]])/(a^2*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3} - \frac{2}{ax^2} + \frac{2}{a^2x} - \frac{2}{a^2(a+x)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{2\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2a^2d} + \frac{2\log(\sin(c+dx))}{a^2d} - \frac{2\log(1+\sin(c+dx))}{a^2d}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 0.75

$$\frac{4\csc(c+dx) - \csc^2(c+dx) + 4\log(\sin(c+dx)) - 4\log(1+\sin(c+dx))}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*Csc[c + d*x] - Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - 4*Log[1 + Sin[c + d*x]])/(2*a^2*d)
```

Maple [A]
time = 0.25, size = 49, normalized size = 0.75

method	result	size
derivativedivides	$\frac{-2\ln(1+\sin(dx+c)) - \frac{1}{2\sin(dx+c)^2} + \frac{2}{\sin(dx+c)} + 2\ln(\sin(dx+c))}{d\,a^2}$	49
default	$\frac{-2\ln(1+\sin(dx+c)) - \frac{1}{2\sin(dx+c)^2} + \frac{2}{\sin(dx+c)} + 2\ln(\sin(dx+c))}{d\,a^2}$	49
risch	$\frac{2i\left(-i\,\mathrm{e}^{2i(dx+c)} + 2\,\mathrm{e}^{3i(dx+c)} - 2\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^2\left(\mathrm{e}^{2i(dx+c)} - 1\right)^2} - \frac{4\ln\left(\mathrm{e}^{i(dx+c)} + i\right)}{a^2d} + \frac{2\ln\left(\mathrm{e}^{2i(dx+c)} - 1\right)}{a^2d}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-2*ln(1+sin(d*x+c))-1/2/sin(d*x+c)^2+2/sin(d*x+c)+2*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.85

$$-\frac{\frac{4\,\log(\sin(dx+c)+1)}{a^2} - \frac{4\,\log(\sin(dx+c))}{a^2} - \frac{4\,\sin(dx+c)-1}{a^2\,\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*log(sin(d*x + c) + 1)/a^2 - 4*log(sin(d*x + c))/a^2 - (4*sin(d*x +
c) - 1)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.37, size = 76, normalized size = 1.17

$$\frac{4\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-4\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\sin\left(dx+c\right)+1\right)-4\sin\left(dx+c\right)+1}{2\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 4*(cos(d*x + c)^2 - 1)*
log(sin(d*x + c) + 1) - 4*sin(d*x + c) + 1)/(a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^{3}\left(c+dx\right)}{\sin^{2}\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 4.13, size = 115, normalized size = 1.77

$$-\frac{\frac{32\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{2}}-\frac{16\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{2}}+\frac{a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-8a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{4}}+\frac{24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(32*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 16*log(abs(tan(1/2*d*x +
1/2*c)))/a^2 + (a^2*tan(1/2*d*x + 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^
4 + (24*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d
*x + 1/2*c)^2))/d
```

Mupad [B]
time = 6.53, size = 103, normalized size = 1.58

$$\frac{2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^{2}d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{8a^{2}d}-\frac{4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^{2}d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{2}d}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{1}{8}\right)}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (2*log(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (4*l
og(tan(c/2 + (d*x)/2) + 1))/(a^2*d) + tan(c/2 + (d*x)/2)/(a^2*d) + (cot(c/2
 + (d*x)/2)^2*(tan(c/2 + (d*x)/2) - 1/8))/(a^2*d)
```

3.67 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4a^2d}$$

```
[Out] -1/2*csc(d*x+c)^2/a^2/d+2/3*csc(d*x+c)^3/a^2/d-1/4*csc(d*x+c)^4/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2786, 45}

$$-\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*Csc[c + d*x]^2/(a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4
/(4*a^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^5}-\frac{2a}{x^4}+\frac{1}{x^3}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4a^2d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 0.69

$$\frac{\csc^4(c+dx)(-6+3\cos(2(c+dx))+8\sin(c+dx))}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
[Out] (Csc[c + d*x]^4*(-6 + 3*Cos[2*(c + d*x)] + 8*Sin[c + d*x]))/(12*a^2*d)
```

Maple [A]
time = 0.25, size = 39, normalized size = 0.71

method	result	size
derivativedivides	$\frac{\frac{2}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}-\frac{1}{4\sin(dx+c)^4}}{d\,a^2}$	39
default	$\frac{\frac{2}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}-\frac{1}{4\sin(dx+c)^4}}{d\,a^2}$	39
risch	$\frac{2\,e^{6i(dx+c)}-8\,e^{4i(dx+c)}-\frac{16ie^{5i(dx+c)}}{3}+2\,e^{2i(dx+c)}+\frac{16ie^{3i(dx+c)}}{3}}{d\,a^2\left(e^{2i(dx+c)}-1\right)^4}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2-1/4/sin(d*x+c)^4)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.65

$$-\frac{6\,\sin\left(dx+c\right)^2-8\,\sin\left(dx+c\right)+3}{12\,a^2d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

Fricas [A]
time = 0.35, size = 57, normalized size = 1.04

$$\frac{6\,\cos\left(dx+c\right)^2+8\,\sin\left(dx+c\right)-9}{12\,\left(a^2d\cos\left(dx+c\right)^4-2\,a^2d\cos\left(dx+c\right)^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*cos(d*x + c)^2 + 8*sin(d*x + c) - 9)/(a^2*d*cos(d*x + c)^4 - 2*a^2*
d*cos(d*x + c)^2 + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^5(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
[Out] Integral(cot(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 5.70, size = 36, normalized size = 0.65

$$-\frac{6\sin(dx+c)^2 - 8\sin(dx+c) + 3}{12\,a^2 d\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

Mupad [B]
time = 6.33, size = 36, normalized size = 0.65

$$-\frac{\frac{\sin(c+d\,x)^2}{2} - \frac{2\,\sin(c+d\,x)}{3} + \frac{1}{4}}{a^2\,d\sin(c+d\,x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a*sin(c + d*x))^2,x)
[Out] -(sin(c + d*x)^2/2 - (2*sin(c + d*x))/3 + 1/4)/(a^2*d*sin(c + d*x)^4)
```

3.68 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$\frac{\csc^2(c+dx)}{2a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{\csc^6(c+dx)}{6a^2d}$$

```
[Out] 1/2*csc(d*x+c)^2/a^2/d-2/3*csc(d*x+c)^3/a^2/d+2/5*csc(d*x+c)^5/a^2/d-1/6*cs
c(d*x+c)^6/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 76}

$$-\frac{\csc^6(c+dx)}{6a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5
)/(5*a^2*d) - Csc[c + d*x]^6/(6*a^2*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4}{x^7} - \frac{2a^3}{x^6} + \frac{2a}{x^4} - \frac{1}{x^3}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^2(c+dx)}{2a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d} - \frac{\csc^6(c+dx)}{6a^2d}$$

Mathematica [A]
time = 0.05, size = 73, normalized size = 1.00

$$\frac{\csc^2(c+dx)}{2a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d} - \frac{\csc^6(c+dx)}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5
)/(5*a^2*d) - Csc[c + d*x]^6/(6*a^2*d)
```

Maple [A]
time = 0.32, size = 49, normalized size = 0.67

method	result
derivativedivides	$\frac{\frac{2}{5\sin(dx+c)^5}+\frac{1}{2\sin(dx+c)^2}-\frac{2}{3\sin(dx+c)^3}-\frac{1}{6\sin(dx+c)^6}}{d\,a^2}$
default	$\frac{\frac{2}{5\sin(dx+c)^5}+\frac{1}{2\sin(dx+c)^2}-\frac{2}{3\sin(dx+c)^3}-\frac{1}{6\sin(dx+c)^6}}{d\,a^2}$
risch	$-\frac{2\left(15\,\mathrm{e}^{10i(dx+c)}-60\,\mathrm{e}^{8i(dx+c)}-40i\mathrm{e}^{9i(dx+c)}+10\,\mathrm{e}^{6i(dx+c)}+24i\mathrm{e}^{7i(dx+c)}-60\,\mathrm{e}^{4i(dx+c)}-24i\mathrm{e}^{5i(dx+c)}+15\,\mathrm{e}^{2i(dx+c)}+4\right.}{15d\,a^2\left(\mathrm{e}^{2i(dx+c)}-1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(2/5/sin(d*x+c)^5+1/2/sin(d*x+c)^2-2/3/sin(d*x+c)^3-1/6/sin(d*x+c)^
6)
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.63

$$\frac{15\,\sin\left(dx+c\right)^4 - 20\,\sin\left(dx+c\right)^3 + 12\,\sin\left(dx+c\right) - 5}{30\,a^2 d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 12*sin(d*x + c) - 5)/(a^2*d*s
in(d*x + c)^6)
```

Fricas [A]
time = 0.36, size = 94, normalized size = 1.29

$$-\frac{15\cos\left(dx+c\right)^4-30\cos\left(dx+c\right)^2+4\left(5\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)+10}{30\left(a^2d\cos\left(dx+c\right)^6-3a^2d\cos\left(dx+c\right)^4+3a^2d\cos\left(dx+c\right)^2-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(15*cos(d*x + c)^4 - 30*cos(d*x + c)^2 + 4*(5*cos(d*x + c)^2 - 2)*sin
(d*x + c) + 10)/(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*co
s(d*x + c)^2 - a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^7\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**7/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 26.12, size = 46, normalized size = 0.63

$$\frac{15\sin\left(dx+c\right)^4-20\sin\left(dx+c\right)^3+12\sin\left(dx+c\right)-5}{30a^2d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 12*sin(d*x + c) - 5)/(a^2*d*s
in(d*x + c)^6)
```

Mupad [B]
time = 6.37, size = 46, normalized size = 0.63

$$\frac{15\sin\left(c+d\,x\right)^4-20\sin\left(c+d\,x\right)^3+12\,\sin\left(c+d\,x\right)-5}{30\,a^2\,d\sin\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (12*sin(c + d*x) - 20*sin(c + d*x)^3 + 15*sin(c + d*x)^4 - 5)/(30*a^2*d*sin
(c + d*x)^6)
```

3.69 $\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=127

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}+\frac{\csc^4(c+dx)}{4a^2d}-\frac{4\csc^5(c+dx)}{5a^2d}+\frac{\csc^6(c+dx)}{6a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{\csc^8(c+dx)}{8a^2d}$$

```
[Out] -1/2*csc(d*x+c)^2/a^2/d+2/3*csc(d*x+c)^3/a^2/d+1/4*csc(d*x+c)^4/a^2/d-4/5*c
sc(d*x+c)^5/a^2/d+1/6*csc(d*x+c)^6/a^2/d+2/7*csc(d*x+c)^7/a^2/d-1/8*csc(d*x
+c)^8/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$-\frac{\csc^8(c+dx)}{8a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}+\frac{\csc^6(c+dx)}{6a^2d}-\frac{4\csc^5(c+dx)}{5a^2d}+\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*Csc[c + d*x]^2/(a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) + Csc[c + d*x]^4
/(4*a^2*d) - (4*Csc[c + d*x]^5)/(5*a^2*d) + Csc[c + d*x]^6/(6*a^2*d) + (2*C
sc[c + d*x]^7)/(7*a^2*d) - Csc[c + d*x]^8/(8*a^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\mathrm{Subst}\left(\int \frac{(a-x)^4(a+x)^2}{x^9}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(\frac{a^6}{x^9} - \frac{2a^5}{x^8} - \frac{a^4}{x^7} + \frac{4a^3}{x^6} - \frac{a^2}{x^5} - \frac{2a}{x^4} + \frac{1}{x^3}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^2(c+dx)}{2a^2d} + \frac{2\csc^3(c+dx)}{3a^2d} + \frac{\csc^4(c+dx)}{4a^2d} - \frac{4\csc^5(c+dx)}{5a^2d} + \frac{\csc^6(c+d}{6a^2d}$$

Mathematica [A]
time = 0.10, size = 78, normalized size = 0.61

$$\frac{\csc^2(c+dx)\left(-420+560\csc(c+dx)+210\csc^2(c+dx)-672\csc^3(c+dx)+140\csc^4(c+dx)+240\csc^5(c+dx)-105\csc^6(c+dx)\right)}{840a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^2*(-420 + 560*Csc[c + d*x] + 210*Csc[c + d*x]^2 - 672*Csc[c +
 d*x]^3 + 140*Csc[c + d*x]^4 + 240*Csc[c + d*x]^5 - 105*Csc[c + d*x]^6))/(8
40*a^2*d)
```

Maple [A]
time = 0.34, size = 79, normalized size = 0.62

method	result
derivativedivides	$\frac{\frac{1}{6\sin(dx+c)^6}+\frac{2}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}+\frac{2}{7\sin(dx+c)^7}-\frac{4}{5\sin(dx+c)^5}-\frac{1}{8\sin(dx+c)^8}+\frac{1}{4\sin(dx+c)^4}}{d\,a^2}$
default	$\frac{\frac{1}{6\sin(dx+c)^6}+\frac{2}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}+\frac{2}{7\sin(dx+c)^7}-\frac{4}{5\sin(dx+c)^5}-\frac{1}{8\sin(dx+c)^8}+\frac{1}{4\sin(dx+c)^4}}{d\,a^2}$
risch	$\frac{2\,e^{14i(dx+c)}-8\,e^{12i(dx+c)}-\frac{16ie^{13i(dx+c)}}{3}+\frac{10\,e^{10i(dx+c)}}{3}+\frac{16ie^{11i(dx+c)}}{15}-\frac{80\,e^{8i(dx+c)}}{3}-\frac{1376ie^{9i(dx+c)}}{105}+\frac{10\,e^{6i(dx+c)}}{3}+1}{d\,a^2\left(e^{2i(dx+c)}-1\right)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/6/sin(d*x+c)^6+2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2+2/7/sin(d*x+c)^
7-4/5/sin(d*x+c)^5-1/8/sin(d*x+c)^8+1/4/sin(d*x+c)^4)
```

Maxima [A]
time = 0.29, size = 76, normalized size = 0.60

$$-\frac{420\sin(dx+c)^6-560\sin(dx+c)^5-210\sin(dx+c)^4+672\sin(dx+c)^3-140\sin(dx+c)^2-240\sin(dx+c)+105}{840\,a^2d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/840*(420*sin(d*x + c)^6 - 560*sin(d*x + c)^5 - 210*sin(d*x + c)^4 + 672*
sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 240*sin(d*x + c) + 105)/(a^2*d*sin(d*
x + c)^8)
```

Fricas [A]
time = 0.35, size = 127, normalized size = 1.00

$$\frac{420 \cos(dx+c)^6 - 1050 \cos(dx+c)^4 + 700 \cos(dx+c)^2 + 16 \left(35 \cos(dx+c)^4 - 28 \cos(dx+c)^2 + 8\right) \sin(dx+c) - 175}{840 \left(a^2 d \cos(dx+c)^8 - 4 a^2 d \cos(dx+c)^6 + 6 a^2 d \cos(dx+c)^4 - 4 a^2 d \cos(dx+c)^2 + a^2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(420*cos(d*x + c)^6 - 1050*cos(d*x + c)^4 + 700*cos(d*x + c)^2 + 16*(
35*cos(d*x + c)^4 - 28*cos(d*x + c)^2 + 8)*sin(d*x + c) - 175)/(a^2*d*cos(d
*x + c)^8 - 4*a^2*d*cos(d*x + c)^6 + 6*a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d
*x + c)^2 + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^9(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**9/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 11.05, size = 76, normalized size = 0.60

$$-\frac{420 \sin(dx+c)^6 - 560 \sin(dx+c)^5 - 210 \sin(dx+c)^4 + 672 \sin(dx+c)^3 - 140 \sin(dx+c)^2 - 240 \sin(dx+c) + 105}{840 a^2 d \sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(420*sin(d*x + c)^6 - 560*sin(d*x + c)^5 - 210*sin(d*x + c)^4 + 672*
sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 240*sin(d*x + c) + 105)/(a^2*d*sin(d*
x + c)^8)
```

Mupad [B]
time = 6.54, size = 76, normalized size = 0.60

$$\frac{-420 \sin(c+dx)^6 + 560 \sin(c+dx)^5 + 210 \sin(c+dx)^4 - 672 \sin(c+dx)^3 + 140 \sin(c+dx)^2 + 240 \sin(c+dx) - 105}{840 a^2 d \sin(c+dx)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^9/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (240*sin(c + d*x) + 140*sin(c + d*x)^2 - 672*sin(c + d*x)^3 + 210*sin(c + d
*x)^4 + 560*sin(c + d*x)^5 - 420*sin(c + d*x)^6 - 105)/(840*a^2*d*sin(c + d
*x)^8)
```

3.70 $\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=145

$$\frac{\csc^2(c+dx)}{2a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{2a^2d}+\frac{6\csc^5(c+dx)}{5a^2d}-\frac{6\csc^7(c+dx)}{7a^2d}+\frac{\csc^8(c+dx)}{4a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-$$

```
[Out] 1/2*csc(d*x+c)^2/a^2/d-2/3*csc(d*x+c)^3/a^2/d-1/2*csc(d*x+c)^4/a^2/d+6/5*cs
c(d*x+c)^5/a^2/d-6/7*csc(d*x+c)^7/a^2/d+1/4*csc(d*x+c)^8/a^2/d+2/9*csc(d*x+
c)^9/a^2/d-1/10*csc(d*x+c)^10/a^2/d
```

Rubi [A]

time = 0.06, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$-\frac{\csc^{10}(c+dx)}{10a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}+\frac{\csc^8(c+dx)}{4a^2d}-\frac{6\csc^7(c+dx)}{7a^2d}+\frac{6\csc^5(c+dx)}{5a^2d}-\frac{\csc^4(c+dx)}{2a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(2
*a^2*d) + (6*Csc[c + d*x]^5)/(5*a^2*d) - (6*Csc[c + d*x]^7)/(7*a^2*d) + Csc
[c + d*x]^8/(4*a^2*d) + (2*Csc[c + d*x]^9)/(9*a^2*d) - Csc[c + d*x]^10/(10*
a^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^5(a+x)^3}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^8}{x^{11}} - \frac{2a^7}{x^{10}} - \frac{2a^6}{x^9} + \frac{6a^5}{x^8} - \frac{6a^3}{x^6} + \frac{2a^2}{x^5} + \frac{2a}{x^4} - \frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^2(c+dx)}{2a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} - \frac{\csc^4(c+dx)}{2a^2d} + \frac{6\csc^5(c+dx)}{5a^2d} - \frac{6\csc^7(c+d}{7a^2d}$$

Mathematica [A]
time = 0.14, size = 88, normalized size = 0.61

$$\frac{\csc^2(c+dx)\left(630 - 840\csc(c+dx) - 630\csc^2(c+dx) + 1512\csc^3(c+dx) - 1080\csc^5(c+dx) + 315\csc^6(c+dx) + 280\csc^7(c+dx) - 126\csc^8(c+dx)\right)}{1260a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^2*(630 - 840*Csc[c + d*x] - 630*Csc[c + d*x]^2 + 1512*Csc[c +
 d*x]^3 - 1080*Csc[c + d*x]^5 + 315*Csc[c + d*x]^6 + 280*Csc[c + d*x]^7 - 1
26*Csc[c + d*x]^8))/(1260*a^2*d)
```

Maple [A]
time = 0.46, size = 89, normalized size = 0.61

method	result
derivativedivides	$\frac{\frac{2}{9\sin(dx+c)^9} - \frac{2}{3\sin(dx+c)^3} + \frac{1}{2\sin(dx+c)^2} + \frac{1}{4\sin(dx+c)^8} - \frac{1}{2\sin(dx+c)^4} - \frac{1}{10\sin(dx+c)^{10}} + \frac{6}{5\sin(dx+c)^5} - \frac{6}{7\sin(dx+c)^7}}{d\,a^2}$
default	$\frac{\frac{2}{9\sin(dx+c)^9} - \frac{2}{3\sin(dx+c)^3} + \frac{1}{2\sin(dx+c)^2} + \frac{1}{4\sin(dx+c)^8} - \frac{1}{2\sin(dx+c)^4} - \frac{1}{10\sin(dx+c)^{10}} + \frac{6}{5\sin(dx+c)^5} - \frac{6}{7\sin(dx+c)^7}}{d\,a^2}$
risch	$-\frac{2(315\,e^{18i(dx+c)} - 1260\,e^{16i(dx+c)} + 840ie^{3i(dx+c)} + 1260\,e^{14i(dx+c)} - 840ie^{17i(dx+c)} - 8820\,e^{12i(dx+c)} + 168ie^{5i(dx+c)} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(2/9/sin(d*x+c)^9-2/3/sin(d*x+c)^3+1/2/sin(d*x+c)^2+1/4/sin(d*x+c)^
8-1/2/sin(d*x+c)^4-1/10/sin(d*x+c)^10+6/5/sin(d*x+c)^5-6/7/sin(d*x+c)^7)
```

Maxima [A]
time = 0.28, size = 86, normalized size = 0.59

$$\frac{630\sin(dx+c)^8 - 840\sin(dx+c)^7 - 630\sin(dx+c)^6 + 1512\sin(dx+c)^5 - 1080\sin(dx+c)^3 + 315\sin(dx+c)^2 + 280\sin(dx+c) - 126}{1260\,a^2d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(630*sin(d*x + c)^8 - 840*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 1512
*sin(d*x + c)^5 - 1080*sin(d*x + c)^3 + 315*sin(d*x + c)^2 + 280*sin(d*x +
c) - 126)/(a^2*d*sin(d*x + c)^10)
```

Fricas [A]
time = 0.36, size = 162, normalized size = 1.12

$$-\frac{630 \cos(dx+c)^8 - 1890 \cos(dx+c)^6 + 1890 \cos(dx+c)^4 - 945 \cos(dx+c)^2 + 8 \left(105 \cos(dx+c)^6 - 126 \cos(dx+c)^4 + 72 \cos(dx+c)^2 - 16\right) \sin(dx+c) + 189}{1260 \left(a^2 d \cos(dx+c)^{10} - 5 a^2 d \cos(dx+c)^8 + 10 a^2 d \cos(dx+c)^6 - 10 a^2 d \cos(dx+c)^4 + 5 a^2 d \cos(dx+c)^2 - a^2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/1260*(630*cos(d*x + c)^8 - 1890*cos(d*x + c)^6 + 1890*cos(d*x + c)^4 - 9
45*cos(d*x + c)^2 + 8*(105*cos(d*x + c)^6 - 126*cos(d*x + c)^4 + 72*cos(d*x
 + c)^2 - 16)*sin(d*x + c) + 189)/(a^2*d*cos(d*x + c)^10 - 5*a^2*d*cos(d*x
+ c)^8 + 10*a^2*d*cos(d*x + c)^6 - 10*a^2*d*cos(d*x + c)^4 + 5*a^2*d*cos(d*
x + c)^2 - a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**11/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 14.52, size = 86, normalized size = 0.59

$$\frac{630 \sin(dx+c)^8 - 840 \sin(dx+c)^7 - 630 \sin(dx+c)^6 + 1512 \sin(dx+c)^5 - 1080 \sin(dx+c)^3 + 315 \sin(dx+c)^2 + 280 \sin(dx+c) - 126}{1260 a^2 d \sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1260*(630*sin(d*x + c)^8 - 840*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 1512
*sin(d*x + c)^5 - 1080*sin(d*x + c)^3 + 315*sin(d*x + c)^2 + 280*sin(d*x +
c) - 126)/(a^2*d*sin(d*x + c)^10)
```

Mupad [B]
time = 6.63, size = 85, normalized size = 0.59

$$\frac{\frac{\sin(c+d\,x)^8}{2} - \frac{2\sin(c+d\,x)^7}{3} - \frac{\sin(c+d\,x)^6}{2} + \frac{6\sin(c+d\,x)^5}{5} - \frac{6\sin(c+d\,x)^3}{7} + \frac{\sin(c+d\,x)^2}{4} + \frac{2\,\sin(c+d\,x)}{9} - \frac{1}{10}}{a^2\,d\sin\left(c+d\,x\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^11/(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((2*sin(c + d*x))/9 + sin(c + d*x)^2/4 - (6*sin(c + d*x)^3)/7 + (6*sin(c +
d*x)^5)/5 - sin(c + d*x)^6/2 - (2*sin(c + d*x)^7)/3 + sin(c + d*x)^8/2 - 1/
10)/(a^2*d*sin(c + d*x)^10)
```

3.71 $\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=199

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}+\frac{3\csc^4(c+dx)}{4a^2d}-\frac{8\csc^5(c+dx)}{5a^2d}-\frac{\csc^6(c+dx)}{3a^2d}+\frac{12\csc^7(c+dx)}{7a^2d}-\frac{\csc^8(c+dx}{4a^2d}$$

```
[Out] -1/2*csc(d*x+c)^2/a^2/d+2/3*csc(d*x+c)^3/a^2/d+3/4*csc(d*x+c)^4/a^2/d-8/5*c
sc(d*x+c)^5/a^2/d-1/3*csc(d*x+c)^6/a^2/d+12/7*csc(d*x+c)^7/a^2/d-1/4*csc(d*
x+c)^8/a^2/d-8/9*csc(d*x+c)^9/a^2/d+3/10*csc(d*x+c)^10/a^2/d+2/11*csc(d*x+c
)^11/a^2/d-1/12*csc(d*x+c)^12/a^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$-\frac{\csc^{12}(c+dx)}{12a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}+\frac{3\csc^{10}(c+dx)}{10a^2d}-\frac{8\csc^9(c+dx)}{9a^2d}-\frac{\csc^8(c+dx)}{4a^2d}+\frac{12\csc^7(c+dx)}{7a^2d}-\frac{\csc^6(c+dx)}{3a^2d}-\frac{8\csc^5(c+dx)}{5a^2d}+\frac{3\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*Csc[c + d*x]^2/(a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) + (3*Csc[c + d*x
]^4)/(4*a^2*d) - (8*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(3*a^2*d) +
(12*Csc[c + d*x]^7)/(7*a^2*d) - Csc[c + d*x]^8/(4*a^2*d) - (8*Csc[c + d*x]^
9)/(9*a^2*d) + (3*Csc[c + d*x]^10)/(10*a^2*d) + (2*Csc[c + d*x]^11)/(11*a^2
*d) - Csc[c + d*x]^12/(12*a^2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^6(a+x)^4}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^{10}}{x^{13}} - \frac{2a^9}{x^{12}} - \frac{3a^8}{x^{11}} + \frac{8a^7}{x^{10}} + \frac{2a^6}{x^9} - \frac{12a^5}{x^8} + \frac{2a^4}{x^7} + \frac{8a^3}{x^6} - \frac{3a^2}{x^5} - \frac{2a}{x^4} + \frac{1}{x^3}\right) dx\right.}{d}$$
$$= -\frac{\csc^2(c+dx)}{2a^2d} + \frac{2\csc^3(c+dx)}{3a^2d} + \frac{3\csc^4(c+dx)}{4a^2d} - \frac{8\csc^5(c+dx)}{5a^2d} - \frac{\csc^6(c+}{3a^2d}$$

Mathematica [A]
time = 0.22, size = 118, normalized size = 0.59

$$-\frac{\csc^2(c+dx)\left(6930-9240\csc(c+dx)-10395\csc^2(c+dx)+22176\csc^3(c+dx)+4620\csc^4(c+dx)-23760\csc^5(c+dx)+3465\csc^6(c+dx)+12320\csc^7(c+dx)-4158\csc^8(c+dx)-2520\csc^9(c+dx)+1155\csc^{10}(c+dx)\right)}{13860a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/13860*(Csc[c + d*x]^2*(6930 - 9240*Csc[c + d*x] - 10395*Csc[c + d*x]^2 +
 22176*Csc[c + d*x]^3 + 4620*Csc[c + d*x]^4 - 23760*Csc[c + d*x]^5 + 3465*C
sc[c + d*x]^6 + 12320*Csc[c + d*x]^7 - 4158*Csc[c + d*x]^8 - 2520*Csc[c + d
*x]^9 + 1155*Csc[c + d*x]^10))/(a^2*d)
```

Maple [A]
time = 0.58, size = 119, normalized size = 0.60

method	result
derivativedivides	$\frac{-\frac{8}{9\sin(dx+c)^9}-\frac{1}{12\sin(dx+c)^{12}}-\frac{1}{2\sin(dx+c)^2}-\frac{1}{4\sin(dx+c)^8}+\frac{2}{3\sin(dx+c)^3}+\frac{2}{11\sin(dx+c)^{11}}+\frac{3}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^6}-\frac{}{5\,\mathrm{s}}}{d\,a^2}$
default	$\frac{-\frac{8}{9\sin(dx+c)^9}-\frac{1}{12\sin(dx+c)^{12}}-\frac{1}{2\sin(dx+c)^2}-\frac{1}{4\sin(dx+c)^8}+\frac{2}{3\sin(dx+c)^3}+\frac{2}{11\sin(dx+c)^{11}}+\frac{3}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^6}-\frac{}{5\,\mathrm{s}}}{d\,a^2}$
risch	$\frac{2\,e^{22i(dx+c)}-8\,e^{20i(dx+c)}+\frac{16i e^{3i(dx+c)}}{3}+\frac{46\,e^{18i(dx+c)}}{3}-\frac{16i e^{19i(dx+c)}}{5}-96\,e^{16i(dx+c)}-\frac{16i e^{21i(dx+c)}}{3}+\frac{84\,e^{14i(dx+c)}}{5}-\frac{1}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-8/9/sin(d*x+c)^9-1/12/sin(d*x+c)^12-1/2/sin(d*x+c)^2-1/4/sin(d*x+
c)^8+2/3/sin(d*x+c)^3+2/11/sin(d*x+c)^11+3/4/sin(d*x+c)^4-1/3/sin(d*x+c)^6-
8/5/sin(d*x+c)^5+12/7/sin(d*x+c)^7+3/10/sin(d*x+c)^10)
```

Maxima [A]
time = 0.28, size = 116, normalized size = 0.58

$$-\frac{6930\sin(dx+c)^{10}-9240\sin(dx+c)^9-10395\sin(dx+c)^8+22176\sin(dx+c)^7+4620\sin(dx+c)^6-23760\sin(dx+c)^5+3465\sin(dx+c)^4+12320\sin(dx+c)^3-4158\sin(dx+c)^2-2520\sin(dx+c)+1155}{13860\,a^2d\sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/13860*(6930*sin(d*x + c)^10 - 9240*sin(d*x + c)^9 - 10395*sin(d*x + c)^8
 + 22176*sin(d*x + c)^7 + 4620*sin(d*x + c)^6 - 23760*sin(d*x + c)^5 + 3465
*sin(d*x + c)^4 + 12320*sin(d*x + c)^3 - 4158*sin(d*x + c)^2 - 2520*sin(d*x
 + c) + 1155)/(a^2*d*sin(d*x + c)^12)
```

Fricas [A]
time = 0.38, size = 195, normalized size = 0.98

$$\frac{6930\,\cos\left(dx+c\right)^{10}-24255\,\cos\left(dx+c\right)^{8}+32340\,\cos\left(dx+c\right)^{6}-24255\,\cos\left(dx+c\right)^{4}+9702\,\cos\left(dx+c\right)^{2}+8\left(1155\,\cos\left(dx+c\right)^{8}-1848\,\cos\left(dx+c\right)^{6}+1584\,\cos\left(dx+c\right)^{4}-704\,\cos\left(dx+c\right)^{2}+128\right)\sin\left(dx+c\right)-1617}{13860\left(a^{2}d\cos\left(dx+c\right)^{12}-6\,a^{2}d\cos\left(dx+c\right)^{10}+15\,a^{2}d\cos\left(dx+c\right)^{8}-20\,a^{2}d\cos\left(dx+c\right)^{6}+15\,a^{2}d\cos\left(dx+c\right)^{4}-6\,a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/13860*(6930*cos(d*x + c)^10 - 24255*cos(d*x + c)^8 + 32340*cos(d*x + c)^6
 - 24255*cos(d*x + c)^4 + 9702*cos(d*x + c)^2 + 8*(1155*cos(d*x + c)^8 - 18
48*cos(d*x + c)^6 + 1584*cos(d*x + c)^4 - 704*cos(d*x + c)^2 + 128)*sin(d*x
 + c) - 1617)/(a^2*d*cos(d*x + c)^12 - 6*a^2*d*cos(d*x + c)^10 + 15*a^2*d*c
os(d*x + c)^8 - 20*a^2*d*cos(d*x + c)^6 + 15*a^2*d*cos(d*x + c)^4 - 6*a^2*d
*cos(d*x + c)^2 + a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**13/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [A]
time = 10.22, size = 116, normalized size = 0.58

$$-\frac{6930\,\sin\left(dx+c\right)^{10}-9240\,\sin\left(dx+c\right)^{9}-10395\,\sin\left(dx+c\right)^{8}+22176\,\sin\left(dx+c\right)^{7}+4620\,\sin\left(dx+c\right)^{6}-23760\,\sin\left(dx+c\right)^{5}+3465\,\sin\left(dx+c\right)^{4}+12320\,\sin\left(dx+c\right)^{3}-4158\,\sin\left(dx+c\right)^{2}-2520\,\sin\left(dx+c\right)+1155}{13860\,a^{2}d\sin\left(dx+c\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/13860*(6930*sin(d*x + c)^10 - 9240*sin(d*x + c)^9 - 10395*sin(d*x + c)^8
 + 22176*sin(d*x + c)^7 + 4620*sin(d*x + c)^6 - 23760*sin(d*x + c)^5 + 3465
*sin(d*x + c)^4 + 12320*sin(d*x + c)^3 - 4158*sin(d*x + c)^2 - 2520*sin(d*x
 + c) + 1155)/(a^2*d*sin(d*x + c)^12)
```

Mupad [B]
time = 6.85, size = 116, normalized size = 0.58

$$-\frac{\frac{\sin(c+d\,x)^{10}}{2}-\frac{2\sin(c+d\,x)^9}{3}-\frac{3\sin(c+d\,x)^8}{4}+\frac{8\sin(c+d\,x)^7}{5}+\frac{\sin(c+d\,x)^6}{3}-\frac{12\sin(c+d\,x)^5}{7}+\frac{\sin(c+d\,x)^4}{4}+\frac{8\sin(c+d\,x)^3}{9}-\frac{3\sin(c+d\,x)^2}{10}-\frac{2\sin(c+d\,x)}{11}+\frac{1}{12}}{a^2\,d\sin\left(c+d\,x\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^13/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -((8*sin(c + d*x)^3)/9 - (3*sin(c + d*x)^2)/10 - (2*sin(c + d*x))/11 + sin(
c + d*x)^4/4 - (12*sin(c + d*x)^5)/7 + sin(c + d*x)^6/3 + (8*sin(c + d*x)^7
)/5 - (3*sin(c + d*x)^8)/4 - (2*sin(c + d*x)^9)/3 + sin(c + d*x)^10/2 + 1/1
2)/(a^2*d*sin(c + d*x)^12)
```

3.72 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=171

$$\frac{\tanh^{-1}(\sin(c+dx))}{128a^3d}+\frac{1}{128ad(a-a\sin(c+dx))^2}+\frac{a^2}{40d(a+a\sin(c+dx))^5}-\frac{7a}{64d(a+a\sin(c+dx))^4}+\frac{}{6d(a+}$$

```
[Out] 1/128*arctanh(sin(d*x+c))/a^3/d+1/128/a/d/(a-a*sin(d*x+c))^2+1/40*a^2/d/(a+
a*sin(d*x+c))^5-7/64*a/d/(a+a*sin(d*x+c))^4+1/6/d/(a+a*sin(d*x+c))^3-5/64/a
/d/(a+a*sin(d*x+c))^2-1/32/d/(a^3-a^3*sin(d*x+c))-5/128/d/(a^3+a^3*sin(d*x+
c))
```

Rubi [A]
time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2786, 90, 212}

$$-\frac{1}{32d\,(a^3-a^3\sin(c+dx))}-\frac{5}{128d\,(a^3\sin(c+dx)+a^3)}+\frac{\tanh^{-1}(\sin(c+dx))}{128a^3d}+\frac{a^2}{40d(a\sin(c+dx)+a)^5}-\frac{7a}{64d(a\sin(c+dx)+a)^4}+\frac{1}{6d(a\sin(c+dx)+a)^3}+\frac{1}{128ad(a-a\sin(c+dx))^2}-\frac{5}{64ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(128*a^3*d) + 1/(128*a*d*(a - a*Sin[c + d*x])^2) + a^
2/(40*d*(a + a*Sin[c + d*x])^5) - (7*a)/(64*d*(a + a*Sin[c + d*x])^4) + 1/(
6*d*(a + a*Sin[c + d*x])^3) - 5/(64*a*d*(a + a*Sin[c + d*x])^2) - 1/(32*d*(
a^3 - a^3*Sin[c + d*x])) - 5/(128*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
```

```
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{64a(a-x)^3} - \frac{1}{32a^2(a-x)^2} - \frac{a^2}{8(a+x)^6} + \frac{7a}{16(a+x)^5} - \frac{1}{2(a+x)^4} + \frac{5}{32a(a+x)^3} + \frac{}{128}\right.\right.}{d}$$

$$= \frac{1}{128ad(a-a\sin(c+dx))^2} + \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{7a}{64d(a+a\sin(c+dx))}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{128a^3d} + \frac{1}{128ad(a-a\sin(c+dx))^2} + \frac{a^2}{40d(a+a\sin(c+dx))^5} -$$

Mathematica [A]
time = 0.45, size = 102, normalized size = 0.60

$$\frac{15\tanh^{-1}(\sin(c+dx)) - \frac{112+351\sin(c+dx)+157\sin^2(c+dx)-540\sin^3(c+dx)-620\sin^4(c+dx)+45\sin^5(c+dx)+15\sin^6(c+dx)}{(-1+\sin(c+dx))^2(1+\sin(c+dx))^5}}{1920a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] - (112 + 351*Sin[c + d*x] + 157*Sin[c + d*x]^2 -
540*Sin[c + d*x]^3 - 620*Sin[c + d*x]^4 + 45*Sin[c + d*x]^5 + 15*Sin[c + d*
x]^6)/((-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^5))/(1920*a^3*d)
```

Maple [A]
time = 0.32, size = 115, normalized size = 0.67

method	result
derivativedivides	$\frac{\frac{1}{40(1+\sin(dx+c))^5}-\frac{7}{64(1+\sin(dx+c))^4}+\frac{1}{6(1+\sin(dx+c))^3}-\frac{5}{64(1+\sin(dx+c))^2}-\frac{5}{128(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{256}+\frac{1}{128(\sin(dx}}{d\,a^3}$
default	$\frac{\frac{1}{40(1+\sin(dx+c))^5}-\frac{7}{64(1+\sin(dx+c))^4}+\frac{1}{6(1+\sin(dx+c))^3}-\frac{5}{64(1+\sin(dx+c))^2}-\frac{5}{128(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{256}+\frac{1}{128(\sin(dx}}{d\,a^3}$
risch	$-\frac{i\left(15\,e^{i(dx+c)}-828ie^{8i(dx+c)}+2390\,e^{3i(dx+c)}-90ie^{2i(dx+c)}-3870ie^{4i(dx+c)}-7183\,e^{5i(dx+c)}+828ie^{6i(dx+c)}+90ie^{12i(}\right.}{960\left(e^{i(dx+c)}-i\right)^4\left(e^{i(dx+c)}+i\right)^{10}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/40/(1+sin(d*x+c))^5-7/64/(1+sin(d*x+c))^4+1/6/(1+sin(d*x+c))^3-5
/64/(1+sin(d*x+c))^2-5/128/(1+sin(d*x+c))+1/256*ln(1+sin(d*x+c))+1/128/(sin
(d*x+c)-1)^2+1/32/(sin(d*x+c)-1)-1/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 188, normalized size = 1.10

$$-\frac{\frac{2\left(15\sin(dx+c)^6+45\sin(dx+c)^5-620\sin(dx+c)^4-540\sin(dx+c)^3+157\sin(dx+c)^2+351\sin(dx+c)+112\right)}{a^3\sin(dx+c)^7+3a^3\sin(dx+c)^6+a^3\sin(dx+c)^5-5a^3\sin(dx+c)^4-5a^3\sin(dx+c)^3+a^3\sin(dx+c)^2+3a^3\sin(dx+c)+a^3}-\frac{15\log(\sin(dx+c)+1)}{a^3}+\frac{15\log(\sin(dx+c)-1)}{a^3}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3840*(2*(15*sin(d*x + c)^6 + 45*sin(d*x + c)^5 - 620*sin(d*x + c)^4 - 54
0*sin(d*x + c)^3 + 157*sin(d*x + c)^2 + 351*sin(d*x + c) + 112)/(a^3*sin(d*
x + c)^7 + 3*a^3*sin(d*x + c)^6 + a^3*sin(d*x + c)^5 - 5*a^3*sin(d*x + c)^4
 - 5*a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) -
15*log(sin(d*x + c) + 1)/a^3 + 15*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [A]
time = 0.37, size = 248, normalized size = 1.45

$$-\frac{30\cos(dx+c)^6+1150\cos(dx+c)^4-2076\cos(dx+c)^2-15\left(3\cos(dx+c)^6-4\cos(dx+c)^4+\left(\cos(dx+c)^6-4\cos(dx+c)^4\right)\sin(dx+c)\right)\log(\sin(dx+c)+1)+15\left(3\cos(dx+c)^6-4\cos(dx+c)^4+\left(\cos(dx+c)^6-4\cos(dx+c)^4\right)\sin(dx+c)\right)\log(-\sin(dx+c)+1)-18\left(5\cos(dx+c)^4+50\cos(dx+c)^2-16\right)\sin(dx+c)+672}{3840\left(3a^3d\cos(dx+c)^6-4a^3d\cos(dx+c)^4+\left(a^3d\cos(dx+c)^6-4a^3d\cos(dx+c)^4\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3840*(30*cos(d*x + c)^6 + 1150*cos(d*x + c)^4 - 2076*cos(d*x + c)^2 - 15
*(3*cos(d*x + c)^6 - 4*cos(d*x + c)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)
*sin(d*x + c))*log(sin(d*x + c) + 1) + 15*(3*cos(d*x + c)^6 - 4*cos(d*x + c
)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)*sin(d*x + c))*log(-sin(d*x + c) +
 1) - 18*(5*cos(d*x + c)^4 + 50*cos(d*x + c)^2 - 16)*sin(d*x + c) + 672)/(3
*a^3*d*cos(d*x + c)^6 - 4*a^3*d*cos(d*x + c)^4 + (a^3*d*cos(d*x + c)^6 - 4*
a^3*d*cos(d*x + c)^4)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^5(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A]

time = 12.50, size = 136, normalized size = 0.80

$$\frac{\frac{60 \log\left(\left|\sin(dx+c)+1\right|\right)}{a^3} - \frac{60 \log\left(\left|\sin(dx+c)-1\right|\right)}{a^3} + \frac{30\left(3 \sin(dx+c)^2+10 \sin(dx+c)-9\right)}{a^3(\sin(dx+c)-1)^2} - \frac{137 \sin(dx+c)^5+1285 \sin(dx+c)^4+4970 \sin(dx+c)^3+6010 \sin(dx+c)^2+3245 \sin(dx+c)+673}{a^3(\sin(dx+c)+1)^5}}{15360\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15360*(60*log(abs(sin(d*x + c) + 1))/a^3 - 60*log(abs(sin(d*x + c) - 1))/
a^3 + 30*(3*sin(d*x + c)^2 + 10*sin(d*x + c) - 9)/(a^3*(sin(d*x + c) - 1)^2
) - (137*sin(d*x + c)^5 + 1285*sin(d*x + c)^4 + 4970*sin(d*x + c)^3 + 6010*
sin(d*x + c)^2 + 3245*sin(d*x + c) + 673)/(a^3*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]

time = 10.05, size = 418, normalized size = 2.44

$$\frac{-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{32}-\frac{17\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{96}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{32}+\frac{527\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{960}+\frac{901\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{80}+\frac{711\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{80}+\frac{901\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{80}+\frac{527\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{960}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{32}-\frac{17\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{96}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{32}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+6a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+11a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-4a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-39a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-38a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+27a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+72a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+27a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-38a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}-39a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-4a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+11a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+6a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4/32 - (3*tan(c/2 + (d*x)/2)^2)/32 - (17*tan(c/2 + (d*x
)/2)^3)/96 - tan(c/2 + (d*x)/2)/64 + (527*tan(c/2 + (d*x)/2)^5)/960 + (901*
tan(c/2 + (d*x)/2)^6)/80 + (711*tan(c/2 + (d*x)/2)^7)/80 + (901*tan(c/2 + (
d*x)/2)^8)/80 + (527*tan(c/2 + (d*x)/2)^9)/960 + tan(c/2 + (d*x)/2)^10/32 -
 (17*tan(c/2 + (d*x)/2)^11)/96 - (3*tan(c/2 + (d*x)/2)^12)/32 - tan(c/2 + (
d*x)/2)^13/64)/(d*(11*a^3*tan(c/2 + (d*x)/2)^2 - 4*a^3*tan(c/2 + (d*x)/2)^3
 - 39*a^3*tan(c/2 + (d*x)/2)^4 - 38*a^3*tan(c/2 + (d*x)/2)^5 + 27*a^3*tan(c
/2 + (d*x)/2)^6 + 72*a^3*tan(c/2 + (d*x)/2)^7 + 27*a^3*tan(c/2 + (d*x)/2)^8
 - 38*a^3*tan(c/2 + (d*x)/2)^9 - 39*a^3*tan(c/2 + (d*x)/2)^10 - 4*a^3*tan(c
/2 + (d*x)/2)^11 + 11*a^3*tan(c/2 + (d*x)/2)^12 + 6*a^3*tan(c/2 + (d*x)/2)^
13 + a^3*tan(c/2 + (d*x)/2)^14 + a^3 + 6*a^3*tan(c/2 + (d*x)/2))) + atanh(t
an(c/2 + (d*x)/2))/(64*a^3*d)
```

3.73 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$-\frac{\tanh^{-1}(\sin(c+dx))}{32a^3d}+\frac{a}{16d(a+a\sin(c+dx))^4}-\frac{1}{6d(a+a\sin(c+dx))^3}+\frac{3}{32ad(a+a\sin(c+dx))^2}+\frac{}{32d\,(a^3}$$

```
[Out] -1/32*arctanh(sin(d*x+c))/a^3/d+1/16*a/d/(a+a*sin(d*x+c))^4-1/6/d/(a+a*sin(
d*x+c))^3+3/32/a/d/(a+a*sin(d*x+c))^2+1/32/d/(a^3-a^3*sin(d*x+c))+1/16/d/(a
^3+a^3*sin(d*x+c))
```

Rubi [A]

time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2786, 90, 212}

$$\frac{1}{32d\,(a^3-a^3\sin(c+dx))}+\frac{1}{16d\,(a^3\sin(c+dx)+a^3)}-\frac{\tanh^{-1}(\sin(c+dx))}{32a^3d}+\frac{a}{16d(a\sin(c+dx)+a)^4}-\frac{1}{6d(a\sin(c+dx)+a)^3}+\frac{3}{32ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/32*ArcTanh[Sin[c + d*x]]/(a^3*d) + a/(16*d*(a + a*Sin[c + d*x])^4) - 1/(
6*d*(a + a*Sin[c + d*x])^3) + 3/(32*a*d*(a + a*Sin[c + d*x])^2) + 1/(32*d*(
a^3 - a^3*Sin[c + d*x])) + 1/(16*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{32a^2(a-x)^2} - \frac{a}{4(a+x)^5} + \frac{1}{2(a+x)^4} - \frac{3}{16a(a+x)^3} - \frac{1}{16a^2(a+x)^2} - \frac{1}{32a^2(a^2-x^2)}\right)\right.}{d}$$

$$= \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{6d(a+a\sin(c+dx))^3} + \frac{3}{32ad(a+a\sin(c+dx))^2}$$

$$= -\frac{\tanh^{-1}(\sin(c+dx))}{32a^3d} + \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{6d(a+a\sin(c+dx))^3} +$$

Mathematica [A]
time = 0.25, size = 82, normalized size = 0.65

$$-\frac{3\tanh^{-1}(\sin(c+dx)) - \frac{3}{1-\sin(c+dx)} - \frac{6}{(1+\sin(c+dx))^4} + \frac{16}{(1+\sin(c+dx))^3} - \frac{9}{(1+\sin(c+dx))^2} - \frac{6}{1+\sin(c+dx)}}{96a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/96*(3*ArcTanh[Sin[c + d*x]] - 3/(1 - Sin[c + d*x]) - 6/(1 + Sin[c + d*x]
)^4 + 16/(1 + Sin[c + d*x])^3 - 9/(1 + Sin[c + d*x])^2 - 6/(1 + Sin[c + d*x
]))/(a^3*d)
```

Maple [A]
time = 0.28, size = 91, normalized size = 0.72

method	result
derivativedivides	$\frac{-\frac{1}{32(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{64}+\frac{1}{16(1+\sin(dx+c))^4}-\frac{1}{6(1+\sin(dx+c))^3}+\frac{3}{32(1+\sin(dx+c))^2}+\frac{1}{16+16\sin(dx+c)}-\frac{\ln(1+\sin(d}{64}}{d\,a^3}$
default	$\frac{-\frac{1}{32(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{64}+\frac{1}{16(1+\sin(dx+c))^4}-\frac{1}{6(1+\sin(dx+c))^3}+\frac{3}{32(1+\sin(dx+c))^2}+\frac{1}{16+16\sin(dx+c)}-\frac{\ln(1+\sin(d}{64}}{d\,a^3}$
risch	$\frac{i\left(-310\,e^{5i(dx+c)}-162ie^{4i(dx+c)}+88\,e^{3i(dx+c)}-18ie^{2i(dx+c)}+3\,e^{i(dx+c)}+88\,e^{7i(dx+c)}+162ie^{6i(dx+c)}+18ie^{8i(dx+c)}+3\right.}{48\left(e^{i(dx+c)}+i\right)^8\left(e^{i(dx+c)}-i\right)^2 d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/32/(sin(d*x+c)-1)+1/64*ln(sin(d*x+c)-1)+1/16/(1+sin(d*x+c))^4-1
/6/(1+sin(d*x+c))^3+3/32/(1+sin(d*x+c))^2+1/16/(1+sin(d*x+c))-1/64*ln(1+sin
(d*x+c)))
```

Maxima [A]
time = 0.28, size = 146, normalized size = 1.16

$$\frac{\frac{2\left(3\,\sin(dx+c)^4+9\,\sin(dx+c)^3-25\,\sin(dx+c)^2-27\,\sin(dx+c)-8\right)}{a^3\sin(dx+c)^5+3\,a^3\sin(dx+c)^4+2\,a^3\sin(dx+c)^3-2\,a^3\sin(dx+c)^2-3\,a^3\sin(dx+c)-a^3}-\frac{3\,\log(\sin(dx+c)+1)}{a^3}+\frac{3\,\log(\sin(dx+c)-1)}{a^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/192*(2*(3*sin(d*x + c)^4 + 9*sin(d*x + c)^3 - 25*sin(d*x + c)^2 - 27*sin(
d*x + c) - 8)/(a^3*sin(d*x + c)^5 + 3*a^3*sin(d*x + c)^4 + 2*a^3*sin(d*x +
c)^3 - 2*a^3*sin(d*x + c)^2 - 3*a^3*sin(d*x + c) - a^3) - 3*log(sin(d*x + c
) + 1)/a^3 + 3*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [A]
time = 0.38, size = 226, normalized size = 1.79

$$\frac{6\,\cos(dx+c)^4+38\,\cos(dx+c)^2-3\left(3\,\cos(dx+c)^4-4\,\cos(dx+c)^2+\left(\cos(dx+c)^4-4\,\cos(dx+c)^2\right)\sin(dx+c)\right)\log(\sin(dx+c)+1)+3\left(3\,\cos(dx+c)^4-4\,\cos(dx+c)^2+\left(\cos(dx+c)^4-4\,\cos(dx+c)^2\right)\sin(dx+c)\right)\log(-\sin(dx+c)+1)-18\left(\cos(dx+c)^2+2\right)\sin(dx+c)-60}{192\left(3\,a^3d\cos(dx+c)^4-4\,a^3d\cos(dx+c)^2+\left(a^3d\cos(dx+c)^4-4\,a^3d\cos(dx+c)^2\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/192*(6*cos(d*x + c)^4 + 38*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^4 - 4*cos(d
*x + c)^2 + (cos(d*x + c)^4 - 4*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x +
 c) + 1) + 3*(3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4 - 4*cos
(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 18*(cos(d*x + c)^2 + 2)
*sin(d*x + c) - 60)/(3*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2 + (a^3
*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 9.47, size = 114, normalized size = 0.90

$$-\frac{\frac{12\,\log(|\sin(dx+c)+1|)}{a^3}-\frac{12\,\log(|\sin(dx+c)-1|)}{a^3}+\frac{12\,(\sin(dx+c)+1)}{a^3(\sin(dx+c)-1)}-\frac{25\,\sin(dx+c)^4+148\,\sin(dx+c)^3+366\,\sin(dx+c)^2+260\,\sin(dx+c)+65}{a^3(\sin(dx+c)+1)^4}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/768*(12*log(abs(sin(d*x + c) + 1))/a^3 - 12*log(abs(sin(d*x + c) - 1))/a
^3 + 12*(sin(d*x + c) + 1)/(a^3*(sin(d*x + c) - 1)) - (25*sin(d*x + c)^4 +
148*sin(d*x + c)^3 + 366*sin(d*x + c)^2 + 260*sin(d*x + c) + 65)/(a^3*(sin(
d*x + c) + 1)^4))/d
```

Mupad [B]
time = 9.93, size = 302, normalized size = 2.40

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{16}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{8}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{37\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{8}+\frac{101\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{24}+\frac{37\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{8}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-14\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-28\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-14\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^3\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{16\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)/16 + (3*tan(c/2 + (d*x)/2)^2)/8 + (5*tan(c/2 + (d*x)/2)
^3)/6 + (37*tan(c/2 + (d*x)/2)^4)/8 + (101*tan(c/2 + (d*x)/2)^5)/24 + (37*t
an(c/2 + (d*x)/2)^6)/8 + (5*tan(c/2 + (d*x)/2)^7)/6 + (3*tan(c/2 + (d*x)/2)
^8)/8 + tan(c/2 + (d*x)/2)^9/16)/(d*(13*a^3*tan(c/2 + (d*x)/2)^2 + 8*a^3*ta
n(c/2 + (d*x)/2)^3 - 14*a^3*tan(c/2 + (d*x)/2)^4 - 28*a^3*tan(c/2 + (d*x)/2
)^5 - 14*a^3*tan(c/2 + (d*x)/2)^6 + 8*a^3*tan(c/2 + (d*x)/2)^7 + 13*a^3*tan
(c/2 + (d*x)/2)^8 + 6*a^3*tan(c/2 + (d*x)/2)^9 + a^3*tan(c/2 + (d*x)/2)^10
+ a^3 + 6*a^3*tan(c/2 + (d*x)/2))) - atanh(tan(c/2 + (d*x)/2))/(16*a^3*d)
```

3.74 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\sin(c+dx)\right)}$$

```
[Out] 1/8*arctanh(sin(d*x+c))/a^3/d+1/6/d/(a+a*sin(d*x+c))^3-1/8/a/d/(a+a*sin(d*x
+c))^2-1/8/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2786, 78, 212}

$$-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{8ad(a\sin(c+dx)+a)^2}+\frac{1}{6d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(8*a^3*d) + 1/(6*d*(a + a*Sin[c + d*x])^3) - 1/(8*a*d
*(a + a*Sin[c + d*x])^2) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^4}+\frac{1}{4a(a+x)^3}+\frac{1}{8a^2(a+x)^2}+\frac{1}{8a^2(a^2-x^2)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\sin(c+dx)\right)}+$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-$$

Mathematica [A]
time = 0.10, size = 52, normalized size = 0.63

$$\frac{3\tanh^{-1}(\sin(c+dx))-\frac{2+9\sin(c+dx)+3\sin^2(c+dx)}{(1+\sin(c+dx))^3}}{24a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]] - (2 + 9*Sin[c + d*x] + 3*Sin[c + d*x]^2)/(1 + Sin
[c + d*x])^3)/(24*a^3*d)
```

Maple [A]
time = 0.26, size = 67, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\frac{1}{6(1+\sin(dx+c))^3}-\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{8(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{16}-\frac{\ln(\sin(dx+c)-1)}{16}}{d\,a^3}$	67
default	$\frac{\frac{1}{6(1+\sin(dx+c))^3}-\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{8(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{16}-\frac{\ln(\sin(dx+c)-1)}{16}}{d\,a^3}$	67
risch	$-\frac{i\left(3\,e^{i(dx+c)}-14\,e^{3i(dx+c)}-18i e^{2i(dx+c)}+18i e^{4i(dx+c)}+3\,e^{5i(dx+c)}\right)}{12d\,a^3\left(e^{i(dx+c)}+i\right)^6}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{8a^3d}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{8a^3d}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/6/(1+sin(d*x+c))^3-1/8/(1+sin(d*x+c))^2-1/8/(1+sin(d*x+c))+1/16*
ln(1+sin(d*x+c))-1/16*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 98, normalized size = 1.20

$$-\frac{\frac{2\left(3\sin\left(dx+c\right)^{2}+9\sin\left(dx+c\right)+2\right)}{a^{3}\sin\left(dx+c\right)^{3}+3a^{3}\sin\left(dx+c\right)^{2}+3a^{3}\sin\left(dx+c\right)+a^{3}}-\frac{3\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}}+\frac{3\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(3*sin(d*x + c)^2 + 9*sin(d*x + c) + 2)/(a^3*sin(d*x + c)^3 + 3*a^
3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) - 3*log(sin(d*x + c) + 1)/a^3
+ 3*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(74) = 148.
time = 0.36, size = 154, normalized size = 1.88

$$-\frac{6\cos\left(dx+c\right)^{2}-3\left(3\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-4\right)\sin\left(dx+c\right)-4\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(3\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-4\right)\sin\left(dx+c\right)-4\right)\log\left(-\sin\left(dx+c\right)+1\right)-18\sin\left(dx+c\right)-10}{48\left(3a^{3}d\cos\left(dx+c\right)^{2}-4a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-4a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(6*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*
x + c) - 4)*log(sin(d*x + c) + 1) + 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 -
 4)*sin(d*x + c) - 4)*log(-sin(d*x + c) + 1) - 18*sin(d*x + c) - 10)/(3*a^3
*d*cos(d*x + c)^2 - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 4*a^3*d)*sin(d*x + c)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x)
 + 1), x)/a**3
```

Giac [A]
time = 13.21, size = 81, normalized size = 0.99

$$\frac{\frac{6\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{3}}-\frac{6\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^{3}}-\frac{11\sin\left(dx+c\right)^{3}+45\sin\left(dx+c\right)^{2}+69\sin\left(dx+c\right)+19}{a^{3}\left(\sin\left(dx+c\right)+1\right)^{3}}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(sin(d*x + c) + 1))/a^3 - 6*log(abs(sin(d*x + c) - 1))/a^3 -
 (11*sin(d*x + c)^3 + 45*sin(d*x + c)^2 + 69*sin(d*x + c) + 19)/(a^3*(sin(d
*x + c) + 1)^3))/d
```

Mupad [B]
time = 8.82, size = 186, normalized size = 2.27

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^3\,d}+\frac{-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+15\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+20\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+15\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(4*a^3*d) + (tan(c/2 + (d*x)/2)^2/2 - tan(c/2 + (
d*x)/2)/4 + tan(c/2 + (d*x)/2)^3/6 + tan(c/2 + (d*x)/2)^4/2 - tan(c/2 + (d*
x)/2)^5/4)/(d*(15*a^3*tan(c/2 + (d*x)/2)^2 + 20*a^3*tan(c/2 + (d*x)/2)^3 +
15*a^3*tan(c/2 + (d*x)/2)^4 + 6*a^3*tan(c/2 + (d*x)/2)^5 + a^3*tan(c/2 + (d
*x)/2)^6 + a^3 + 6*a^3*tan(c/2 + (d*x)/2)))
```

3.75 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=74

$$\frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(1+\sin(c+dx))}{a^3d}+\frac{1}{2ad(a+a\sin(c+dx))^2}+\frac{1}{d\,(a^3+a^3\sin(c+dx))}$$

```
[Out] ln(sin(d*x+c))/a^3/d-ln(1+sin(d*x+c))/a^3/d+1/2/a/d/(a+a*sin(d*x+c))^2+1/d/
(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2786, 46}

$$\frac{1}{d\,(a^3\sin(c+dx)+a^3)}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[1 + Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) + 1/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{1}{a(a+x)^3} - \frac{1}{a^2(a+x)^2} - \frac{1}{a^3(a+x)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(1+\sin(c+dx))}{a^3d} + \frac{1}{2ad(a+a\sin(c+dx))^2} + \frac{}{d\,(a^3+}$$

Mathematica [A]
time = 0.13, size = 52, normalized size = 0.70

$$\frac{2\log(\sin(c+dx)) - 2\log(1+\sin(c+dx)) + \frac{3+2\sin(c+dx)}{(1+\sin(c+dx))^2}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*Log[Sin[c + d*x]] - 2*Log[1 + Sin[c + d*x]] + (3 + 2*Sin[c + d*x])/(1 +
Sin[c + d*x])^2)/(2*a^3*d)
```

Maple [A]
time = 0.20, size = 49, normalized size = 0.66

method	result	size
derivativedivides	$\frac{\ln(\sin(dx+c))+\frac{1}{2(1+\sin(dx+c))^2}+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{a^3d}$	49
default	$\frac{\ln(\sin(dx+c))+\frac{1}{2(1+\sin(dx+c))^2}+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{a^3d}$	49
risch	$\frac{2i\left(-e^{i(dx+c)}+3ie^{2i(dx+c)}+e^{3i(dx+c)}\right)}{d\,a^3\left(e^{i(dx+c)}+i\right)^4} - \frac{2\ln\left(e^{i(dx+c)}+i\right)}{a^3d} + \frac{\ln\left(e^{2i(dx+c)}-1\right)}{d\,a^3}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/d*(ln(sin(d*x+c))+1/2/(1+sin(d*x+c))^2+1/(1+sin(d*x+c))-ln(1+sin(d*x+
c)))
```

Maxima [A]
time = 0.28, size = 72, normalized size = 0.97

$$\frac{\frac{2\sin(dx+c)+3}{a^3\sin(dx+c)^2+2\,a^3\sin(dx+c)+a^3} - \frac{2\log(\sin(dx+c)+1)}{a^3} + \frac{2\log(\sin(dx+c))}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((2*sin(d*x + c) + 3)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) -
 2*log(sin(d*x + c) + 1)/a^3 + 2*log(sin(d*x + c))/a^3)/d
```

Fricas [A]
time = 0.35, size = 104, normalized size = 1.41

$$\frac{2\left(\cos\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)-2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-2\left(\cos\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)-2\,\sin\left(dx+c\right)-3}{2\left(a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\sin\left(dx+c\right)-2\,a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(1/2*sin(d*x + c)) - 2*(cos
(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 2*sin(d*x + c) -
3)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x)
 + 1), x)/a**3
```

Giac [A]
time = 9.82, size = 59, normalized size = 0.80

$$-\frac{\frac{2\,\log(|\sin(dx+c)+1|)}{a^{3}}-\frac{2\,\log(|\sin(dx+c)|)}{a^{3}}-\frac{2\,\sin(dx+c)+3}{a^{3}(\sin(dx+c)+1)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c) + 1))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 - (2*
sin(d*x + c) + 3)/(a^3*(sin(d*x + c) + 1)^2))/d
```

Mupad [B]
time = 6.65, size = 148, normalized size = 2.00

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^{3}\,d}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+4\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+6\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+4\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}\right)}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - (4*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)
/2)^2 + 4*tan(c/2 + (d*x)/2)^3)/(d*(6*a^3*tan(c/2 + (d*x)/2)^2 + 4*a^3*tan(
c/2 + (d*x)/2)^3 + a^3*tan(c/2 + (d*x)/2)^4 + a^3 + 4*a^3*tan(c/2 + (d*x)/2
))) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d)
```

3.76 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=86

$$\frac{3\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{5\log(\sin(c+dx))}{a^3d}-\frac{5\log(1+\sin(c+dx))}{a^3d}+\frac{2}{d\,(a^3+a^3\sin(c+dx))}$$

```
[Out] 3*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^3/d+5*ln(sin(d*x+c))/a^3/d-5*ln(1+sin
(d*x+c))/a^3/d+2/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2786, 78}

$$\frac{2}{d\,(a^3\sin(c+dx)+a^3)}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{5\log(\sin(c+dx))}{a^3d}-\frac{5\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (5*Log[Sin[c + d*x]])
/(a^3*d) - (5*Log[1 + Sin[c + d*x]])/(a^3*d) + 2/(d*(a^3 + a^3*Sin[c + d*x]
))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{3}{a^2x^2}+\frac{5}{a^3x}-\frac{2}{a^2(a+x)^2}-\frac{5}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{3\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{5\log(\sin(c+dx))}{a^3d}-\frac{5\log(1+\sin(c+dx))}{a^3d}+$$

Mathematica [A]
time = 0.13, size = 61, normalized size = 0.71

$$\frac{6\csc(c+dx)-\csc^2(c+dx)+10\log(\sin(c+dx))-10\log(1+\sin(c+dx))+\frac{4}{1+\sin(c+dx)}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Csc[c + d*x] - Csc[c + d*x]^2 + 10*Log[Sin[c + d*x]] - 10*Log[1 + Sin[c
+ d*x]] + 4/(1 + Sin[c + d*x]))/(2*a^3*d)
```

Maple [A]
time = 0.31, size = 61, normalized size = 0.71

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{3}{\sin(dx+c)}+5\ln(\sin(dx+c))+\frac{2}{1+\sin(dx+c)}-5\ln(1+\sin(dx+c))}{d\,a^3}$	61
default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{3}{\sin(dx+c)}+5\ln(\sin(dx+c))+\frac{2}{1+\sin(dx+c)}-5\ln(1+\sin(dx+c))}{d\,a^3}$	61
risch	$\frac{2i\left(5i\,\mathrm{e}^{4i(dx+c)}+5\,\mathrm{e}^{5i(dx+c)}-5i\,\mathrm{e}^{2i(dx+c)}-8\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}\right)}{\left(\mathrm{e}^{2i(dx+c)}-1\right)^2\left(\mathrm{e}^{i(dx+c)}+i\right)^2 d\,a^3}-\frac{10\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^3d}+\frac{5\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d\,a^3}$	137

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/2/sin(d*x+c)^2+3/sin(d*x+c)+5*ln(sin(d*x+c))+2/(1+sin(d*x+c))-5
*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 80, normalized size = 0.93

$$\frac{\frac{10\,\sin(dx+c)^2+5\,\sin(dx+c)-1}{a^3\sin(dx+c)^3+a^3\sin(dx+c)^2}-\frac{10\,\log(\sin(dx+c)+1)}{a^3}+\frac{10\,\log(\sin(dx+c))}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((10*sin(d*x + c)^2 + 5*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^3 + a^3*sin
(d*x + c)^2) - 10*log(sin(d*x + c) + 1)/a^3 + 10*log(sin(d*x + c))/a^3)/d
```

Fricas [A]
time = 0.37, size = 147, normalized size = 1.71

$$\frac{10\cos(dx+c)^2+10\left(\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-10\left(\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-1\right)\log(\sin(dx+c)+1)-5\sin(dx+c)-9}{2\left(a^3d\cos(dx+c)^2-a^3d+\left(a^3d\cos(dx+c)^2-a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(10*cos(d*x + c)^2 + 10*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x
+ c) - 1)*log(1/2*sin(d*x + c)) - 10*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)
*sin(d*x + c) - 1)*log(sin(d*x + c) + 1) - 5*sin(d*x + c) - 9)/(a^3*d*cos(d
*x + c)^2 - a^3*d + (a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 7.56, size = 154, normalized size = 1.79

$$-\frac{\frac{80\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{40\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{30\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+53\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2a^3}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(80*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 40*log(abs(tan(1/2*d*x +
1/2*c)))/a^3 - (30*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*x + 1/2*c)^3 + 53*
tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c
)^2 + tan(1/2*d*x + 1/2*c))^2*a^3) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*t
an(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 6.70, size = 169, normalized size = 1.97

$$\frac{5\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,a^3\,d}-\frac{10\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}+\frac{-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\frac{23\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}}{d\,\left(4\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+8\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}+\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*log(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^2/(8*a^3*d) - (10*
log(tan(c/2 + (d*x)/2) + 1))/(a^3*d) + (5*tan(c/2 + (d*x)/2) + (23*tan(c/2
+ (d*x)/2)^2)/2 - 10*tan(c/2 + (d*x)/2)^3 - 1/2)/(d*(4*a^3*tan(c/2 + (d*x)/
2)^2 + 8*a^3*tan(c/2 + (d*x)/2)^3 + 4*a^3*tan(c/2 + (d*x)/2)^4)) + (3*tan(c
/2 + (d*x)/2))/(2*a^3*d)
```

3.77 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=96

$$\frac{4\csc(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{\csc^4(c+dx)}{4a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(1+\sin(c+dx))}{a^3d}$$

```
[Out] 4*csc(d*x+c)/a^3/d-2*csc(d*x+c)^2/a^3/d+csc(d*x+c)^3/a^3/d-1/4*csc(d*x+c)^4
/a^3/d+4*ln(sin(d*x+c))/a^3/d-4*ln(1+sin(d*x+c))/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$-\frac{\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{4\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Csc[c + d*x])/(a^3*d) - (2*Csc[c + d*x]^2)/(a^3*d) + Csc[c + d*x]^3/(a^3
*d) - Csc[c + d*x]^4/(4*a^3*d) + (4*Log[Sin[c + d*x]])/(a^3*d) - (4*Log[1 +
 Sin[c + d*x]])/(a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^5}-\frac{3}{x^4}+\frac{4}{ax^3}-\frac{4}{a^2x^2}+\frac{4}{a^3x}-\frac{4}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{4\csc(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{\csc^4(c+dx)}{4a^3d}+\frac{4\log(\sin(c+}{a^3d}$$

Mathematica [A]
time = 0.21, size = 69, normalized size = 0.72

$$\frac{16\csc(c+dx)-8\csc^2(c+dx)+4\csc^3(c+dx)-\csc^4(c+dx)+16\log(\sin(c+dx))-16\log(1+\sin(c+dx))}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (16*Csc[c + d*x] - 8*Csc[c + d*x]^2 + 4*Csc[c + d*x]^3 - Csc[c + d*x]^4 + 1
6*Log[Sin[c + d*x]] - 16*Log[1 + Sin[c + d*x]])/(4*a^3*d)
```

Maple [A]
time = 0.30, size = 67, normalized size = 0.70

method	result
derivativedivides	$\frac{-\frac{1}{4\sin(dx+c)^4}+\frac{1}{\sin(dx+c)^3}-\frac{2}{\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+4\ln(\sin(dx+c))-4\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{-\frac{1}{4\sin(dx+c)^4}+\frac{1}{\sin(dx+c)^3}-\frac{2}{\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+4\ln(\sin(dx+c))-4\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$\frac{4i\left(-2i\,e^{6i(dx+c)}+2\,e^{7i(dx+c)}+5i\,e^{4i(dx+c)}-8\,e^{5i(dx+c)}-2i\,e^{2i(dx+c)}+8\,e^{3i(dx+c)}-2\,e^{i(dx+c)}\right)}{d\,a^3\left(e^{2i(dx+c)}-1\right)^4}-\frac{8\ln\left(e^{i(dx+c)}+i\right)}{a^3d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/4/sin(d*x+c)^4+1/sin(d*x+c)^3-2/sin(d*x+c)^2+4/sin(d*x+c)+4*ln(
sin(d*x+c))-4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 75, normalized size = 0.78

$$-\frac{\frac{16\,\log(\sin(dx+c)+1)}{a^3}-\frac{16\,\log(\sin(dx+c))}{a^3}-\frac{16\,\sin(dx+c)^3-8\,\sin(dx+c)^2+4\,\sin(dx+c)-1}{a^3\sin(dx+c)^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(16*log(sin(d*x + c) + 1)/a^3 - 16*log(sin(d*x + c))/a^3 - (16*sin(d*x
 + c)^3 - 8*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^4))/d
```

Fricas [A]
time = 0.39, size = 131, normalized size = 1.36

$$\frac{8\cos\left(dx+c\right)^{2}+16\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-16\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\log\left(\sin\left(dx+c\right)+1\right)-4\left(4\cos\left(dx+c\right)^{2}-5\right)\sin\left(dx+c\right)-9}{4\left(a^{3}d\cos\left(dx+c\right)^{4}-2a^{3}d\cos\left(dx+c\right)^{2}+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*cos(d*x + c)^2 + 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*
sin(d*x + c)) - 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(sin(d*x + c)
 + 1) - 4*(4*cos(d*x + c)^2 - 5)*sin(d*x + c) - 9)/(a^3*d*cos(d*x + c)^4 -
2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{5}\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 8.40, size = 174, normalized size = 1.81

$$-\frac{\frac{1536\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{3}}-\frac{768\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{3}}+\frac{1600\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-456\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+108\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3}{a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}}+\frac{3\left(a^{9}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-8a^{9}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+36a^{9}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-152a^{9}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{12}}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/192*(1536*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 768*log(abs(tan(1/2*d
*x + 1/2*c)))/a^3 + (1600*tan(1/2*d*x + 1/2*c)^4 - 456*tan(1/2*d*x + 1/2*c)
^3 + 108*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 3)/(a^3*tan(1/2
*d*x + 1/2*c)^4) + 3*(a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*
c)^3 + 36*a^9*tan(1/2*d*x + 1/2*c)^2 - 152*a^9*tan(1/2*d*x + 1/2*c))/a^12)/
d
```

Mupad [B]
time = 6.72, size = 171, normalized size = 1.78

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{8a^3d}-\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16a^3d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64a^3d}+\frac{4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3d}-\frac{8\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^3d}+\frac{19\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^3d}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(38\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{1}{4}\right)}{16a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + a*sin(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(8*a^3*d) - (9*tan(c/2 + (d*x)/2)^2)/(16*a^3*d) - tan(
c/2 + (d*x)/2)^4/(64*a^3*d) + (4*log(tan(c/2 + (d*x)/2)))/(a^3*d) - (8*log(
tan(c/2 + (d*x)/2) + 1))/(a^3*d) + (19*tan(c/2 + (d*x)/2))/(8*a^3*d) + (cot
(c/2 + (d*x)/2)^4*(2*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^2 + 38*tan(c
/2 + (d*x)/2)^3 - 1/4))/(16*a^3*d)
```

3.78 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=73

$$\frac{\csc^3(c+dx)}{3a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{3\csc^5(c+dx)}{5a^3d}-\frac{\csc^6(c+dx)}{6a^3d}$$

```
[Out] 1/3*csc(d*x+c)^3/a^3/d-3/4*csc(d*x+c)^4/a^3/d+3/5*csc(d*x+c)^5/a^3/d-1/6*cs
c(d*x+c)^6/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2786, 45}

$$-\frac{\csc^6(c+dx)}{6a^3d}+\frac{3\csc^5(c+dx)}{5a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + (3*Csc[c + d*x]^5
)/(5*a^3*d) - Csc[c + d*x]^6/(6*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^7} - \frac{3a^2}{x^6} + \frac{3a}{x^5} - \frac{1}{x^4}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^3(c+dx)}{3a^3d} - \frac{3\csc^4(c+dx)}{4a^3d} + \frac{3\csc^5(c+dx)}{5a^3d} - \frac{\csc^6(c+dx)}{6a^3d}$$

Mathematica [A]
time = 0.07, size = 48, normalized size = 0.66

$$\frac{\csc^3(c+dx)\left(20 - 45\csc(c+dx) + 36\csc^2(c+dx) - 10\csc^3(c+dx)\right)}{60a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(20 - 45*Csc[c + d*x] + 36*Csc[c + d*x]^2 - 10*Csc[c + d*x]
^3))/(60*a^3*d)
```

Maple [A]
time = 0.30, size = 49, normalized size = 0.67

method	result	size
derivativedivides	$\frac{\frac{3}{5\sin(dx+c)^5}+\frac{1}{3\sin(dx+c)^3}-\frac{3}{4\sin(dx+c)^4}-\frac{1}{6\sin(dx+c)^6}}{d\,a^3}$	49
default	$\frac{\frac{3}{5\sin(dx+c)^5}+\frac{1}{3\sin(dx+c)^3}-\frac{3}{4\sin(dx+c)^4}-\frac{1}{6\sin(dx+c)^6}}{d\,a^3}$	49
risch	$-\frac{4i\left(-45i\,e^{8i(dx+c)}+10\,e^{9i(dx+c)}+130i\,e^{6i(dx+c)}-102\,e^{7i(dx+c)}-45i\,e^{4i(dx+c)}+102\,e^{5i(dx+c)}-10\,e^{3i(dx+c)}\right)}{15d\,a^3\left(e^{2i(dx+c)}-1\right)^6}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(3/5/sin(d*x+c)^5+1/3/sin(d*x+c)^3-3/4/sin(d*x+c)^4-1/6/sin(d*x+c)^
6)
```

Maxima [A]
time = 0.29, size = 46, normalized size = 0.63

$$\frac{20\,\sin\left(dx+c\right)^3 - 45\,\sin\left(dx+c\right)^2 + 36\,\sin\left(dx+c\right) - 10}{60\,a^3d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(20*sin(d*x + c)^3 - 45*sin(d*x + c)^2 + 36*sin(d*x + c) - 10)/(a^3*d*
sin(d*x + c)^6)
```

Fricas [A]
time = 0.34, size = 84, normalized size = 1.15

$$-\frac{45\cos\left(dx+c\right)^{2}-4\left(5\cos\left(dx+c\right)^{2}-14\right)\sin\left(dx+c\right)-55}{60\left(a^{3}d\cos\left(dx+c\right)^{6}-3\,a^{3}d\cos\left(dx+c\right)^{4}+3\,a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(45*cos(d*x + c)^2 - 4*(5*cos(d*x + c)^2 - 14)*sin(d*x + c) - 55)/(a^
3*d*cos(d*x + c)^6 - 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^2 - a^3*
d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^{7}\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**7/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 7.55, size = 46, normalized size = 0.63

$$\frac{20\sin\left(dx+c\right)^{3}-45\sin\left(dx+c\right)^{2}+36\sin\left(dx+c\right)-10}{60\,a^{3}d\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(20*sin(d*x + c)^3 - 45*sin(d*x + c)^2 + 36*sin(d*x + c) - 10)/(a^3*d*
sin(d*x + c)^6)
```

Mupad [B]
time = 6.69, size = 46, normalized size = 0.63

$$\frac{20\sin\left(c+d\,x\right)^{3}-45\sin\left(c+d\,x\right)^{2}+36\,\sin\left(c+d\,x\right)-10}{60\,a^{3}\,d\sin\left(c+d\,x\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (36*sin(c + d*x) - 45*sin(c + d*x)^2 + 20*sin(c + d*x)^3 - 10)/(60*a^3*d*si
n(c + d*x)^6)
```

3.79 $\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{2\csc^5(c+dx)}{5a^3d}-\frac{\csc^6(c+dx)}{3a^3d}+\frac{3\csc^7(c+dx)}{7a^3d}-\frac{\csc^8(c+dx)}{8a^3d}$$

```
[Out] -1/3*csc(d*x+c)^3/a^3/d+3/4*csc(d*x+c)^4/a^3/d-2/5*csc(d*x+c)^5/a^3/d-1/3*c
sc(d*x+c)^6/a^3/d+3/7*csc(d*x+c)^7/a^3/d-1/8*csc(d*x+c)^8/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 76}

$$-\frac{\csc^8(c+dx)}{8a^3d}+\frac{3\csc^7(c+dx)}{7a^3d}-\frac{\csc^6(c+dx)}{3a^3d}-\frac{2\csc^5(c+dx)}{5a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{\csc^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/3*Csc[c + d*x]^3/(a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - (2*Csc[c + d*x
]^5)/(5*a^3*d) - Csc[c + d*x]^6/(3*a^3*d) + (3*Csc[c + d*x]^7)/(7*a^3*d) -
Csc[c + d*x]^8/(8*a^3*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^4(a+x)}{x^9}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^5}{x^9} - \frac{3a^4}{x^8} + \frac{2a^3}{x^7} + \frac{2a^2}{x^6} - \frac{3a}{x^5} + \frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3a^3d} + \frac{3\csc^4(c+dx)}{4a^3d} - \frac{2\csc^5(c+dx)}{5a^3d} - \frac{\csc^6(c+dx)}{3a^3d} + \frac{3\csc^7(c+}{7a^3d}$$

Mathematica [A]
time = 0.06, size = 68, normalized size = 0.62

$$-\frac{\csc^3(c+dx)\,(280-630\csc(c+dx)+336\csc^2(c+dx)+280\csc^3(c+dx)-360\csc^4(c+dx)+105\csc^5(c+dx))}{840a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/840*(Csc[c + d*x]^3*(280 - 630*Csc[c + d*x] + 336*Csc[c + d*x]^2 + 280*C
sc[c + d*x]^3 - 360*Csc[c + d*x]^4 + 105*Csc[c + d*x]^5))/(a^3*d)
```

Maple [A]
time = 0.37, size = 69, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{1}{3\sin(dx+c)^3}-\frac{1}{3\sin(dx+c)^6}+\frac{3}{4\sin(dx+c)^4}-\frac{2}{5\sin(dx+c)^5}-\frac{1}{8\sin(dx+c)^8}+\frac{3}{7\sin(dx+c)^7}}{d\,a^3}$
default	$\frac{-\frac{1}{3\sin(dx+c)^3}-\frac{1}{3\sin(dx+c)^6}+\frac{3}{4\sin(dx+c)^4}-\frac{2}{5\sin(dx+c)^5}-\frac{1}{8\sin(dx+c)^8}+\frac{3}{7\sin(dx+c)^7}}{d\,a^3}$
risch	$\frac{4i(-315i e^{12i(dx+c)}+70\,e^{13i(dx+c)}+700i e^{10i(dx+c)}-686\,e^{11i(dx+c)}+70i e^{8i(dx+c)}+268\,e^{9i(dx+c)}+700i e^{6i(dx+c)}-268\,e^{7}}{105d\,a^3\left(e^{2i(dx+c)}-1\right)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/3/sin(d*x+c)^3-1/3/sin(d*x+c)^6+3/4/sin(d*x+c)^4-2/5/sin(d*x+c)
^5-1/8/sin(d*x+c)^8+3/7/sin(d*x+c)^7)
```

Maxima [A]
time = 0.28, size = 66, normalized size = 0.61

$$-\frac{280\sin(dx+c)^5-630\sin(dx+c)^4+336\sin(dx+c)^3+280\sin(dx+c)^2-360\sin(dx+c)+105}{840\,a^3d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/840*(280*sin(d*x + c)^5 - 630*sin(d*x + c)^4 + 336*sin(d*x + c)^3 + 280*
sin(d*x + c)^2 - 360*sin(d*x + c) + 105)/(a^3*d*sin(d*x + c)^8)
```

Fricas [A]

time = 0.35, size = 117, normalized size = 1.07

$$\frac{630 \cos(dx+c)^4 - 980 \cos(dx+c)^2 - 8 \left(35 \cos(dx+c)^4 - 112 \cos(dx+c)^2 + 32\right) \sin(dx+c) + 245}{840 \left(a^3 d \cos(dx+c)^8 - 4 a^3 d \cos(dx+c)^6 + 6 a^3 d \cos(dx+c)^4 - 4 a^3 d \cos(dx+c)^2 + a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(630*cos(d*x + c)^4 - 980*cos(d*x + c)^2 - 8*(35*cos(d*x + c)^4 - 112
*cos(d*x + c)^2 + 32)*sin(d*x + c) + 245)/(a^3*d*cos(d*x + c)^8 - 4*a^3*d*c
os(d*x + c)^6 + 6*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^9(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**9/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A]

time = 6.48, size = 66, normalized size = 0.61

$$-\frac{280 \sin(dx+c)^5 - 630 \sin(dx+c)^4 + 336 \sin(dx+c)^3 + 280 \sin(dx+c)^2 - 360 \sin(dx+c) + 105}{840 a^3 d \sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/840*(280*sin(d*x + c)^5 - 630*sin(d*x + c)^4 + 336*sin(d*x + c)^3 + 280*
sin(d*x + c)^2 - 360*sin(d*x + c) + 105)/(a^3*d*sin(d*x + c)^8)
```

Mupad [B]

time = 6.63, size = 66, normalized size = 0.61

$$-\frac{280 \sin(c+d\,x)^5 - 630 \sin(c+d\,x)^4 + 336 \sin(c+d\,x)^3 + 280 \sin(c+d\,x)^2 - 360 \sin(c+d\,x) + 105}{840\,a^3\,d \sin(c+d\,x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^9/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -(280*sin(c + d*x)^2 - 360*sin(c + d*x) + 336*sin(c + d*x)^3 - 630*sin(c +
d*x)^4 + 280*sin(c + d*x)^5 + 105)/(840*a^3*d*sin(c + d*x)^8)
```

3.80 $\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{\csc^3(c+dx)}{3a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{\csc^5(c+dx)}{5a^3d}+\frac{5\csc^6(c+dx)}{6a^3d}-\frac{5\csc^7(c+dx)}{7a^3d}-\frac{\csc^8(c+dx)}{8a^3d}+\frac{\csc^9(c+dx)}{3a^3d}-$$

```
[Out] 1/3*csc(d*x+c)^3/a^3/d-3/4*csc(d*x+c)^4/a^3/d+1/5*csc(d*x+c)^5/a^3/d+5/6*cs
c(d*x+c)^6/a^3/d-5/7*csc(d*x+c)^7/a^3/d-1/8*csc(d*x+c)^8/a^3/d+1/3*csc(d*x+
c)^9/a^3/d-1/10*csc(d*x+c)^10/a^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$-\frac{\csc^{10}(c+dx)}{10a^3d}+\frac{\csc^9(c+dx)}{3a^3d}-\frac{\csc^8(c+dx)}{8a^3d}-\frac{5\csc^7(c+dx)}{7a^3d}+\frac{5\csc^6(c+dx)}{6a^3d}+\frac{\csc^5(c+dx)}{5a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + Csc[c + d*x]^5/(5
*a^3*d) + (5*Csc[c + d*x]^6)/(6*a^3*d) - (5*Csc[c + d*x]^7)/(7*a^3*d) - Csc
[c + d*x]^8/(8*a^3*d) + Csc[c + d*x]^9/(3*a^3*d) - Csc[c + d*x]^10/(10*a^3*
d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^5(a+x)^2}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^7}{x^{11}} - \frac{3a^6}{x^{10}} + \frac{a^5}{x^9} + \frac{5a^4}{x^8} - \frac{5a^3}{x^7} - \frac{a^2}{x^6} + \frac{3a}{x^5} - \frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^3(c+dx)}{3a^3d} - \frac{3\csc^4(c+dx)}{4a^3d} + \frac{\csc^5(c+dx)}{5a^3d} + \frac{5\csc^6(c+dx)}{6a^3d} - \frac{5\csc^7(c+dx)}{7a^3d}$$

Mathematica [A]
time = 0.08, size = 88, normalized size = 0.61

$$\frac{\csc^3(c+dx)\left(280-630\csc(c+dx)+168\csc^2(c+dx)+700\csc^3(c+dx)-600\csc^4(c+dx)-105\csc^5(c+dx)+280\csc^6(c+dx)-84\csc^7(c+dx)\right)}{840a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^3,x]
[Out] (Csc[c + d*x]^3*(280 - 630*Csc[c + d*x] + 168*Csc[c + d*x]^2 + 700*Csc[c +
d*x]^3 - 600*Csc[c + d*x]^4 - 105*Csc[c + d*x]^5 + 280*Csc[c + d*x]^6 - 84*
Csc[c + d*x]^7))/(840*a^3*d)
```

Maple [A]
time = 0.45, size = 89, normalized size = 0.61

method	result
derivativedivides	$\frac{-\frac{1}{8\sin(dx+c)^8}+\frac{5}{6\sin(dx+c)^6}+\frac{1}{3\sin(dx+c)^9}-\frac{5}{7\sin(dx+c)^7}-\frac{3}{4\sin(dx+c)^4}+\frac{1}{5\sin(dx+c)^5}-\frac{1}{10\sin(dx+c)^{10}}+\frac{1}{3\sin(dx+c)^3}}{d\,a^3}$
default	$\frac{-\frac{1}{8\sin(dx+c)^8}+\frac{5}{6\sin(dx+c)^6}+\frac{1}{3\sin(dx+c)^9}-\frac{5}{7\sin(dx+c)^7}-\frac{3}{4\sin(dx+c)^4}+\frac{1}{5\sin(dx+c)^5}-\frac{1}{10\sin(dx+c)^{10}}+\frac{1}{3\sin(dx+c)^3}}{d\,a^3}$
risch	$-\frac{4i\left(-315i e^{16i(dx+c)}+70\,e^{17i(dx+c)}+490i e^{14i(dx+c)}-658\,e^{15i(dx+c)}+35i e^{12i(dx+c)}-90\,e^{13i(dx+c)}+2268i e^{10i(dx+c)}-14\right.}{105d\,a^3\left(e^{2i(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(-1/8/sin(d*x+c)^8+5/6/sin(d*x+c)^6+1/3/sin(d*x+c)^9-5/7/sin(d*x+c)
^7-3/4/sin(d*x+c)^4+1/5/sin(d*x+c)^5-1/10/sin(d*x+c)^10+1/3/sin(d*x+c)^3)
```

Maxima [A]
time = 0.28, size = 86, normalized size = 0.59

$$\frac{280\sin(dx+c)^7-630\sin(dx+c)^6+168\sin(dx+c)^5+700\sin(dx+c)^4-600\sin(dx+c)^3-105\sin(dx+c)^2+280\sin(dx+c)-84}{840\,a^3d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(280*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 168*sin(d*x + c)^5 + 700*s
in(d*x + c)^4 - 600*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 280*sin(d*x + c)
- 84)/(a^3*d*sin(d*x + c)^10)
```

Fricas [**A**]
time = 0.41, size = 152, normalized size = 1.05

$$-\frac{630\cos(dx+c)^6-1190\cos(dx+c)^4+595\cos(dx+c)^2-8\left(35\cos(dx+c)^6-126\cos(dx+c)^4+72\cos(dx+c)^2-16\right)\sin(dx+c)-119}{840\left(a^3d\cos(dx+c)^{10}-5a^3d\cos(dx+c)^8+10a^3d\cos(dx+c)^6-10a^3d\cos(dx+c)^4+5a^3d\cos(dx+c)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/840*(630*cos(d*x + c)^6 - 1190*cos(d*x + c)^4 + 595*cos(d*x + c)^2 - 8*(
35*cos(d*x + c)^6 - 126*cos(d*x + c)^4 + 72*cos(d*x + c)^2 - 16)*sin(d*x +
c) - 119)/(a^3*d*cos(d*x + c)^10 - 5*a^3*d*cos(d*x + c)^8 + 10*a^3*d*cos(d*
x + c)^6 - 10*a^3*d*cos(d*x + c)^4 + 5*a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**11/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.05, size = 86, normalized size = 0.59

$$\frac{280\sin(dx+c)^7-630\sin(dx+c)^6+168\sin(dx+c)^5+700\sin(dx+c)^4-600\sin(dx+c)^3-105\sin(dx+c)^2+280\sin(dx+c)-84}{840a^3d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(280*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 168*sin(d*x + c)^5 + 700*s
in(d*x + c)^4 - 600*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 280*sin(d*x + c)
- 84)/(a^3*d*sin(d*x + c)^10)
```

Mupad [**B**]
time = 6.81, size = 86, normalized size = 0.59

$$\frac{280\sin(c+dx)^7-630\sin(c+dx)^6+168\sin(c+dx)^5+700\sin(c+dx)^4-600\sin(c+dx)^3-105\sin(c+dx)^2+280\sin(c+dx)-84}{840a^3d\sin(c+dx)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^11/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (280*sin(c + d*x) - 105*sin(c + d*x)^2 - 600*sin(c + d*x)^3 + 700*sin(c + d
*x)^4 + 168*sin(c + d*x)^5 - 630*sin(c + d*x)^6 + 280*sin(c + d*x)^7 - 84)/
(840*a^3*d*sin(c + d*x)^10)
```

3.81 $\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{4\csc^6(c+dx)}{3a^3d}+\frac{6\csc^7(c+dx)}{7a^3d}+\frac{3\csc^8(c+dx)}{4a^3d}-\frac{8\csc^9(c+dx)}{9a^3d}+\frac{3\csc^{11}(c-}{11a^3c}$$

```
[Out] -1/3*csc(d*x+c)^3/a^3/d+3/4*csc(d*x+c)^4/a^3/d-4/3*csc(d*x+c)^6/a^3/d+6/7*c
sc(d*x+c)^7/a^3/d+3/4*csc(d*x+c)^8/a^3/d-8/9*csc(d*x+c)^9/a^3/d+3/11*csc(d*
x+c)^11/a^3/d-1/12*csc(d*x+c)^12/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$-\frac{\csc^{12}(c+dx)}{12a^3d}+\frac{3\csc^{11}(c+dx)}{11a^3d}-\frac{8\csc^9(c+dx)}{9a^3d}+\frac{3\csc^8(c+dx)}{4a^3d}+\frac{6\csc^7(c+dx)}{7a^3d}-\frac{4\csc^6(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{\csc^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/3*Csc[c + d*x]^3/(a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - (4*Csc[c + d*x
]^6)/(3*a^3*d) + (6*Csc[c + d*x]^7)/(7*a^3*d) + (3*Csc[c + d*x]^8)/(4*a^3*d
) - (8*Csc[c + d*x]^9)/(9*a^3*d) + (3*Csc[c + d*x]^11)/(11*a^3*d) - Csc[c +
 d*x]^12/(12*a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^6(a+x)^3}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{13}} - \frac{3a^8}{x^{12}} + \frac{8a^6}{x^{10}} - \frac{6a^5}{x^9} - \frac{6a^4}{x^8} + \frac{8a^3}{x^7} - \frac{3a}{x^5} + \frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3a^3d} + \frac{3\csc^4(c+dx)}{4a^3d} - \frac{4\csc^6(c+dx)}{3a^3d} + \frac{6\csc^7(c+dx)}{7a^3d} + \frac{3\csc^8(c+}{4a^3d}$$

Mathematica [A]
time = 0.09, size = 88, normalized size = 0.61

$$\frac{\csc^3(c+dx)\left(-924+2079\csc(c+dx)-3696\csc^3(c+dx)+2376\csc^4(c+dx)+2079\csc^5(c+dx)-2464\csc^6(c+dx)+756\csc^8(c+dx)-231\csc^9(c+dx)\right)}{2772a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^3,x]
[Out] (Csc[c + d*x]^3*(-924 + 2079*Csc[c + d*x] - 3696*Csc[c + d*x]^3 + 2376*Csc[
c + d*x]^4 + 2079*Csc[c + d*x]^5 - 2464*Csc[c + d*x]^6 + 756*Csc[c + d*x]^8
 - 231*Csc[c + d*x]^9))/(2772*a^3*d)
```

Maple [A]
time = 0.57, size = 89, normalized size = 0.61

method	result
derivativedivides	$\frac{\frac{3}{4\sin(dx+c)^4}-\frac{8}{9\sin(dx+c)^9}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{12\sin(dx+c)^{12}}+\frac{3}{4\sin(dx+c)^8}-\frac{4}{3\sin(dx+c)^6}+\frac{6}{7\sin(dx+c)^7}+\frac{3}{11\sin(dx+c)^{11}}}{d\,a^3}$
default	$\frac{\frac{3}{4\sin(dx+c)^4}-\frac{8}{9\sin(dx+c)^9}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{12\sin(dx+c)^{12}}+\frac{3}{4\sin(dx+c)^8}-\frac{4}{3\sin(dx+c)^6}+\frac{6}{7\sin(dx+c)^7}+\frac{3}{11\sin(dx+c)^{11}}}{d\,a^3}$
risch	$4i\left(-2079i e^{20i(dx+c)}+462\,e^{21i(dx+c)}-2079i e^{4i(dx+c)}-4158\,e^{19i(dx+c)}+9702i e^{12i(dx+c)}-2376\,e^{17i(dx+c)}+27720i e^{10i(dx\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(3/4/sin(d*x+c)^4-8/9/sin(d*x+c)^9-1/3/sin(d*x+c)^3-1/12/sin(d*x+c)
^12+3/4/sin(d*x+c)^8-4/3/sin(d*x+c)^6+6/7/sin(d*x+c)^7+3/11/sin(d*x+c)^11)
```

Maxima [A]
time = 0.28, size = 86, normalized size = 0.59

$$-\frac{924\sin(dx+c)^9-2079\sin(dx+c)^8+3696\sin(dx+c)^6-2376\sin(dx+c)^5-2079\sin(dx+c)^4+2464\sin(dx+c)^3-756\sin(dx+c)+231}{2772\,a^3d\sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2772*(924*sin(d*x + c)^9 - 2079*sin(d*x + c)^8 + 3696*sin(d*x + c)^6 - 2
376*sin(d*x + c)^5 - 2079*sin(d*x + c)^4 + 2464*sin(d*x + c)^3 - 756*sin(d*
x + c) + 231)/(a^3*d*sin(d*x + c)^12)
```

Fricas [A]
time = 0.38, size = 185, normalized size = 1.28

$$\frac{2079 \cos(dx+c)^8 - 4620 \cos(dx+c)^6 + 3465 \cos(dx+c)^4 - 1386 \cos(dx+c)^2 - 4\left(231 \cos(dx+c)^8 - 924 \cos(dx+c)^6 + 792 \cos(dx+c)^4 - 352 \cos(dx+c)^2 + 64\right)\sin(dx+c) + 231}{2772\left(a^3 d\cos(dx+c)^{12} - 6a^3 d\cos(dx+c)^{10} + 15a^3 d\cos(dx+c)^8 - 20a^3 d\cos(dx+c)^6 + 15a^3 d\cos(dx+c)^4 - 6a^3 d\cos(dx+c)^2 + a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2772*(2079*cos(d*x + c)^8 - 4620*cos(d*x + c)^6 + 3465*cos(d*x + c)^4 - 1
386*cos(d*x + c)^2 - 4*(231*cos(d*x + c)^8 - 924*cos(d*x + c)^6 + 792*cos(d
*x + c)^4 - 352*cos(d*x + c)^2 + 64)*sin(d*x + c) + 231)/(a^3*d*cos(d*x + c
)^12 - 6*a^3*d*cos(d*x + c)^10 + 15*a^3*d*cos(d*x + c)^8 - 20*a^3*d*cos(d*x
 + c)^6 + 15*a^3*d*cos(d*x + c)^4 - 6*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**13/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [A]
time = 6.73, size = 86, normalized size = 0.59

$$-\frac{924 \sin(dx+c)^9 - 2079 \sin(dx+c)^8 + 3696 \sin(dx+c)^6 - 2376 \sin(dx+c)^5 - 2079 \sin(dx+c)^4 + 2464 \sin(dx+c)^3 - 756 \sin(dx+c) + 231}{2772\, a^3 d\sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2772*(924*sin(d*x + c)^9 - 2079*sin(d*x + c)^8 + 3696*sin(d*x + c)^6 - 2
376*sin(d*x + c)^5 - 2079*sin(d*x + c)^4 + 2464*sin(d*x + c)^3 - 756*sin(d*
x + c) + 231)/(a^3*d*sin(d*x + c)^12)
```

Mupad [B]
time = 6.82, size = 85, normalized size = 0.59

$$\frac{-\frac{\sin(c+d\,x)^9}{3}+\frac{3\sin(c+d\,x)^8}{4}-\frac{4\sin(c+d\,x)^6}{3}+\frac{6\sin(c+d\,x)^5}{7}+\frac{3\sin(c+d\,x)^4}{4}-\frac{8\sin(c+d\,x)^3}{9}+\frac{3\sin(c+d\,x)}{11}-\frac{1}{12}}{a^3\,d\sin\left(c+d\,x\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^13/(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((3*sin(c + d*x))/11 - (8*sin(c + d*x)^3)/9 + (3*sin(c + d*x)^4)/4 + (6*sin
(c + d*x)^5)/7 - (4*sin(c + d*x)^6)/3 + (3*sin(c + d*x)^8)/4 - sin(c + d*x)
^9/3 - 1/12)/(a^3*d*sin(c + d*x)^12)
```

3.82 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=195

$$-\frac{\tanh^{-1}(\sin(c+dx))}{128a^4d}+\frac{a^2}{48d(a+a\sin(c+dx))^6}-\frac{7a}{80d(a+a\sin(c+dx))^5}+\frac{1}{8d(a+a\sin(c+dx))^4}-\frac{}{96ad(a}$$

```
[Out] -1/128*arctanh(sin(d*x+c))/a^4/d+1/48*a^2/d/(a+a*sin(d*x+c))^6-7/80*a/d/(a+
a*sin(d*x+c))^5+1/8/d/(a+a*sin(d*x+c))^4-5/96/a/d/(a+a*sin(d*x+c))^3+1/256/
d/(a^2-a^2*sin(d*x+c))^2-5/256/d/(a^2+a^2*sin(d*x+c))^2-3/256/d/(a^4-a^4*si
n(d*x+c))-1/256/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2786, 90, 212}

$$-\frac{3}{256d\,(a^4-a^4\sin(c+dx))}-\frac{1}{256d\,(a^4\sin(c+dx)+a^4)}-\frac{\tanh^{-1}(\sin(c+dx))}{128a^4d}+\frac{a^2}{48d(a\sin(c+dx)+a)^6}+\frac{1}{256d\,(a^2-a^2\sin(c+dx))^2}-\frac{5}{256d\,(a^2\sin(c+dx)+a^2)^2}-\frac{7a}{80d(a\sin(c+dx)+a)^5}+\frac{1}{8d(a\sin(c+dx)+a)^4}-\frac{5}{96ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/128*ArcTanh[Sin[c + d*x]]/(a^4*d) + a^2/(48*d*(a + a*Sin[c + d*x])^6) -
(7*a)/(80*d*(a + a*Sin[c + d*x])^5) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 5/(9
6*a*d*(a + a*Sin[c + d*x])^3) + 1/(256*d*(a^2 - a^2*Sin[c + d*x])^2) - 5/(2
56*d*(a^2 + a^2*Sin[c + d*x])^2) - 3/(256*d*(a^4 - a^4*Sin[c + d*x])) - 1/(
256*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
```

```
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^7}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{128a^2(a-x)^3} - \frac{3}{256a^3(a-x)^2} - \frac{a^2}{8(a+x)^7} + \frac{7a}{16(a+x)^6} - \frac{1}{2(a+x)^5} + \frac{5}{32a(a+x)^4} + \frac{1}{12}\right.\right.}{d}$$

$$= \frac{a^2}{48d(a+a\sin(c+dx))^6} - \frac{7a}{80d(a+a\sin(c+dx))^5} + \frac{1}{8d(a+a\sin(c+dx))^4} -$$

$$= -\frac{\tanh^{-1}(\sin(c+dx))}{128a^4d} + \frac{a^2}{48d(a+a\sin(c+dx))^6} - \frac{7a}{80d(a+a\sin(c+dx))^5} + \frac{}{8}$$

Mathematica [A]
time = 0.95, size = 112, normalized size = 0.57

$$-\frac{30\tanh^{-1}(\sin(c+dx)) - \frac{2(-48-177\sin(c+dx)-132\sin^2(c+dx)+257\sin^3(c+dx)+440\sin^4(c+dx)+65\sin^5(c+dx)+60\sin^6(c+dx)+15\sin^7(c+dx))}{(-1+\sin(c+dx))^2(1+\sin(c+dx))^6}}{3840a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/3840*(30*ArcTanh[Sin[c + d*x]] - (2*(-48 - 177*Sin[c + d*x] - 132*Sin[c
+ d*x]^2 + 257*Sin[c + d*x]^3 + 440*Sin[c + d*x]^4 + 65*Sin[c + d*x]^5 + 60
*Sin[c + d*x]^6 + 15*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^2*(1 + Sin[c + d
*x])^6))/(a^4*d)
```

Maple [A]
time = 0.32, size = 127, normalized size = 0.65

method	result
derivativedivides	$\frac{\frac{1}{256(\sin(dx+c)-1)^2}+\frac{3}{256(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{256}+\frac{1}{48(1+\sin(dx+c))^6}-\frac{7}{80(1+\sin(dx+c))^5}+\frac{1}{8(1+\sin(dx+c))^4}-\frac{5}{96(1+\sin(d}}{d\,a^4}$
default	$\frac{\frac{1}{256(\sin(dx+c)-1)^2}+\frac{3}{256(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{256}+\frac{1}{48(1+\sin(dx+c))^6}-\frac{7}{80(1+\sin(dx+c))^5}+\frac{1}{8(1+\sin(dx+c))^4}-\frac{5}{96(1+\sin(d}}{d\,a^4}$
risch	$\frac{i(4133\,e^{7i(dx+c)}+5727\,e^{11i(dx+c)}+365\,e^{3i(dx+c)}-15\,e^{i(dx+c)}-5727\,e^{5i(dx+c)}-4240ie^{4i(dx+c)}+11656ie^{6i(dx+c)}-8928ie^{8}}{960\left(e^{i(dx+c)}-i\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(1/256/(sin(d*x+c)-1)^2+3/256/(sin(d*x+c)-1)+1/256*ln(sin(d*x+c)-1)
+1/48/(1+sin(d*x+c))^6-7/80/(1+sin(d*x+c))^5+1/8/(1+sin(d*x+c))^4-5/96/(1+s
in(d*x+c))^3-5/256/(1+sin(d*x+c))^2-1/256/(1+sin(d*x+c))-1/256*ln(1+sin(d*x
+c)))
```

Maxima [A]
time = 0.29, size = 213, normalized size = 1.09

$$\frac{\frac{2\left(15\,\sin(dx+c)^7+60\,\sin(dx+c)^6+65\,\sin(dx+c)^5+440\,\sin(dx+c)^4+257\,\sin(dx+c)^3-132\,\sin(dx+c)^2-177\,\sin(dx+c)-48\right)}{a^4\sin(dx+c)^8+4\,a^4\sin(dx+c)^7+4\,a^4\sin(dx+c)^6-4\,a^4\sin(dx+c)^5-10\,a^4\sin(dx+c)^4-4\,a^4\sin(dx+c)^3+4\,a^4\sin(dx+c)^2+4\,a^4\sin(dx+c)+a^4}-\frac{15\,\log(\sin(dx+c)+1)}{a^4}+\frac{15\,\log(\sin(dx+c)-1)}{a^4}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3840*(2*(15*sin(d*x + c)^7 + 60*sin(d*x + c)^6 + 65*sin(d*x + c)^5 + 440*
sin(d*x + c)^4 + 257*sin(d*x + c)^3 - 132*sin(d*x + c)^2 - 177*sin(d*x + c)
 - 48)/(a^4*sin(d*x + c)^8 + 4*a^4*sin(d*x + c)^7 + 4*a^4*sin(d*x + c)^6 -
4*a^4*sin(d*x + c)^5 - 10*a^4*sin(d*x + c)^4 - 4*a^4*sin(d*x + c)^3 + 4*a^4
*sin(d*x + c)^2 + 4*a^4*sin(d*x + c) + a^4) - 15*log(sin(d*x + c) + 1)/a^4
+ 15*log(sin(d*x + c) - 1)/a^4)/d
```

Fricas [A]
time = 0.38, size = 290, normalized size = 1.49

$$-\frac{120\,\cos(dx+c)^6-1240\,\cos(dx+c)^4+1856\,\cos(dx+c)^2+15\left(\cos(dx+c)^8-8\,\cos(dx+c)^6+8\,\cos(dx+c)^4-4\left(\cos(dx+c)^6-2\,\cos(dx+c)^4\right)\sin(dx+c)\right)\log(\sin(dx+c)+1)-15\left(\cos(dx+c)^8-8\,\cos(dx+c)^6+8\,\cos(dx+c)^4-4\left(\cos(dx+c)^6-2\,\cos(dx+c)^4\right)\sin(dx+c)\right)\log(-\sin(dx+c)+1)+2\left(15\,\cos(dx+c)^6-110\,\cos(dx+c)^4+432\,\cos(dx+c)^2-160\right)\sin(dx+c)-640}{3840\left(a^4d\cos(dx+c)^8-8\,a^4d\cos(dx+c)^6+8\,a^4d\cos(dx+c)^4-4\left(a^4d\cos(dx+c)^6-2\,a^4d\cos(dx+c)^4\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3840*(120*cos(d*x + c)^6 - 1240*cos(d*x + c)^4 + 1856*cos(d*x + c)^2 + 1
5*(cos(d*x + c)^8 - 8*cos(d*x + c)^6 + 8*cos(d*x + c)^4 - 4*(cos(d*x + c)^6
 - 2*cos(d*x + c)^4)*sin(d*x + c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)
^8 - 8*cos(d*x + c)^6 + 8*cos(d*x + c)^4 - 4*(cos(d*x + c)^6 - 2*cos(d*x +
c)^4)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(15*cos(d*x + c)^6 - 110*cos
(d*x + c)^4 + 432*cos(d*x + c)^2 - 160)*sin(d*x + c) - 640)/(a^4*d*cos(d*x
+ c)^8 - 8*a^4*d*cos(d*x + c)^6 + 8*a^4*d*cos(d*x + c)^4 - 4*(a^4*d*cos(d*x
 + c)^6 - 2*a^4*d*cos(d*x + c)^4)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^5(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**4,x)
```

[Out] Integral(tan(c + d*x)**5/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4

Giac [A]
time = 24.28, size = 146, normalized size = 0.75

$$-\frac{\frac{60\log\left(|\sin(dx+c)+1|\right)}{a^4}-\frac{60\log\left(|\sin(dx+c)-1|\right)}{a^4}+\frac{30\left(3\sin(dx+c)^2-12\sin(dx+c)+7\right)}{a^4(\sin(dx+c)-1)^2}-\frac{147\sin(dx+c)^6+822\sin(dx+c)^5+1605\sin(dx+c)^4+340\sin(dx+c)^3-675\sin(dx+c)^2-522\sin(dx+c)-117}{a^4(\sin(dx+c)+1)^6}}{15360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="giac")

[Out] -1/15360*(60*log(abs(sin(d*x + c) + 1))/a^4 - 60*log(abs(sin(d*x + c) - 1))/a^4 + 30*(3*sin(d*x + c)^2 - 12*sin(d*x + c) + 7)/(a^4*(sin(d*x + c) - 1)^2) - (147*sin(d*x + c)^6 + 822*sin(d*x + c)^5 + 1605*sin(d*x + c)^4 + 340*sin(d*x + c)^3 - 675*sin(d*x + c)^2 - 522*sin(d*x + c) - 117)/(a^4*(sin(d*x + c) + 1)^6))/d

Mupad [B]
time = 10.66, size = 476, normalized size = 2.44

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{8}+\frac{73\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{12}-\frac{139\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{320}+\frac{1073\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{120}+\frac{10277\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{960}+\frac{237\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{10}+\frac{10277\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{960}+\frac{1073\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{120}-\frac{139\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{320}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{12}+\frac{73\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{192}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{8}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+8a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+24a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+24a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-36a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-120a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-88a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+88a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+198a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+88a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-88a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-120a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}-36a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+24a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+24a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+8a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^4\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^5/(a + a*sin(c + d*x))^4,x)

[Out] (tan(c/2 + (d*x)/2)/64 + tan(c/2 + (d*x)/2)^2/8 + (73*tan(c/2 + (d*x)/2)^3)/192 + (5*tan(c/2 + (d*x)/2)^4)/12 - (139*tan(c/2 + (d*x)/2)^5)/320 + (1073*tan(c/2 + (d*x)/2)^6)/120 + (10277*tan(c/2 + (d*x)/2)^7)/960 + (237*tan(c/2 + (d*x)/2)^8)/10 + (10277*tan(c/2 + (d*x)/2)^9)/960 + (1073*tan(c/2 + (d*x)/2)^10)/120 - (139*tan(c/2 + (d*x)/2)^11)/320 + (5*tan(c/2 + (d*x)/2)^12)/12 + (73*tan(c/2 + (d*x)/2)^13)/192 + tan(c/2 + (d*x)/2)^14/8 + tan(c/2 + (d*x)/2)^15/64)/(d*(24*a^4*tan(c/2 + (d*x)/2)^2 + 24*a^4*tan(c/2 + (d*x)/2)^3 - 36*a^4*tan(c/2 + (d*x)/2)^4 - 120*a^4*tan(c/2 + (d*x)/2)^5 - 88*a^4*tan(c/2 + (d*x)/2)^6 + 88*a^4*tan(c/2 + (d*x)/2)^7 + 198*a^4*tan(c/2 + (d*x)/2)^8 + 88*a^4*tan(c/2 + (d*x)/2)^9 - 88*a^4*tan(c/2 + (d*x)/2)^10 - 120*a^4*tan(c/2 + (d*x)/2)^11 - 36*a^4*tan(c/2 + (d*x)/2)^12 + 24*a^4*tan(c/2 + (d*x)/2)^13 + 24*a^4*tan(c/2 + (d*x)/2)^14 + 8*a^4*tan(c/2 + (d*x)/2)^15 + a^4*tan(c/2 + (d*x)/2)^16 + a^4 + 8*a^4*tan(c/2 + (d*x)/2))) - atanh(tan(c/2 + (d*x)/2))/(64*a^4*d)

3.83 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=132

$$\frac{a}{20d(a+a\sin(c+dx))^5}-\frac{1}{8d(a+a\sin(c+dx))^4}+\frac{1}{16ad(a+a\sin(c+dx))^3}+\frac{1}{32d\left(a^2+a^2\sin(c+dx)\right)^2}+\frac{1}{64}$$

```
[Out] 1/20*a/d/(a+a*sin(d*x+c))^5-1/8/d/(a+a*sin(d*x+c))^4+1/16/a/d/(a+a*sin(d*x+
c))^3+1/32/d/(a^2+a^2*sin(d*x+c))^2+1/64/d/(a^4-a^4*sin(d*x+c))+1/64/d/(a^4
+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$\frac{1}{64d\left(a^4-a^4\sin(c+dx)\right)}+\frac{1}{64d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{32d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{a}{20d(a\sin(c+dx)+a)^5}-\frac{1}{8d(a\sin(c+dx)+a)^4}+\frac{1}{16ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] a/(20*d*(a + a*Sin[c + d*x])^5) - 1/(8*d*(a + a*Sin[c + d*x])^4) + 1/(16*a*
d*(a + a*Sin[c + d*x])^3) + 1/(32*d*(a^2 + a^2*Sin[c + d*x])^2) + 1/(64*d*(
a^4 - a^4*Sin[c + d*x])) + 1/(64*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{64a^3(a-x)^2} - \frac{a}{4(a+x)^6} + \frac{1}{2(a+x)^5} - \frac{3}{16a(a+x)^4} - \frac{1}{16a^2(a+x)^3} - \frac{1}{64a^3(a+x)^2}\right) dx\right.}{d}$$
$$= \frac{a}{20d(a+a\sin(c+dx))^5} - \frac{1}{8d(a+a\sin(c+dx))^4} + \frac{1}{16ad(a+a\sin(c+dx))^3} +$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.38

$$-\frac{1+4\sin(c+dx)+5\sin^2(c+dx)}{20a^4d(-1+\sin(c+dx))(1+\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/20*(1 + 4*Sin[c + d*x] + 5*Sin[c + d*x]^2)/(a^4*d*(-1 + Sin[c + d*x])*(1
 + Sin[c + d*x])^5)
```

Maple [A]
time = 0.25, size = 81, normalized size = 0.61

method	result	size
derivativedivides	$\frac{-\frac{1}{64(\sin(dx+c)-1)}+\frac{1}{20(1+\sin(dx+c))^5}-\frac{1}{8(1+\sin(dx+c))^4}+\frac{1}{16(1+\sin(dx+c))^3}+\frac{1}{32(1+\sin(dx+c))^2}+\frac{1}{64+64\sin(dx+c)}}{d\,a^4}$	81
default	$\frac{-\frac{1}{64(\sin(dx+c)-1)}+\frac{1}{20(1+\sin(dx+c))^5}-\frac{1}{8(1+\sin(dx+c))^4}+\frac{1}{16(1+\sin(dx+c))^3}+\frac{1}{32(1+\sin(dx+c))^2}+\frac{1}{64+64\sin(dx+c)}}{d\,a^4}$	81
risch	$-\frac{4\left(8i\,e^{7i(dx+c)}+5\,e^{8i(dx+c)}-8i\,e^{5i(dx+c)}-14\,e^{6i(dx+c)}+5\,e^{4i(dx+c)}\right)}{5\left(e^{i(dx+c)}+i\right)^{10}\left(e^{i(dx+c)}-i\right)^2 d\,a^4}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-1/64/(sin(d*x+c)-1)+1/20/(1+sin(d*x+c))^5-1/8/(1+sin(d*x+c))^4+1/
16/(1+sin(d*x+c))^3+1/32/(1+sin(d*x+c))^2+1/64/(1+sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 95, normalized size = 0.72

$$-\frac{5\sin(dx+c)^2+4\sin(dx+c)+1}{20\left(a^4\sin(dx+c)^6+4\,a^4\sin(dx+c)^5+5\,a^4\sin(dx+c)^4-5\,a^4\sin(dx+c)^2-4\,a^4\sin(dx+c)-a^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/20*(5*sin(d*x + c)^2 + 4*sin(d*x + c) + 1)/((a^4*sin(d*x + c)^6 + 4*a^4*
sin(d*x + c)^5 + 5*a^4*sin(d*x + c)^4 - 5*a^4*sin(d*x + c)^2 - 4*a^4*sin(d*
x + c) - a^4)*d)
```

Fricas [A]
time = 0.36, size = 102, normalized size = 0.77

$$-\frac{5 \cos(dx+c)^2 - 4 \sin(dx+c) - 6}{20 \left(a^4 d \cos(dx+c)^6 - 8 a^4 d \cos(dx+c)^4 + 8 a^4 d \cos(dx+c)^2 - 4 \left(a^4 d \cos(dx+c)^4 - 2 a^4 d \cos(dx+c)^2\right) \sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/20*(5*cos(d*x + c)^2 - 4*sin(d*x + c) - 6)/(a^4*d*cos(d*x + c)^6 - 8*a^4
*d*cos(d*x + c)^4 + 8*a^4*d*cos(d*x + c)^2 - 4*(a^4*d*cos(d*x + c)^4 - 2*a^
4*d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 16.61, size = 76, normalized size = 0.58

$$-\frac{\frac{5}{a^4(\sin(dx+c)-1)} - \frac{5 \sin(dx+c)^4+30 \sin(dx+c)^3+80 \sin(dx+c)^2+50 \sin(dx+c)+11}{a^4(\sin(dx+c)+1)^5}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/320*(5/(a^4*(sin(d*x + c) - 1)) - (5*sin(d*x + c)^4 + 30*sin(d*x + c)^3
+ 80*sin(d*x + c)^2 + 50*sin(d*x + c) + 11)/(a^4*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 7.51, size = 172, normalized size = 1.30

$$\frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+\frac{56\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{5}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}+4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{a^4\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^2\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + a*sin(c + d*x))^4,x)
```

```
[Out] (4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + (32*cos(c/2 + (d*x)/2)^5*sin
(c/2 + (d*x)/2)^7)/5 + (56*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)/5 + (
32*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/5 + 4*cos(c/2 + (d*x)/2)^8*si
n(c/2 + (d*x)/2)^4)/(a^4*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^2*(cos
(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^10)
```

3.84 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=105

$$\frac{\tanh^{-1}(\sin(c+dx))}{16a^4d}+\frac{1}{8d(a+a\sin(c+dx))^4}-\frac{1}{12ad(a+a\sin(c+dx))^3}-\frac{1}{16d\left(a^2+a^2\sin(c+dx)\right)^2}-\frac{}{16d\,(}$$

```
[Out] 1/16*arctanh(sin(d*x+c))/a^4/d+1/8/d/(a+a*sin(d*x+c))^4-1/12/a/d/(a+a*sin(d
*x+c))^3-1/16/d/(a^2+a^2*sin(d*x+c))^2-1/16/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2786, 78, 212}

$$-\frac{1}{16d\left(a^4\sin(c+dx)+a^4\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{16a^4d}-\frac{1}{16d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{1}{12ad(a\sin(c+dx)+a)^3}+\frac{1}{8d(a\sin(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(16*a^4*d) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 1/(12*a
*d*(a + a*Sin[c + d*x])^3) - 1/(16*d*(a^2 + a^2*Sin[c + d*x])^2) - 1/(16*d*
(a^4 + a^4*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^5}+\frac{1}{4a(a+x)^4}+\frac{1}{8a^2(a+x)^3}+\frac{1}{16a^3(a+x)^2}+\frac{1}{16a^3(a^2-x^2)}\right)\,dx, x, a\sin\right.}{d}$$
$$= \frac{1}{8d(a+a\sin(c+dx))^4}-\frac{1}{12ad(a+a\sin(c+dx))^3}-\frac{1}{16d\,(a^2+a^2\sin(c+dx))^2}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{16a^4d}+\frac{1}{8d(a+a\sin(c+dx))^4}-\frac{1}{12ad(a+a\sin(c+dx))^3}-\frac{}{16}$$

Mathematica [A]
time = 0.18, size = 62, normalized size = 0.59

$$\frac{3\tanh^{-1}(\sin(c+dx))-\frac{4+19\sin(c+dx)+12\sin^2(c+dx)+3\sin^3(c+dx)}{(1+\sin(c+dx))^4}}{48a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]] - (4 + 19*Sin[c + d*x] + 12*Sin[c + d*x]^2 + 3*Sin
[c + d*x]^3)/(1 + Sin[c + d*x])^4)/(48*a^4*d)
```

Maple [A]
time = 0.32, size = 79, normalized size = 0.75

method	result
derivativedivides	$\frac{\frac{1}{8(1+\sin(dx+c))^4}-\frac{1}{12(1+\sin(dx+c))^3}-\frac{1}{16(1+\sin(dx+c))^2}-\frac{1}{16(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{32}-\frac{\ln(\sin(dx+c)-1)}{32}}{d\,a^4}$
default	$\frac{\frac{1}{8(1+\sin(dx+c))^4}-\frac{1}{12(1+\sin(dx+c))^3}-\frac{1}{16(1+\sin(dx+c))^2}-\frac{1}{16(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{32}-\frac{\ln(\sin(dx+c)-1)}{32}}{d\,a^4}$
risch	$-\frac{i\left(-3\,e^{i(dx+c)}+85\,e^{3i(dx+c)}+24ie^{2i(dx+c)}-80ie^{4i(dx+c)}-85\,e^{5i(dx+c)}+24ie^{6i(dx+c)}+3\,e^{7i(dx+c)}\right)}{24d\,a^4\left(e^{i(dx+c)}+i\right)^8}+\frac{\ln(e^{i(dx+c)}+i)}{16d\,a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(1/8/(1+sin(d*x+c))^4-1/12/(1+sin(d*x+c))^3-1/16/(1+sin(d*x+c))^2-1
/16/(1+sin(d*x+c))+1/32*ln(1+sin(d*x+c))-1/32*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 121, normalized size = 1.15

$$-\frac{\frac{2\left(3\sin(dx+c)^3+12\sin(dx+c)^2+19\sin(dx+c)+4\right)}{a^4\sin(dx+c)^4+4a^4\sin(dx+c)^3+6a^4\sin(dx+c)^2+4a^4\sin(dx+c)+a^4}-\frac{3\log(\sin(dx+c)+1)}{a^4}+\frac{3\log(\sin(dx+c)-1)}{a^4}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(3*sin(d*x + c)^3 + 12*sin(d*x + c)^2 + 19*sin(d*x + c) + 4)/(a^4*
sin(d*x + c)^4 + 4*a^4*sin(d*x + c)^3 + 6*a^4*sin(d*x + c)^2 + 4*a^4*sin(d*
x + c) + a^4) - 3*log(sin(d*x + c) + 1)/a^4 + 3*log(sin(d*x + c) - 1)/a^4)/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(95) = 190$.
time = 0.35, size = 198, normalized size = 1.89

$$\frac{24\cos(dx+c)^2+3\left(\cos(dx+c)^4-8\cos(dx+c)^2-4\left(\cos(dx+c)^2-2\right)\sin(dx+c)+8\right)\log(\sin(dx+c)+1)-3\left(\cos(dx+c)^4-8\cos(dx+c)^2-4\left(\cos(dx+c)^2-2\right)\sin(dx+c)+8\right)\log(-\sin(dx+c)+1)+2\left(3\cos(dx+c)^2-22\right)\sin(dx+c)-32}{96\left(a^4d\cos(dx+c)^4-8a^4d\cos(dx+c)^2+8a^4d-4\left(a^4d\cos(dx+c)^2-2a^4d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/96*(24*cos(d*x + c)^2 + 3*(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x
 + c)^2 - 2)*sin(d*x + c) + 8)*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^4 -
8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)*log(-sin(d*x +
c) + 1) + 2*(3*cos(d*x + c)^2 - 22)*sin(d*x + c) - 32)/(a^4*d*cos(d*x + c)^
4 - 8*a^4*d*cos(d*x + c)^2 + 8*a^4*d - 4*(a^4*d*cos(d*x + c)^2 - 2*a^4*d)*s
in(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d*x)
**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 17.15, size = 91, normalized size = 0.87

$$\frac{\frac{12\log(|\sin(dx+c)+1|)}{a^4}-\frac{12\log(|\sin(dx+c)-1|)}{a^4}-\frac{25\sin(dx+c)^4+124\sin(dx+c)^3+246\sin(dx+c)^2+252\sin(dx+c)+57}{a^4(\sin(dx+c)+1)^4}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/384*(12*log(abs(sin(d*x + c) + 1))/a^4 - 12*log(abs(sin(d*x + c) - 1))/a^
4 - (25*sin(d*x + c)^4 + 124*sin(d*x + c)^3 + 246*sin(d*x + c)^2 + 252*sin(
d*x + c) + 57)/(a^4*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 10.07, size = 240, normalized size = 2.29

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^4\,d}+\frac{-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{8}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{43\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{24}+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{43\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+8\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+28\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+56\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+70\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+56\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+28\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(8*a^4*d) + (tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)/8 + (43*tan(c/2 + (d*x)/2)^3)/24 + (10*tan(c/2 + (d*x)/2)^4)/3 + (43*
tan(c/2 + (d*x)/2)^5)/24 + tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^7/8)/(
d*(28*a^4*tan(c/2 + (d*x)/2)^2 + 56*a^4*tan(c/2 + (d*x)/2)^3 + 70*a^4*tan(c
/2 + (d*x)/2)^4 + 56*a^4*tan(c/2 + (d*x)/2)^5 + 28*a^4*tan(c/2 + (d*x)/2)^6
 + 8*a^4*tan(c/2 + (d*x)/2)^7 + a^4*tan(c/2 + (d*x)/2)^8 + a^4 + 8*a^4*tan(
c/2 + (d*x)/2)))
```

3.85 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=106

$$\frac{4\csc(c+dx)}{a^4d}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{9\log(\sin(c+dx))}{a^4d}-\frac{9\log(1+\sin(c+dx))}{a^4d}+\frac{1}{d\,(a^2+a^2\sin(c+dx))^2}+\frac{}{d\,(a^4+}$$

```
[Out] 4*csc(d*x+c)/a^4/d-1/2*csc(d*x+c)^2/a^4/d+9*ln(sin(d*x+c))/a^4/d-9*ln(1+sin
(d*x+c))/a^4/d+1/d/(a^2+a^2*sin(d*x+c))^2+5/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 78}

$$\frac{5}{d\,(a^4\sin(c+dx)+a^4)}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{4\csc(c+dx)}{a^4d}+\frac{9\log(\sin(c+dx))}{a^4d}-\frac{9\log(\sin(c+dx)+1)}{a^4d}+\frac{1}{d\,(a^2\sin(c+dx)+a^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^4*d) + (9*Log[Sin[c + d*x]])
/(a^4*d) - (9*Log[1 + Sin[c + d*x]])/(a^4*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]
)^2) + 5/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{x^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^3} - \frac{4}{a^3x^2} + \frac{9}{a^4x} - \frac{2}{a^2(a+x)^3} - \frac{5}{a^3(a+x)^2} - \frac{9}{a^4(a+x)}\right) dx, x, a\sin(c+dx)\right.}{d}$$
$$= \frac{4\csc(c+dx)}{a^4d} - \frac{\csc^2(c+dx)}{2a^4d} + \frac{9\log(\sin(c+dx))}{a^4d} - \frac{9\log(1+\sin(c+dx))}{a^4d} + \frac{}{d}$$

Mathematica [A]
time = 0.55, size = 73, normalized size = 0.69

$$\frac{8\csc(c+dx) - \csc^2(c+dx) + 18\log(\sin(c+dx)) - 18\log(1+\sin(c+dx)) + \frac{2}{(1+\sin(c+dx))^2} + \frac{10}{1+\sin(c+dx)}}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (8*Csc[c + d*x] - Csc[c + d*x]^2 + 18*Log[Sin[c + d*x]] - 18*Log[1 + Sin[c
+ d*x]] + 2/(1 + Sin[c + d*x])^2 + 10/(1 + Sin[c + d*x]))/(2*a^4*d)
```

Maple [A]
time = 0.33, size = 71, normalized size = 0.67

method	result
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+9\ln(\sin(dx+c))+\frac{1}{(1+\sin(dx+c))^2}+\frac{5}{1+\sin(dx+c)}-9\ln(1+\sin(dx+c))}{d\,a^4}$
default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+9\ln(\sin(dx+c))+\frac{1}{(1+\sin(dx+c))^2}+\frac{5}{1+\sin(dx+c)}-9\ln(1+\sin(dx+c))}{d\,a^4}$
risch	$\frac{2i\left(27i e^{6i(dx+c)}+9\,e^{7i(dx+c)}-50i e^{4i(dx+c)}-39\,e^{5i(dx+c)}+27i e^{2i(dx+c)}+39\,e^{3i(dx+c)}-9\,e^{i(dx+c)}\right)}{\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}+i\right)^4 d\,a^4} - \frac{18\ln\left(e^{i(dx+c)}+i\right)}{d\,a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-1/2/sin(d*x+c)^2+4/sin(d*x+c)+9*ln(sin(d*x+c))+1/(1+sin(d*x+c))^2
+5/(1+sin(d*x+c))-9*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 103, normalized size = 0.97

$$\frac{\frac{18\sin(dx+c)^3+27\sin(dx+c)^2+6\sin(dx+c)-1}{a^4\sin(dx+c)^4+2a^4\sin(dx+c)^3+a^4\sin(dx+c)^2} - \frac{18\log(\sin(dx+c)+1)}{a^4} + \frac{18\log(\sin(dx+c))}{a^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*((18*sin(d*x + c)^3 + 27*sin(d*x + c)^2 + 6*sin(d*x + c) - 1)/(a^4*sin(
d*x + c)^4 + 2*a^4*sin(d*x + c)^3 + a^4*sin(d*x + c)^2) - 18*log(sin(d*x +
c) + 1)/a^4 + 18*log(sin(d*x + c))/a^4)/d
```

Fricas [A]
time = 0.40, size = 196, normalized size = 1.85

$$-\frac{27 \cos\left(dx+c\right)^{2}-18 \left(\cos\left(dx+c\right)^{4}-3 \cos\left(dx+c\right)^{2}-2 \left(\cos\left(dx+c\right)^{2}-1\right) \sin\left(dx+c\right)+2\right) \log\left(\frac{1}{2} \sin\left(dx+c\right)\right)+18 \left(\cos\left(dx+c\right)^{4}-3 \cos\left(dx+c\right)^{2}-2 \left(\cos\left(dx+c\right)^{2}-1\right) \sin\left(dx+c\right)+2\right) \log\left(\sin\left(dx+c\right)+1\right)+6 \left(3 \cos\left(dx+c\right)^{2}-4\right) \sin\left(dx+c\right)-26}{2 \left(a^{4} d \cos\left(dx+c\right)^{4}-3 a^{4} d \cos\left(dx+c\right)^{2}+2 a^{4} d-2 \left(a^{4} d \cos\left(dx+c\right)^{2}-a^{4} d\right) \sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/2*(27*cos(d*x + c)^2 - 18*(cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 2*(cos(d*
x + c)^2 - 1)*sin(d*x + c) + 2)*log(1/2*sin(d*x + c)) + 18*(cos(d*x + c)^4
- 3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c) + 2)*log(sin(d*x +
 c) + 1) + 6*(3*cos(d*x + c)^2 - 4)*sin(d*x + c) - 26)/(a^4*d*cos(d*x + c)^
4 - 3*a^4*d*cos(d*x + c)^2 + 2*a^4*d - 2*(a^4*d*cos(d*x + c)^2 - a^4*d)*sin
(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{3}\left(c+dx\right)}{\sin^{4}\left(c+dx\right)+4 \sin^{3}\left(c+dx\right)+6 \sin^{2}\left(c+dx\right)+4 \sin\left(c+dx\right)+1}\, dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 7.97, size = 185, normalized size = 1.75

$$-\frac{\frac{144 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a^{4}}-\frac{72 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^{4}}+\frac{108 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-16 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1}{a^{4} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}}+\frac{a^{4} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-16 a^{4} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{8}}-\frac{4 \left(75 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}+272 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+402 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+272 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+75\right)}{a^{4} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^{4}}}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/8*(144*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 72*log(abs(tan(1/2*d*x +
 1/2*c)))/a^4 + (108*tan(1/2*d*x + 1/2*c)^2 - 16*tan(1/2*d*x + 1/2*c) + 1)/
(a^4*tan(1/2*d*x + 1/2*c)^2) + (a^4*tan(1/2*d*x + 1/2*c)^2 - 16*a^4*tan(1/2
```

```
*d*x + 1/2*c))/a^8 - 4*(75*tan(1/2*d*x + 1/2*c)^4 + 272*tan(1/2*d*x + 1/2*c
)^3 + 402*tan(1/2*d*x + 1/2*c)^2 + 272*tan(1/2*d*x + 1/2*c) + 75)/(a^4*(tan
(1/2*d*x + 1/2*c) + 1)^4))/d
```

Mupad [B]
time = 6.61, size = 228, normalized size = 2.15

$$\frac{9\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^4\,d}-\frac{48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{129\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{1}{2}}{d\,\left(4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+24\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{18\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^4\,d}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + a*sin(c + d*x))^4,x)
```

```
[Out] (9*log(tan(c/2 + (d*x)/2)))/(a^4*d) - tan(c/2 + (d*x)/2)^2/(8*a^4*d) - (10*
tan(c/2 + (d*x)/2)^3 - 29*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2) + (12
9*tan(c/2 + (d*x)/2)^4)/2 + 48*tan(c/2 + (d*x)/2)^5 + 1/2)/(d*(4*a^4*tan(c/
2 + (d*x)/2)^2 + 16*a^4*tan(c/2 + (d*x)/2)^3 + 24*a^4*tan(c/2 + (d*x)/2)^4
+ 16*a^4*tan(c/2 + (d*x)/2)^5 + 4*a^4*tan(c/2 + (d*x)/2)^6)) - (18*log(tan(
c/2 + (d*x)/2) + 1))/(a^4*d) + (2*tan(c/2 + (d*x)/2))/(a^4*d)
```

3.86 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=135

$$\frac{8\csc(c+dx)}{a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{8\csc^3(c+dx)}{3a^4d}-\frac{7\csc^4(c+dx)}{4a^4d}+\frac{4\csc^5(c+dx)}{5a^4d}-\frac{\csc^6(c+dx)}{6a^4d}+\frac{8\log(\sin(c+$$

```
[Out] 8*csc(d*x+c)/a^4/d-4*csc(d*x+c)^2/a^4/d+8/3*csc(d*x+c)^3/a^4/d-7/4*csc(d*x+
c)^4/a^4/d+4/5*csc(d*x+c)^5/a^4/d-1/6*csc(d*x+c)^6/a^4/d+8*ln(sin(d*x+c))/a
^4/d-8*ln(1+sin(d*x+c))/a^4/d
```

Rubi [A]
time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$-\frac{\csc^6(c+dx)}{6a^4d}+\frac{4\csc^5(c+dx)}{5a^4d}-\frac{7\csc^4(c+dx)}{4a^4d}+\frac{8\csc^3(c+dx)}{3a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{8\csc(c+dx)}{a^4d}+\frac{8\log(\sin(c+dx))}{a^4d}-\frac{8\log(\sin(c+dx)+1)}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (8*Csc[c + d*x])/(a^4*d) - (4*Csc[c + d*x]^2)/(a^4*d) + (8*Csc[c + d*x]^3)/
(3*a^4*d) - (7*Csc[c + d*x]^4)/(4*a^4*d) + (4*Csc[c + d*x]^5)/(5*a^4*d) - C
sc[c + d*x]^6/(6*a^4*d) + (8*Log[Sin[c + d*x]])/(a^4*d) - (8*Log[1 + Sin[c
+ d*x]])/(a^4*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3}{x^7(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^7}-\frac{4a}{x^6}+\frac{7}{x^5}-\frac{8}{ax^4}+\frac{8}{a^2x^3}-\frac{8}{a^3x^2}+\frac{8}{a^4x}-\frac{8}{a^4(a+x)}\right)dx, x, a\sin(c+dx\right.}{d}$$
$$= \frac{8\csc(c+dx)}{a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{8\csc^3(c+dx)}{3a^4d}-\frac{7\csc^4(c+dx)}{4a^4d}+\frac{4\csc^5(c+d}{5a^4d}$$

Mathematica [A]
time = 0.11, size = 89, normalized size = 0.66

$$\frac{480\csc(c+dx)-240\csc^2(c+dx)+160\csc^3(c+dx)-105\csc^4(c+dx)+48\csc^5(c+dx)-10\csc^6(c+dx)+480\log(\sin(c+dx))-480\log(1+\sin(c+dx))}{60a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (480*Csc[c + d*x] - 240*Csc[c + d*x]^2 + 160*Csc[c + d*x]^3 - 105*Csc[c + d
*x]^4 + 48*Csc[c + d*x]^5 - 10*Csc[c + d*x]^6 + 480*Log[Sin[c + d*x]] - 480
*Log[1 + Sin[c + d*x]])/(60*a^4*d)
```

Maple [A]
time = 0.38, size = 89, normalized size = 0.66

method	result
derivativedivides	$\frac{-\frac{1}{6\sin(dx+c)^6}+\frac{4}{5\sin(dx+c)^5}-\frac{7}{4\sin(dx+c)^4}+\frac{8}{3\sin(dx+c)^3}-\frac{4}{\sin(dx+c)^2}+\frac{8}{\sin(dx+c)}+8\ln(\sin(dx+c))-8\ln(1+\sin(dx+c))}{d\,a^4}$
default	$\frac{-\frac{1}{6\sin(dx+c)^6}+\frac{4}{5\sin(dx+c)^5}-\frac{7}{4\sin(dx+c)^4}+\frac{8}{3\sin(dx+c)^3}-\frac{4}{\sin(dx+c)^2}+\frac{8}{\sin(dx+c)}+8\ln(\sin(dx+c))-8\ln(1+\sin(dx+c))}{d\,a^4}$
risch	$\frac{4i\left(-60i\mathrm{e}^{10i(dx+c)}+60\,\mathrm{e}^{11i(dx+c)}+345i\mathrm{e}^{8i(dx+c)}-380\,\mathrm{e}^{9i(dx+c)}-610i\mathrm{e}^{6i(dx+c)}+936\,\mathrm{e}^{7i(dx+c)}+345i\mathrm{e}^{4i(dx+c)}-936\,\mathrm{e}^{5i(dx\right.}{15d\,a^4\left(\mathrm{e}^{2i(dx+c)}-1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-1/6/sin(d*x+c)^6+4/5/sin(d*x+c)^5-7/4/sin(d*x+c)^4+8/3/sin(d*x+c)
^3-4/sin(d*x+c)^2+8/sin(d*x+c)+8*ln(sin(d*x+c))-8*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 95, normalized size = 0.70

$$-\frac{\frac{480\log(\sin(dx+c)+1)}{a^4}-\frac{480\log(\sin(dx+c))}{a^4}-\frac{480\sin(dx+c)^5-240\sin(dx+c)^4+160\sin(dx+c)^3-105\sin(dx+c)^2+48\sin(dx+c)-10}{a^4\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/60*(480*log(sin(d*x + c) + 1)/a^4 - 480*log(sin(d*x + c))/a^4 - (480*sin
(d*x + c)^5 - 240*sin(d*x + c)^4 + 160*sin(d*x + c)^3 - 105*sin(d*x + c)^2
+ 48*sin(d*x + c) - 10)/(a^4*sin(d*x + c)^6))/d
```

Fricas [A]
time = 0.36, size = 186, normalized size = 1.38

$$\frac{240 \cos\left(dx+c\right)^{4} - 585 \cos\left(dx+c\right)^{2} + 480 \left(\cos\left(dx+c\right)^{6} - 3 \cos\left(dx+c\right)^{4} + 3 \cos\left(dx+c\right)^{2} - 1\right) \log\left(\frac{1}{2} \sin\left(dx+c\right)\right) - 480 \left(\cos\left(dx+c\right)^{6} - 3 \cos\left(dx+c\right)^{4} + 3 \cos\left(dx+c\right)^{2} - 1\right) \log\left(\sin\left(dx+c\right) + 1\right) - 16 \left(30 \cos\left(dx+c\right)^{4} - 70 \cos\left(dx+c\right)^{2} + 43\right) \sin\left(dx+c\right) + 355}{60 \left(a^{4} d \cos\left(dx+c\right)^{6} - 3 a^{4} d \cos\left(dx+c\right)^{4} + 3 a^{4} d \cos\left(dx+c\right)^{2} - a^{4} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/60*(240*cos(d*x + c)^4 - 585*cos(d*x + c)^2 + 480*(cos(d*x + c)^6 - 3*cos
(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 480*(cos(d*x +
c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(sin(d*x + c) + 1) - 16*
(30*cos(d*x + c)^4 - 70*cos(d*x + c)^2 + 43)*sin(d*x + c) + 355)/(a^4*d*cos
(d*x + c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{7}\left(c+dx\right)}{\sin^{4}\left(c+dx\right)+4\sin^{3}\left(c+dx\right)+6\sin^{2}\left(c+dx\right)+4\sin\left(c+dx\right)+1}\, dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**7/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 9.92, size = 232, normalized size = 1.72

$$-\frac{\frac{30720 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a^{4}} - \frac{15360 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^{4}} + \frac{37632 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}-10080 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+2835 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}-880 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+240 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-48 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+5}{a^{4} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}} + \frac{5 a^{20} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}-48 a^{20} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+240 a^{20} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}-880 a^{20} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+2835 a^{20} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-10080 a^{20} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{24}}}{1920 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/1920*(30720*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 15360*log(abs(tan(1
/2*d*x + 1/2*c)))/a^4 + (37632*tan(1/2*d*x + 1/2*c)^6 - 10080*tan(1/2*d*x +
 1/2*c)^5 + 2835*tan(1/2*d*x + 1/2*c)^4 - 880*tan(1/2*d*x + 1/2*c)^3 + 240*
tan(1/2*d*x + 1/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 5)/(a^4*tan(1/2*d*x + 1/
2*c)^6) + (5*a^20*tan(1/2*d*x + 1/2*c)^6 - 48*a^20*tan(1/2*d*x + 1/2*c)^5 +
```

```
 240*a^20*tan(1/2*d*x + 1/2*c)^4 - 880*a^20*tan(1/2*d*x + 1/2*c)^3 + 2835*a
^20*tan(1/2*d*x + 1/2*c)^2 - 10080*a^20*tan(1/2*d*x + 1/2*c))/a^24)/d
```

Mupad [B]
time = 6.81, size = 235, normalized size = 1.74

$$\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^4\,d}-\frac{189\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{128\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{8\,a^4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{40\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{384\,a^4\,d}+\frac{8\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4\,d}-\frac{16\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^4\,d}+\frac{21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^4\,d}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(336\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-\frac{189\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}+\frac{88\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{5}-\frac{1}{6}\right)}{64\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7/(a + a*sin(c + d*x))^4,x)
```

```
[Out] (11*tan(c/2 + (d*x)/2)^3)/(24*a^4*d) - (189*tan(c/2 + (d*x)/2)^2)/(128*a^4*
d) - tan(c/2 + (d*x)/2)^4/(8*a^4*d) + tan(c/2 + (d*x)/2)^5/(40*a^4*d) - tan
(c/2 + (d*x)/2)^6/(384*a^4*d) + (8*log(tan(c/2 + (d*x)/2)))/(a^4*d) - (16*l
og(tan(c/2 + (d*x)/2) + 1))/(a^4*d) + (21*tan(c/2 + (d*x)/2))/(4*a^4*d) + (
cot(c/2 + (d*x)/2)^6*((8*tan(c/2 + (d*x)/2))/5 - 8*tan(c/2 + (d*x)/2)^2 + (
88*tan(c/2 + (d*x)/2)^3)/3 - (189*tan(c/2 + (d*x)/2)^4)/2 + 336*tan(c/2 + (
d*x)/2)^5 - 1/6))/(64*a^4*d)
```

3.87 $\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=127

$$-\frac{4\sec^5(c+dx)}{5a^4d}+\frac{12\sec^7(c+dx)}{7a^4d}-\frac{8\sec^9(c+dx)}{9a^4d}+\frac{\tan^3(c+dx)}{3a^4d}+\frac{9\tan^5(c+dx)}{5a^4d}+\frac{16\tan^7(c+dx)}{7a^4d}+\frac{8\tan^9}{9}$$

```
[Out] -4/5*sec(d*x+c)^5/a^4/d+12/7*sec(d*x+c)^7/a^4/d-8/9*sec(d*x+c)^9/a^4/d+1/3*
tan(d*x+c)^3/a^4/d+9/5*tan(d*x+c)^5/a^4/d+16/7*tan(d*x+c)^7/a^4/d+8/9*tan(d
*x+c)^9/a^4/d
```

Rubi [A]
time = 0.23, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2790, 2687, 276, 2686, 14}

$$\frac{8\tan^9(c+dx)}{9a^4d}+\frac{16\tan^7(c+dx)}{7a^4d}+\frac{9\tan^5(c+dx)}{5a^4d}+\frac{\tan^3(c+dx)}{3a^4d}-\frac{8\sec^9(c+dx)}{9a^4d}+\frac{12\sec^7(c+dx)}{7a^4d}-\frac{4\sec^5(c+dx)}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*Sec[c + d*x]^5)/(5*a^4*d) + (12*Sec[c + d*x]^7)/(7*a^4*d) - (8*Sec[c +
d*x]^9)/(9*a^4*d) + Tan[c + d*x]^3/(3*a^4*d) + (9*Tan[c + d*x]^5)/(5*a^4*d)
 + (16*Tan[c + d*x]^7)/(7*a^4*d) + (8*Tan[c + d*x]^9)/(9*a^4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{\int \left(a^4\sec^8(c+dx)\tan^2(c+dx) - 4a^4\sec^7(c+dx)\tan^3(c+dx) + 6a^4\sec^6(c+\cdots\right.}{\cdots}\\
&= \frac{\int \sec^8(c+dx)\tan^2(c+dx)\,dx}{a^4} + \frac{\int \sec^4(c+dx)\tan^6(c+dx)\,dx}{a^4} - \frac{4\int \sec^7(c+\cdots}{\cdots}\\
&= \frac{\mathrm{Subst}\left(\int x^6(1+x^2)\,dx, x, \tan(c+dx)\right)}{a^4d} + \frac{\mathrm{Subst}\left(\int x^2(1+x^2)^3\,dx, x, \tan(c+d\cdots\right.}{a^4d}\\
&= \frac{\mathrm{Subst}\left(\int (x^6+x^8)\,dx, x, \tan(c+dx)\right)}{a^4d} + \frac{\mathrm{Subst}\left(\int (x^2+3x^4+3x^6+x^8)\,dx, x, \cdots\right.}{a^4d}\\
&= -\frac{4\sec^5(c+dx)}{5a^4d} + \frac{12\sec^7(c+dx)}{7a^4d} - \frac{8\sec^9(c+dx)}{9a^4d} + \frac{\tan^3(c+dx)}{3a^4d} + \frac{9\tan^5(c\cdots}{5a^4\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.27, size = 124, normalized size = 0.98

$$\frac{\sec(c+dx)(16128+1554\cos(c+dx)-16896\cos(2(c+dx))-999\cos(3(c+dx))+2816\cos(4(c+dx))+37\cos(5(c+dx))+34944\sin(c+dx)+1776\sin(2(c+dx))-9504\sin(3(c+dx))-296\sin(4(c+dx))+352\sin(5(c+dx)))}{80640a^4d(1+\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]*(16128 + 1554*Cos[c + d*x] - 16896*Cos[2*(c + d*x)] - 999*Cos
[3*(c + d*x)] + 2816*Cos[4*(c + d*x)] + 37*Cos[5*(c + d*x)] + 34944*Sin[c +
 d*x] + 1776*Sin[2*(c + d*x)] - 9504*Sin[3*(c + d*x)] - 296*Sin[4*(c + d*x)
] + 352*Sin[5*(c + d*x)]))/(80640*a^4*d*(1 + Sin[c + d*x])^4)
```

Maple [A]
time = 0.27, size = 158, normalized size = 1.24

method	result
risch	$-\frac{4i\left(504i\,e^{5i(dx+c)}+315\,e^{6i(dx+c)}-528i\,e^{3i(dx+c)}-777\,e^{4i(dx+c)}+88i\,e^{i(dx+c)}+297\,e^{2i(dx+c)}-11\right)}{315\left(e^{i(dx+c)}-i\right)\left(e^{i(dx+c)}+i\right)^{9}d\,a^{4}}$
derivativedivides	$\frac{-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}-\frac{116}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{62}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{83}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d\,a^{4}}$
default	$\frac{-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}-\frac{116}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{62}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{83}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d\,a^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^4*(-1/128/(tan(1/2*d*x+1/2*c)-1)-2/9/(tan(1/2*d*x+1/2*c)+1)^9+1/(tan(
1/2*d*x+1/2*c)+1)^8-29/14/(tan(1/2*d*x+1/2*c)+1)^7+31/12/(tan(1/2*d*x+1/2*c
)+1)^6-83/40/(tan(1/2*d*x+1/2*c)+1)^5+17/16/(tan(1/2*d*x+1/2*c)+1)^4-29/96/
(tan(1/2*d*x+1/2*c)+1)^3+1/64/(tan(1/2*d*x+1/2*c)+1)^2+1/128/(tan(1/2*d*x+1
/2*c)+1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(113) = 226$.
time $= 0.29$, size $= 356$, normalized size $= 2.80$

$$\frac{8\left(\frac{16\sin(dx+c)}{\cos(dx+c)+1}+\frac{54\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{201\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{294\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{378\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{210\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{105\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+2\right)}{315\left(a^{4}+\frac{8a^{4}\sin(dx+c)}{\cos(dx+c)+1}+\frac{27a^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{48a^{4}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{42a^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{42a^{4}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{48a^{4}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{27a^{4}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{8a^{4}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{a^{4}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 8/315*(16*sin(d*x + c)/(cos(d*x + c) + 1) + 54*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 201*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 294*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 210*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 105*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 2)/((
a^4 + 8*a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 27*a^4*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 48*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 42*a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 42*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 -
 48*a^4*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 27*a^4*sin(d*x + c)^8/(cos(d*
x + c) + 1)^8 - 8*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^4*sin(d*x + c
)^10/(cos(d*x + c) + 1)^10)*d)
```

Fricas [**A**]
time $= 0.40$, size $= 129$, normalized size $= 1.02$

$$\frac{88\cos(dx+c)^{4}-220\cos(dx+c)^{2}+\left(22\cos(dx+c)^{4}-165\cos(dx+c)^{2}+175\right)\sin(dx+c)+140}{315\left(a^{4}d\cos(dx+c)^{5}-8a^{4}d\cos(dx+c)^{3}+8a^{4}d\cos(dx+c)-4\left(a^{4}d\cos(dx+c)^{3}-2a^{4}d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/315*(88*cos(d*x + c)^4 - 220*cos(d*x + c)^2 + (22*cos(d*x + c)^4 - 165*co
s(d*x + c)^2 + 175)*sin(d*x + c) + 140)/(a^4*d*cos(d*x + c)^5 - 8*a^4*d*cos
(d*x + c)^3 + 8*a^4*d*cos(d*x + c) - 4*(a^4*d*cos(d*x + c)^3 - 2*a^4*d*cos(
d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^2(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(tan(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 14.26, size = 146, normalized size = 1.15

$$-\frac{\frac{315}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{315\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+3150\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1050\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+630\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-8064\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6006\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5274\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-846\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-59}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^9}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/5040*(315/(a^4*(tan(1/2*d*x + 1/2*c) - 1)) - (315*tan(1/2*d*x + 1/2*c)^8
 + 3150*tan(1/2*d*x + 1/2*c)^7 + 1050*tan(1/2*d*x + 1/2*c)^6 + 630*tan(1/2*
d*x + 1/2*c)^5 - 8064*tan(1/2*d*x + 1/2*c)^4 - 6006*tan(1/2*d*x + 1/2*c)^3
- 5274*tan(1/2*d*x + 1/2*c)^2 - 846*tan(1/2*d*x + 1/2*c) - 59)/(a^4*(tan(1/
2*d*x + 1/2*c) + 1)^9))/d
```

Mupad [B]
time = 7.58, size = 231, normalized size = 1.82

$$\frac{\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{315}+\frac{128\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{315}+\frac{48\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{35}+\frac{536\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{105}+\frac{112\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}+\frac{48\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}}{a^4\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,{\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + a*sin(c + d*x))^4,x)
```

```
[Out] ((16*cos(c/2 + (d*x)/2)^10)/315 + (128*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)
/2))/315 + (8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7)/3 + (16*cos(c/2 +
(d*x)/2)^4*sin(c/2 + (d*x)/2)^6)/3 + (48*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*
x)/2)^5)/5 + (112*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4)/15 + (536*cos(
c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3)/105 + (48*cos(c/2 + (d*x)/2)^8*sin(c
/2 + (d*x)/2)^2)/35)/(a^4*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(
c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^9)
```

3.88 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=108

$$\frac{4\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{\cot(c+dx)}{a^4d}-\frac{2\cot(c+dx)}{5a^4d(1+\csc(c+dx))^3}+\frac{31\cot(c+dx)}{15a^4d(1+\csc(c+dx))^2}-\frac{104\cot(c+dx)}{15a^4d(1+\csc(c+dx))}$$

```
[Out] 4*arctanh(cos(d*x+c))/a^4/d-94/15*cot(d*x+c)/a^4/d+2/5*cot(d*x+c)/a^4/d/(1+
sin(d*x+c))^3+13/15*cot(d*x+c)/a^4/d/(1+sin(d*x+c))^2+4*cot(d*x+c)/a^4/d/(1
+sin(d*x+c))
```

Rubi [A]
time = 0.25, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2788, 3855, 3852, 8, 3862, 4007, 4004, 3879}

$$-\frac{\cot(c+dx)}{a^4d}+\frac{4\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{104\cot(c+dx)}{15a^4d(\csc(c+dx)+1)}+\frac{31\cot(c+dx)}{15a^4d(\csc(c+dx)+1)^2}-\frac{2\cot(c+dx)}{5a^4d(\csc(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*ArcTanh[Cos[c + d*x]])/(a^4*d) - Cot[c + d*x]/(a^4*d) - (2*Cot[c + d*x])
/(5*a^4*d*(1 + Csc[c + d*x])^3) + (31*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c +
d*x])^2) - (104*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(\frac{9}{a^2} - \frac{4\csc(c+dx)}{a^2} + \frac{\csc^2(c+dx)}{a^2} - \frac{2}{a^2(1+\csc(c+dx))^3} + \frac{9}{a^2(1+\csc(c+dx))^2} - \frac{16}{a^2(1+\csc(c+dx)}\right.}{a^2}$$

$$= \frac{9x}{a^4} + \frac{\int \csc^2(c+dx)\,dx}{a^4} - \frac{2\int \frac{1}{(1+\csc(c+dx))^3}\,dx}{a^4} - \frac{4\int \csc(c+dx)\,dx}{a^4} + \frac{9\int \frac{}{(1+\mathrm{cs}}}{}$$

$$= \frac{9x}{a^4} + \frac{4\tanh^{-1}(\cos(c+dx))}{a^4 d} - \frac{2\cot(c+dx)}{5a^4 d(1+\csc(c+dx))^3} + \frac{3\cot(c+dx)}{a^4 d(1+\csc(c+dx))}$$

$$= \frac{2x}{a^4} + \frac{4\tanh^{-1}(\cos(c+dx))}{a^4 d} - \frac{\cot(c+dx)}{a^4 d} - \frac{2\cot(c+dx)}{5a^4 d(1+\csc(c+dx))^3} + \frac{3}{15a^4 d}$$

$$= \frac{4\tanh^{-1}(\cos(c+dx))}{a^4 d} - \frac{\cot(c+dx)}{a^4 d} - \frac{2\cot(c+dx)}{5a^4 d(1+\csc(c+dx))^3} + \frac{31\cot(}{15a^4 d(1+\mathrm{c}}$$

$$= \frac{4\tanh^{-1}(\cos(c+dx))}{a^4 d} - \frac{\cot(c+dx)}{a^4 d} - \frac{2\cot(c+dx)}{5a^4 d(1+\csc(c+dx))^3} + \frac{31\cot(}{15a^4 d(1+\mathrm{c}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 315 vs. 2(108) = 216.
time = 0.28, size = 315, normalized size = 2.92

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(24\sin\left(\frac{1}{2}(c+dx)\right)-12\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+76\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-38\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+316\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4-15\cot\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5+120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5-120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5+15\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\tan\left(\frac{1}{2}(c+dx)\right)\right)}{30d(a+a\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(24*Sin[(c + d*x)/2] - 12*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]) + 76*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^2 - 38*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 316*Sin[(c
+ d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 15*Cot[(c + d*x)/2]*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^5 + 120*Log[Cos[(c + d*x)/2]]*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^5 - 120*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^5 + 15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*Tan[(c
+ d*x)/2]))/(30*d*(a + a*Sin[c + d*x])^4)
```

Maple [**A**]
time = 0.31, size = 119, normalized size = 1.10

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{28}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{36}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{d}{2}\right.}}{2d\,a^4}$

default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{28}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{36}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{dx}{2}+\right.}}{2d\,a^{4}}$
risch	$-\frac{4\left(-320\,e^{4i(dx+c)}+150ie^{5i(dx+c)}+367\,e^{2i(dx+c)}-385ie^{3i(dx+c)}-47+205ie^{i(dx+c)}+30\,e^{6i(dx+c)}\right)}{15\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)^{5}d\,a^{4}}-\frac{4\ln\left(e^{i(dx+c)}-1\right)}{a^{4}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^4*(tan(1/2*d*x+1/2*c)-32/5/(tan(1/2*d*x+1/2*c)+1)^5+16/(tan(1/2*d*x
+1/2*c)+1)^4-88/3/(tan(1/2*d*x+1/2*c)+1)^3+28/(tan(1/2*d*x+1/2*c)+1)^2-36/(
tan(1/2*d*x+1/2*c)+1)-1/tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(102) = 204$.
time = 0.30, size = 288, normalized size = 2.67

$$-\frac{\frac{\frac{491\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{1690\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2570\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1815\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{555\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+15}{\frac{a^{4}\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\,a^{4}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{10\,a^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{5\,a^{4}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{a^{4}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}+\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{4}}-\frac{15\,\sin(dx+c)}{a^{4}(\cos(dx+c)+1)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/30*((491*sin(d*x + c)/(cos(d*x + c) + 1) + 1690*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 2570*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1815*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 555*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15)/(a^4
*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 10*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^4*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 5*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) + 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^4
 - 15*sin(d*x + c)/(a^4*(cos(d*x + c) + 1)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(102) = 204$.
time = 0.37, size = 369, normalized size = 3.42

$$\frac{94\,\cos(dx+c)^{4}+222\,\cos(dx+c)^{3}-115\,\cos(dx+c)^{2}+30\left(\cos(dx+c)^{4}-2\,\cos(dx+c)^{3}-5\,\cos(dx+c)^{2}-\left(\cos(dx+c)^{3}+3\,\cos(dx+c)^{2}-2\,\cos(dx+c)-4\right)\sin(dx+c)+2\,\cos(dx+c)+4\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)-30\left(\cos(dx+c)^{4}-2\,\cos(dx+c)^{3}-5\,\cos(dx+c)^{2}-\left(\cos(dx+c)^{3}+3\,\cos(dx+c)^{2}-2\,\cos(dx+c)-4\right)\sin(dx+c)+2\,\cos(dx+c)+4\right)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+\left(94\,\cos(dx+c)^{3}-128\,\cos(dx+c)^{2}-243\,\cos(dx+c)-6\right)\sin(dx+c)-237\,\cos(dx+c)+6}{15\left(a^{4}d\cos(dx+c)^{4}-2\,a^{4}d\cos(dx+c)^{3}-5\,a^{4}d\cos(dx+c)^{2}+2\,a^{4}d\cos(dx+c)+4\,a^{4}d-\left(a^{4}d\cos(dx+c)^{3}+3\,a^{4}d\cos(dx+c)^{2}-2\,a^{4}d\cos(dx+c)-4\,a^{4}d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/15*(94*cos(d*x + c)^4 + 222*cos(d*x + c)^3 - 115*cos(d*x + c)^2 + 30*(cos
(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - (cos(d*x + c)^3 + 3*cos
(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*log(1/
2*cos(d*x + c) + 1/2) - 30*(cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x +
```

```
c)^2 - (cos(d*x + c)^3 + 3*cos(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d*x +
c) + 2*cos(d*x + c) + 4)*log(-1/2*cos(d*x + c) + 1/2) + (94*cos(d*x + c)^3
- 128*cos(d*x + c)^2 - 243*cos(d*x + c) - 6)*sin(d*x + c) - 237*cos(d*x + c
) + 6)/(a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^3 - 5*a^4*d*cos(d*x + c
)^2 + 2*a^4*d*cos(d*x + c) + 4*a^4*d - (a^4*d*cos(d*x + c)^3 + 3*a^4*d*cos(
d*x + c)^2 - 2*a^4*d*cos(d*x + c) - 4*a^4*d)*sin(d*x + c))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^{2}(c+dx)}{\sin^{4}(c+dx)+4\sin^{3}(c+dx)+6\sin^{2}(c+dx)+4\sin(c+dx)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]

time = 7.34, size = 135, normalized size = 1.25

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{4}}-\frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{4}}-\frac{15\left(8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\frac{4\left(135\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+605\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+385\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+104\right)}{a^{4}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{5}}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/30*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 15*tan(1/2*d*x + 1/2*c)/a^4
 - 15*(8*tan(1/2*d*x + 1/2*c) - 1)/(a^4*tan(1/2*d*x + 1/2*c)) + 4*(135*tan(
1/2*d*x + 1/2*c)^4 + 435*tan(1/2*d*x + 1/2*c)^3 + 605*tan(1/2*d*x + 1/2*c)^
2 + 385*tan(1/2*d*x + 1/2*c) + 104)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]

time = 12.05, size = 203, normalized size = 1.88

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^{4}d}-\frac{37\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+121\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+\frac{514\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{3}+\frac{338\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{3}+\frac{491\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{15}+1}{d\left(2a^{4}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+10a^{4}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+20a^{4}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+20a^{4}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+10a^{4}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a^{4}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}-\frac{4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + a*sin(c + d*x))^4,x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^4*d) - ((491*tan(c/2 + (d*x)/2))/15 + (338*tan(c/2
+ (d*x)/2)^2)/3 + (514*tan(c/2 + (d*x)/2)^3)/3 + 121*tan(c/2 + (d*x)/2)^4 +
 37*tan(c/2 + (d*x)/2)^5 + 1)/(d*(10*a^4*tan(c/2 + (d*x)/2)^2 + 20*a^4*tan(
c/2 + (d*x)/2)^3 + 20*a^4*tan(c/2 + (d*x)/2)^4 + 10*a^4*tan(c/2 + (d*x)/2)^
5 + 2*a^4*tan(c/2 + (d*x)/2)^6 + 2*a^4*tan(c/2 + (d*x)/2))) - (4*log(tan(c/
2 + (d*x)/2)))/(a^4*d)
```

3.89 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{9\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}+\frac{4\cot(c+dx)}{3a^4d(1+\csc(c+dx))^2}-\frac{}{3}$$

```
[Out] 14*arctanh(cos(d*x+c))/a^4/d-33*cot(d*x+c)/a^4/d-11*cot(d*x+c)^3/a^4/d+14*c
ot(d*x+c)*csc(d*x+c)/a^4/d+4/3*cot(d*x+c)*csc(d*x+c)^2/a^4/d/(1+sin(d*x+c))
^2+28/3*cot(d*x+c)*csc(d*x+c)^2/a^4/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2788, 3855, 3852, 8, 3853, 3862, 4004, 3879}

$$-\frac{\cot^3(c+dx)}{3a^4d}-\frac{9\cot(c+dx)}{a^4d}+\frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}-\frac{44\cot(c+dx)}{3a^4d(\csc(c+dx)+1)}+\frac{4\cot(c+dx)}{3a^4d(\csc(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (14*ArcTanh[Cos[c + d*x]])/(a^4*d) - (9*Cot[c + d*x])/(a^4*d) - Cot[c + d*x
]^3/(3*a^4*d) + (2*Cot[c + d*x]*Csc[c + d*x])/(a^4*d) + (4*Cot[c + d*x])/(3
*a^4*d*(1 + Csc[c + d*x])^2) - (44*Cot[c + d*x])/(3*a^4*d*(1 + Csc[c + d*x]
))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(16-12\csc(c+dx)+8\csc^2(c+dx)-4\csc^3(c+dx)+\csc^4(c+dx)+\frac{}{(1+\csc}\right.}{a^4}$$

$$= \frac{16x}{a^4}+\frac{\int \csc^4(c+dx)\,dx}{a^4}-\frac{4\int \csc^3(c+dx)\,dx}{a^4}+\frac{4\int \frac{1}{(1+\csc(c+dx))^2}\,dx}{a^4}+\frac{8\int \csc^2}{}$$

$$= \frac{16x}{a^4}+\frac{12\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}+\frac{4\cot(c+dx)}{3a^4d(1+\csc(c+}$$

$$= \frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{9\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{2\cot(c+dx)\csc(c+}{a^4d}$$

$$= \frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{9\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{2\cot(c+dx)\csc(c+}{a^4d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(120) = 240.
time = 4.61, size = 359, normalized size = 2.99

$$\frac{\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)^5\left(-\cos\left(\tfrac{1}{2}(c+dx)\right)\left(1+\cot\left(\tfrac{1}{2}(c+dx)\right)\right)^3+64\sin\left(\tfrac{1}{2}(c+dx)\right)+12\left(1+\cot\left(\tfrac{1}{2}(c+dx)\right)\right)^3\sin\left(\tfrac{1}{2}(c+dx)\right)-32\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+640\sin\left(\tfrac{1}{2}(c+dx)\right)\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)^2-104\cot\left(\tfrac{1}{2}(c+dx)\right)\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)^3+336\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)^3-336\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)^3+104\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)^3\tan\left(\tfrac{1}{2}(c+dx)\right)-12\cos\left(\tfrac{1}{2}(c+dx)\right)\left(1+\tan\left(\tfrac{1}{2}(c+dx)\right)\right)^3+\sin\left(\tfrac{1}{2}(c+dx)\right)\left(1+\tan\left(\tfrac{1}{2}(c+dx)\right)\right)^3\right)}{24a^4d(1+\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(-(Cos[(c + d*x)/2]*(1 + Cot[(c +
d*x)/2])^3) + 64*Sin[(c + d*x)/2] + 12*(1 + Cot[(c + d*x)/2])^3*Sin[(c + d*
x)/2] - 32*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 640*Sin[(c + d*x)/2]*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 104*Cot[(c + d*x)/2]*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^3 + 336*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^3 - 336*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3 + 104*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*Tan[(c + d*x)/2] -
 12*Cos[(c + d*x)/2]*(1 + Tan[(c + d*x)/2])^3 + Sin[(c + d*x)/2]*(1 + Tan[(
c + d*x)/2])^3))/(24*a^4*d*(1 + Sin[c + d*x])^4)
```

Maple [A]
time = 0.29, size = 143, normalized size = 1.19

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{128}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{64}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{256}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{3\tan\left(\frac{dx}{2}+\right.}}{8d\,a^4}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{128}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{64}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{256}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{3\tan\left(\frac{dx}{2}+\right.}}{8d\,a^4}$

risch	$-\frac{4\left(-119\,\mathrm{e}^{6i(dx+c)}+63i\mathrm{e}^{7i(dx+c)}+204\,\mathrm{e}^{4i(dx+c)}-192i\mathrm{e}^{5i(dx+c)}+21\,\mathrm{e}^{8i(dx+c)}-135\,\mathrm{e}^{2i(dx+c)}+211i\mathrm{e}^{3i(dx+c)}+33-78i\mathrm{e}}{3\left(\mathrm{e}^{2i(dx+c)}-1\right)^3\left(\mathrm{e}^{i(dx+c)}+i\right)^3 d\,a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(1/3*tan(1/2*d*x+1/2*c)^3-4*tan(1/2*d*x+1/2*c)^2+35*tan(1/2*d*x+1
/2*c)-128/3/(tan(1/2*d*x+1/2*c)+1)^3+64/(tan(1/2*d*x+1/2*c)+1)^2-256/(tan(1
/2*d*x+1/2*c)+1)-1/3/tan(1/2*d*x+1/2*c)^3+4/tan(1/2*d*x+1/2*c)^2-35/tan(1/2
*d*x+1/2*c)-112*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(130) = 260$.
time = 0.29, size = 285, normalized size = 2.38

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{72\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{984\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1647\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{873\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-1}{\frac{a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3a^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^4}-\frac{336\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 72*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 984*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1647*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 873*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^4*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 3*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6) + (105*sin(d*x + c)/(cos(d*x + c) + 1) - 12*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^4 - 336*log(sin(d*
x + c)/(cos(d*x + c) + 1))/a^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(130) = 260$.
time = 0.37, size = 445, normalized size = 3.71

$$-\frac{66\cos(dx+c)^5-24\cos(dx+c)^4-147\cos(dx+c)^3+29\cos(dx+c)^2-21\left(\cos(dx+c)^5+2\cos(dx+c)^4-2\cos(dx+c)^3-4\cos(dx+c)^2+\left(\cos(dx+c)^4-\cos(dx+c)^3-3\cos(dx+c)^2+\cos(dx+c)+2\right)\sin(dx+c)+\cos(dx+c)+2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+21\left(\cos(dx+c)^5+2\cos(dx+c)^4-2\cos(dx+c)^3-4\cos(dx+c)^2+\left(\cos(dx+c)^4-\cos(dx+c)^3-3\cos(dx+c)^2+\cos(dx+c)+2\right)\sin(dx+c)+\cos(dx+c)+2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left(66\cos(dx+c)^4+90\cos(dx+c)^3-57\cos(dx+c)^2-86\cos(dx+c)-4\right)\sin(dx+c)+82\cos(dx+c)-4}{3\left(a^4d\cos(dx+c)^5+2a^4d\cos(dx+c)^4-2a^4d\cos(dx+c)^3-4a^4d\cos(dx+c)^2+a^4d\cos(dx+c)+2a^4d+\left(a^4d\cos(dx+c)^4-a^4d\cos(dx+c)^3-3a^4d\cos(dx+c)^2+a^4d\cos(dx+c)+2a^4d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(66*cos(d*x + c)^5 - 24*cos(d*x + c)^4 - 147*cos(d*x + c)^3 + 29*cos(d
*x + c)^2 - 21*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 4*co
s(d*x + c)^2 + (cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 + cos(d*
x + c) + 2)*sin(d*x + c) + cos(d*x + c) + 2)*log(1/2*cos(d*x + c) + 1/2) +
21*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 4*cos(d*x + c)^2
 + (cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 + cos(d*x + c) + 2)*
```

```
sin(d*x + c) + cos(d*x + c) + 2)*log(-1/2*cos(d*x + c) + 1/2) - (66*cos(d*x
 + c)^4 + 90*cos(d*x + c)^3 - 57*cos(d*x + c)^2 - 86*cos(d*x + c) - 4)*sin(
d*x + c) + 82*cos(d*x + c) - 4)/(a^4*d*cos(d*x + c)^5 + 2*a^4*d*cos(d*x + c
)^4 - 2*a^4*d*cos(d*x + c)^3 - 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c)
+ 2*a^4*d + (a^4*d*cos(d*x + c)^4 - a^4*d*cos(d*x + c)^3 - 3*a^4*d*cos(d*x
+ c)^2 + a^4*d*cos(d*x + c) + 2*a^4*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^4(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**4/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 7.79, size = 179, normalized size = 1.49

$$-\frac{\frac{336\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}-\frac{308\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+51\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-723\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-676\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-72\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^3a^4}-\frac{a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-12a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+105a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{12}}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/24*(336*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - (308*tan(1/2*d*x + 1/2*c)^6
 + 51*tan(1/2*d*x + 1/2*c)^5 - 723*tan(1/2*d*x + 1/2*c)^4 - 676*tan(1/2*d*x
 + 1/2*c)^3 - 72*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 1)/((tan
(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c))^3*a^4) - (a^8*tan(1/2*d*x + 1/2
*c)^3 - 12*a^8*tan(1/2*d*x + 1/2*c)^2 + 105*a^8*tan(1/2*d*x + 1/2*c))/a^12)
/d
```

Mupad [B]
time = 7.83, size = 171, normalized size = 1.42

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24a^4d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2a^4d}-\frac{14\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4d}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{291\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{8}+\frac{549\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{8}+41\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8}+\frac{1}{24}\right)}{a^4d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + a*sin(c + d*x))^4,x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^4*d) - tan(c/2 + (d*x)/2)^2/(2*a^4*d) - (14*log(
tan(c/2 + (d*x)/2)))/(a^4*d) + (35*tan(c/2 + (d*x)/2))/(8*a^4*d) - (cot(c/2
 + (d*x)/2)^3*(3*tan(c/2 + (d*x)/2)^2 - (3*tan(c/2 + (d*x)/2))/8 + 41*tan(c
/2 + (d*x)/2)^3 + (549*tan(c/2 + (d*x)/2)^4)/8 + (291*tan(c/2 + (d*x)/2)^5)
/8 + 1/24))/(a^4*d*(tan(c/2 + (d*x)/2) + 1)^3)
```

3.90 $\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=133

$$\frac{27\tanh^{-1}(\cos(c+dx))}{2a^4d}-\frac{16\cot(c+dx)}{a^4d}-\frac{3\cot^3(c+dx)}{a^4d}-\frac{\cot^5(c+dx)}{5a^4d}+\frac{11\cot(c+dx)\csc(c+dx)}{2a^4d}+\frac{\cot(c$$

```
[Out] 27/2*arctanh(cos(d*x+c))/a^4/d-40*cot(d*x+c)/a^4/d-27*cot(d*x+c)^3/a^4/d-41
/5*cot(d*x+c)^5/a^4/d+27/2*cot(d*x+c)*csc(d*x+c)/a^4/d+9*cot(d*x+c)*csc(d*x
+c)^3/a^4/d+8*cot(d*x+c)*csc(d*x+c)^4/a^4/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2788, 3855, 3852, 8, 3853, 3862}

$$-\frac{\cot^5(c+dx)}{5a^4d}-\frac{3\cot^3(c+dx)}{a^4d}-\frac{16\cot(c+dx)}{a^4d}+\frac{27\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{\cot(c+dx)\csc^3(c+dx)}{a^4d}+\frac{11\cot(c+dx)\csc(c+dx)}{2a^4d}-\frac{8\cot(c+dx)}{a^4d(\csc(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (27*ArcTanh[Cos[c + d*x]])/(2*a^4*d) - (16*Cot[c + d*x])/(a^4*d) - (3*Cot[c
 + d*x]^3)/(a^4*d) - Cot[c + d*x]^5/(5*a^4*d) + (11*Cot[c + d*x]*Csc[c + d*
x])/(2*a^4*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(a^4*d) - (8*Cot[c + d*x])/(a
^4*d*(1 + Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(8a^2 - 8a^2\csc(c+dx) + 8a^2\csc^2(c+dx) - 8a^2\csc^3(c+dx) + 7a^2\csc^4(c+\ldots\right.}{a^6}$$

$$= \frac{8x}{a^4} + \frac{\int \csc^6(c+dx)\,dx}{a^4} - \frac{4\int \csc^5(c+dx)\,dx}{a^4} + \frac{7\int \csc^4(c+dx)\,dx}{a^4} - \frac{8\int \csc(\ldots}{\ldots}$$

$$= \frac{8x}{a^4} + \frac{8\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{4\cot(c+dx)\csc(c+dx)}{a^4d} + \frac{\cot(c+dx)\csc^3(c+\ldots}{a^4d}$$

$$= \frac{12\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{16\cot(c+dx)}{a^4d} - \frac{3\cot^3(c+dx)}{a^4d} - \frac{\cot^5(c+dx)}{5a^4d} + \frac{11\ldots}{\ldots}$$

$$= \frac{27\tanh^{-1}(\cos(c+dx))}{2a^4d} - \frac{16\cot(c+dx)}{a^4d} - \frac{3\cot^3(c+dx)}{a^4d} - \frac{\cot^5(c+dx)}{5a^4d} + \frac{11\ldots}{\ldots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 733 vs. 2(133) = 266.
time = 6.07, size = 733, normalized size = 5.51

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (16*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)/(d*(a + a*Sin
[c + d*x])^4) - (33*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
8)/(5*d*(a + a*Sin[c + d*x])^4) + (11*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^8)/(8*d*(a + a*Sin[c + d*x])^4) - (53*Cot[(c + d*x)/2]*
Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(160*d*(a + a*S
in[c + d*x])^4) + (Csc[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^8)/(16*d*(a + a*Sin[c + d*x])^4) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(160*d*(a + a*Sin[c + d*x])^4) + (27
*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2*d*(a + a
*Sin[c + d*x])^4) - (27*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^8)/(2*d*(a + a*Sin[c + d*x])^4) - (11*Sec[(c + d*x)/2]^2*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^8)/(8*d*(a + a*Sin[c + d*x])^4) - (Sec[(c + d*
x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(16*d*(a + a*Sin[c + d*x])
^4) + (33*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*x)/2])/(5*d*(a
 + a*Sin[c + d*x])^4) + (53*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^8*Tan[(c + d*x)/2])/(160*d*(a + a*Sin[c + d*x])^4) + (Sec[(c + d*
x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*x)/2])/(160*d*(a
 + a*Sin[c + d*x])^4)
```

Maple [A]
time = 0.33, size = 165, normalized size = 1.24

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-48\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+222\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{512}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{}{5\tan(\cdots}}{32d\,a^4}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-48\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+222\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{512}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{}{5\tan(\cdots}}{32d\,a^4}$
risch	$-\frac{135i\,e^{9i(dx+c)}-630\,e^{8i(dx+c)}+135\,e^{10i(dx+c)}-610i\,e^{7i(dx+c)}+1260\,e^{6i(dx+c)}+860i\,e^{5i(dx+c)}-1510\,e^{4i(dx+c)}-430i\,e^{3\cdots}}{5\left(e^{2i(dx+c)}-1\right)^5\left(e^{i(dx+c)}+i\right)d\,a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a^4*(1/5*tan(1/2*d*x+1/2*c)^5-2*tan(1/2*d*x+1/2*c)^4+11*tan(1/2*d*x+
1/2*c)^3-48*tan(1/2*d*x+1/2*c)^2+222*tan(1/2*d*x+1/2*c)-512/(tan(1/2*d*x+1/
2*c)+1)-1/5/tan(1/2*d*x+1/2*c)^5+2/tan(1/2*d*x+1/2*c)^4-11/tan(1/2*d*x+1/2*
c)^3+48/tan(1/2*d*x+1/2*c)^2-222/tan(1/2*d*x+1/2*c)-432*ln(tan(1/2*d*x+1/2*
c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(136) = 272$.
time = 0.30, size = 279, normalized size = 2.10

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{45\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{185\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{870\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3670\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-1}{\frac{a^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{1110\sin(dx+c)}{\cos(dx+c)+1}-\frac{240\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{55\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{10\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^4}-\frac{2160\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/160*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 45*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 185*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 870*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 3670*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^4*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) +
 (1110*sin(d*x + c)/(cos(d*x + c) + 1) - 240*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 55*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 10*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^4 - 2160*log(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(136) = 272$.
time = 0.36, size = 439, normalized size = 3.30

$$\frac{424 \cos(dx+c)^6 + 154 \cos(dx+c)^5 - 1060 \cos(dx+c)^4 - 340 \cos(dx+c)^3 + 800 \cos(dx+c)^2 + 135 \left(\cos(dx+c)^6 - 3 \cos(dx+c)^4 + 3 \cos(dx+c)^2 - \left(\cos(dx+c)^5 + \cos(dx+c)^4 - 2 \cos(dx+c)^3 - 2 \cos(dx+c)^2 + \cos(dx+c) + 1\right) \sin(dx+c) - 1\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) - 135 \left(\cos(dx+c)^6 - 3 \cos(dx+c)^4 + 3 \cos(dx+c)^2 - \left(\cos(dx+c)^5 + \cos(dx+c)^4 - 2 \cos(dx+c)^3 - 2 \cos(dx+c)^2 + \cos(dx+c) + 1\right) \sin(dx+c) - 1\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + 2 \left(212 \cos(dx+c)^5 + 135 \cos(dx+c)^4 - 395 \cos(dx+c)^3 - 225 \cos(dx+c)^2 + 175 \cos(dx+c) + 80\right) \sin(dx+c) + 190 \cos(dx+c) - 160}{20 \left(a^4 d \cos(dx+c)^6 - 3 a^4 d \cos(dx+c)^4 + 3 a^4 d \cos(dx+c)^2 - a^4 d - \left(a^4 d \cos(dx+c)^5 + a^4 d \cos(dx+c)^4 - 2 a^4 d \cos(dx+c)^3 - 2 a^4 d \cos(dx+c)^2 + a^4 d \cos(dx+c) + a^4 d\right) \sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/20*(424*cos(d*x + c)^6 + 154*cos(d*x + c)^5 - 1060*cos(d*x + c)^4 - 340*c
os(d*x + c)^3 + 800*cos(d*x + c)^2 + 135*(cos(d*x + c)^6 - 3*cos(d*x + c)^4
 + 3*cos(d*x + c)^2 - (cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 -
 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sin(d*x + c) - 1)*log(1/2*cos(d*x + c
) + 1/2) - 135*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos
(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*
x + c) + 1)*sin(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(212*cos(d*x
 + c)^5 + 135*cos(d*x + c)^4 - 395*cos(d*x + c)^3 - 225*cos(d*x + c)^2 + 17
5*cos(d*x + c) + 80)*sin(d*x + c) + 190*cos(d*x + c) - 160)/(a^4*d*cos(d*x
+ c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d - (a^4*d*c
os(d*x + c)^5 + a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^3 - 2*a^4*d*cos
(d*x + c)^2 + a^4*d*cos(d*x + c) + a^4*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^6(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**6/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 9.18, size = 204, normalized size = 1.53

$$-\frac{\frac{2160\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}+\frac{2560}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{4932\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-1110\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-55\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}-\frac{a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-10a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+55a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-240a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1110a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{20}}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/160*(2160*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 2560/(a^4*(tan(1/2*d*x +
1/2*c) + 1)) - (4932*tan(1/2*d*x + 1/2*c)^5 - 1110*tan(1/2*d*x + 1/2*c)^4 +
 240*tan(1/2*d*x + 1/2*c)^3 - 55*tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x +
1/2*c) - 1)/(a^4*tan(1/2*d*x + 1/2*c)^5) - (a^16*tan(1/2*d*x + 1/2*c)^5 - 1
0*a^16*tan(1/2*d*x + 1/2*c)^4 + 55*a^16*tan(1/2*d*x + 1/2*c)^3 - 240*a^16*t
an(1/2*d*x + 1/2*c)^2 + 1110*a^16*tan(1/2*d*x + 1/2*c))/a^20)/d
```

Mupad [B]
time = 7.67, size = 209, normalized size = 1.57

$$\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{32a^4d}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2a^4d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{16a^4d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160a^4d}-\frac{27\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2a^4d}+\frac{111\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^4d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{367\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{16}+\frac{87\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{16}-\frac{37\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{32}+\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{32}-\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{160}+\frac{1}{160}\right)}{a^4d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + a*sin(c + d*x))^4,x)
```

```
[Out] (11*tan(c/2 + (d*x)/2)^3)/(32*a^4*d) - (3*tan(c/2 + (d*x)/2)^2)/(2*a^4*d) -
 tan(c/2 + (d*x)/2)^4/(16*a^4*d) + tan(c/2 + (d*x)/2)^5/(160*a^4*d) - (27*l
og(tan(c/2 + (d*x)/2)))/(2*a^4*d) + (111*tan(c/2 + (d*x)/2))/(16*a^4*d) - (
cot(c/2 + (d*x)/2)^5*((9*tan(c/2 + (d*x)/2)^2)/32 - (9*tan(c/2 + (d*x)/2))/
160 - (37*tan(c/2 + (d*x)/2)^3)/32 + (87*tan(c/2 + (d*x)/2)^4)/16 + (367*ta
n(c/2 + (d*x)/2)^5)/16 + 1/160))/(a^4*d*(tan(c/2 + (d*x)/2) + 1))
```

3.91 $\int \sqrt{a+a\sin(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=162

$$\frac{11\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{2}\,f}-\frac{27\sec(e+fx)\sqrt{a(1+\sin(e+fx))}}{8f}-\frac{\sec^3(e+fx)\sqrt{a(1+\sin}}{12f}$$

```
[Out] 11/16*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2
)/f*2^(1/2)-27/8*sec(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/f-1/12*sec(f*x+e)^3*(a
*(1+sin(f*x+e)))^(1/2)/f+29/12*(a+a*sin(f*x+e))^(1/2)*tan(f*x+e)/f+5/12*(a*
(1+sin(f*x+e)))^(1/2)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.63, antiderivative size = 195, normalized size of antiderivative = 1.20, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2793, 2725, 4486, 2754, 2766, 2729, 2728, 212, 2957, 2934}

$$\frac{11a^2\cos(e+fx)}{8f(a\sin(e+fx)+a)^{3/2}}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}+\frac{4\sec^3(e+fx)(a\sin(e+fx)+a)^{3/2}}{3af}-\frac{7\sec^3(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{11a\sec(e+fx)}{6f\sqrt{a\sin(e+fx)+a}}+\frac{11\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{8\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(8*Sqrt[2]*f) + (11*a^2*Cos[e + f*x])/(8*f*(a + a*Sin[e + f*x])^(3/2))
 - (2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (11*a*Sec[e + f*x])/(6
*f*Sqrt[a + a*Sin[e + f*x]]) - (7*Sec[e + f*x]^3*Sqrt[a + a*Sin[e + f*x]])/
(3*f) + (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2793

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
```

```
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(b*f*g*(m + p + 2))), x] + Dist[1/(b*
(m + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*
(p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,\tan^4(e+fx)\,dx = \int \sqrt{a+a\sin(e+fx)}\,dx - \int \sec^4(e+fx)\sqrt{a+a\sin(e+fx)}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \int \left(\sec^4(e+fx)\sqrt{a(1+\sin(e+fx))}\right.$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + 2\int \sec^2(e+fx)\sqrt{a(1+\sin(e+fx))}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\sec^3(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{4}{\ }$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{5a\sec(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} - \frac{7\sec^3(e+f}{\ }$$

$$= \frac{5a^2\cos(e+fx)}{8f(a+a\sin(e+fx))^{3/2}} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{11a\sec(e}{6f\sqrt{a+a\sin}}$$

$$= \frac{11a^2\cos(e+fx)}{8f(a+a\sin(e+fx))^{3/2}} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{11a\sec(e}{6f\sqrt{a+a\sin}}$$

$$= \frac{5\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{2}\,f} + \frac{11a^2\cos(e+fx)}{8f(a+a\sin(e+fx)}$$

$$= \frac{11\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{2}\,f} + \frac{11a^2\cos(e+fx}{8f(a+a\sin(e+fx}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.38, size = 394, normalized size = 2.43

$$\frac{\left(\frac{6\sin\left(\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)} - \frac{3\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)} + (33+33i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{fx}{4}\right)\left(\cos\left(\frac{1}{4}(2e+fx)\right)-\sin\left(\frac{1}{4}(2e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2 - 48\cos\left(\frac{fx}{2}\right)\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2 + 48\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\sin\left(\frac{fx}{2}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2 + \frac{4\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} - \frac{36\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)}\right)\sqrt{a(1+\sin(e+fx))}}{24f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] (((6*Sin[(f*x)/2])/(Cos[e/2] + Sin[e/2]) - (3*(Cos[e/2] - Sin[e/2])*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]))/(Cos[e/2] + Sin[e/2]) + (33 + 33*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(f*x)/4]*(Cos[(2*e + f*x)/4] - Sin[(
2*e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 48*Cos[(f*x)/2]*(
Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 48*(Cos[e/2]
 + Sin[e/2])*Sin[(f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
3 - (36*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]))*Sqrt[a*(1 + Sin[e + f*x]])/(24*f*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^3)
```

Maple [A]
time = 1.87, size = 172, normalized size = 1.06

method	result
default	$-\frac{96a^{\frac{5}{2}}\sin(fx+e)(\cos^2(fx+e))+\left(33(a-a\sin(fx+e))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a+20a^{\frac{5}{2}}\right)\sin(fx+e}{48a^{\frac{3}{2}}(\sin(fx+e)-1)\cos(fx+e)\sqrt{a+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/a^(3/2)*(96*a^(5/2)*sin(f*x+e)*cos(f*x+e)^2+(33*(a-a*sin(f*x+e))^(3/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+20*a^(5/2))
*sin(f*x+e)-162*a^(5/2)*cos(f*x+e)^2+33*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a-4*a^(5/2))/(sin(f*x+e)-1)
/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^4, x)
```

Fricas [A]
time = 0.39, size = 217, normalized size = 1.34

$$\frac{33\sqrt{2}\sqrt{a}\cos(fx+e)^3\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a\sin(fx+e)+a}\left(\sqrt{2}\cos(fx+e)-\sqrt{2}\sin(fx+e)+\sqrt{2}\right)\sqrt{a}+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(81\cos(fx+e)^2-2\left(24\cos(fx+e)^2+5\right)\sin(fx+e)+2\right)\sqrt{a\sin(fx+e)+a}}{96f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(33*sqrt(2)*sqrt(a)*cos(f*x + e)^3*log(-(a*cos(f*x + e)^2 + 2*sqrt(a*s
in(f*x + e) + a)*(sqrt(2)*cos(f*x + e) - sqrt(2)*sin(f*x + e) + sqrt(2))*sq
rt(a) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(
f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(81*c
os(f*x + e)^2 - 2*(24*cos(f*x + e)^2 + 5)*sin(f*x + e) + 2)*sqrt(a*sin(f*x
+ e) + a))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\tan^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*tan(e + f*x)**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{tan}\left(e+f\,x\right)}^4\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/2), x)
```

3.92 $\int \sqrt{a+a\sin(e+fx)}\,\tan^2(e+fx)\,dx$

Optimal. Leaf size=101

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}f}+\frac{5\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{f}-\frac{2\sec(e+fx)(a+a\sin(e-$$

```
[Out] -2*sec(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-1/2*arctanh(1/2*cos(f*x+e)*a^(1/2)
*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f*2^(1/2)+5*sec(f*x+e)*(a+a*sin(f*
x+e))^(1/2)/f
```

Rubi [A]
time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2792, 2934, 2728, 212}

$$-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}+\frac{5\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[2]*f)) + (5*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/f - (2*Sec[e +
 f*x]*(a + a*Sin[e + f*x])^(3/2))/(a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2792

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[(a + b*Sin[e + f*x])^m*((b*(m + 1) + a*Sin[e + f*x])/Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
```

```
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,\tan^2(e+fx)\,dx = -\frac{2\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{af} + \frac{2\int\sec^2(e+fx)\sqrt{a+a\,\mathrm{s}}$$

$$= \frac{5\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{f} - \frac{2\sec(e+fx)(a+a\sin(e+}{af}$$

$$= \frac{5\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{f} - \frac{2\sec(e+fx)(a+a\sin(e+}{af}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}f} + \frac{5\sec(e+fx)\sqrt{a+}}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 114, normalized size = 1.13

$$\frac{\sec(e+fx)\left(3+(1-i)\sqrt[4]{-1}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{fx}{4}\right)\left(\cos\left(\frac{1}{4}(2e+fx)\right)-\sin\left(\frac{1}{4}(2e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-2\sin(e+fx)\right)\sqrt{a(1+\sin(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] (Sec[e + f*x]*(3 + (1 - I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(f
*x)/4]*(Cos[(2*e + f*x)/4] - Sin[(2*e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2]) - 2*Sin[e + f*x])*Sqrt[a*(1 + Sin[e + f*x])])/f
```

Maple [A]
time = 2.15, size = 89, normalized size = 0.88

method	result

default	$-\frac{(1+\sin(fx+e))\left(\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{a-a\sin(fx+e)}+4a\sin(fx+e)-6a\right)}{2\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(1+sin(f*x+e))*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*(a-a*sin(f*x+e))^(1/2)+4*a*sin(f*x+e)-6*a)/cos(f*x+e)/(a+a*si
n(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^2, x)
```

Fricas [A]
time = 0.39, size = 184, normalized size = 1.82

$$\frac{\sqrt{2}\,\sqrt{a}\,\cos(fx+e)\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{2}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{a}\,(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\sqrt{a\sin(fx+e)+a}\,(2\sin(fx+e)-3)}{4f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(a)*cos(f*x + e)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a
*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x
+ e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(a*sin(f*x + e) + a)*
(2*sin(f*x + e) - 3))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\,(\sin(e+fx)+1)}\,\tan^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*tan(e + f*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(94) = 188.
time = 10.71, size = 455, normalized size = 4.50

$$\frac{\sqrt{2}\left(\log\left(\frac{2\left(\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}+2\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)+1\right)}{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}+1}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{3}-\log\left(\frac{2\left(\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}-2\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)+1\right)}{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}+1}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{3}-\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{4}+\log\left(\frac{2\left(\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}+2\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)+1\right)}{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}+1}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)-\log\left(\frac{2\left(\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}-2\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)+1\right)}{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}+1}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)-18\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}-\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}}{4\left(\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{3}+\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(log(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 2*tan(-1/8*pi + 1/4*
f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 - log(2*(tan(-1/8*pi +
1/4*f*x + 1/4*e)^2 - 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1
/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1
/4*f*x + 1/4*e)^3 - sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f
*x + 1/4*e)^4 + log(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 2*tan(-1/8*pi + 1
/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e) - log(2*(tan(-1/8*pi +
 1/4*f*x + 1/4*e)^2 - 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi +
1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi +
1/4*f*x + 1/4*e) - 18*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4
*f*x + 1/4*e)^2 - sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((tan(-1/8*p
i + 1/4*f*x + 1/4*e)^3 + tan(-1/8*pi + 1/4*f*x + 1/4*e))*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^2\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/2), x)
```

3.93 $\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=89

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}+\frac{3a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\frac{\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{f}$$

```
[Out] -arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f+3*a*cos(f*x+e
)/f/(a+a*sin(f*x+e))^(1/2)-cot(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2795, 3060, 2852, 212}

$$\frac{3a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f) + (
3*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Sqrt[a + a*S
in[e + f*x]])/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2795

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[(a + b*Sin[e + f*x])^m*((b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x]),
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] && !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{f}+\frac{\int \csc(e+fx)\left(\frac{a}{2}-\frac{3}{2}a\sin(e\ldots\right.}{\ldots}$$

$$=\frac{3a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\frac{\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{f}+\frac{1}{2}\int \ldots$$

$$=\frac{3a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\frac{\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{f}-\frac{a\mathrm{Sub}\ldots}{\ldots}$$

$$=-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}+\frac{3a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\ldots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 206 vs. 2(89) = 178.
time = 0.67, size = 206, normalized size = 2.31

$$\frac{\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(-4\cos\left(\frac{1}{2}(e+fx)\right)+2\cos\left(\frac{3}{2}(e+fx)\right)+4\sin\left(\frac{1}{2}(e+fx)\right)-\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+2\sin\left(\frac{3}{2}(e+fx)\right)\right)}{f\left(1+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(\csc\left(\frac{1}{4}(e+fx)\right)-\sec\left(\frac{1}{4}(e+fx)\right)\right)\left(\csc\left(\frac{1}{4}(e+fx)\right)+\sec\left(\frac{1}{4}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(-4*Cos[(e + f*x)/2] + 2*Cos
[(3*(e + f*x))/2] + 4*Sin[(e + f*x)/2] - Log[1 + Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]]*Sin[e + f*x] + Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[
e + f*x] + 2*Sin[(3*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x
)/4] - Sec[(e + f*x)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

Maple [A]
time = 2.00, size = 125, normalized size = 1.40

method	result
default	$\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(\sin(fx+e)\left(2\sqrt{a-a\sin\left(fx+e\right)}\,a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx\right.}}{\sqrt{a}}\right.\right.\right.}{\sin(fx+e)a^{\frac{3}{2}}\cos(fx+e)\sqrt{a+a\sin\left(fx+e\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*(2*(a-a*sin(f*x+e))^(1
/2)*a^(3/2)-arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a^2)-(a-a*sin(f*x+e))^(
1/2)*a^(3/2))/sin(f*x+e)/a^(3/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cot(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(85) = 170$.
time = 0.39, size = 306, normalized size = 3.44

$$\frac{\left(\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(fx+e)^{3}-7a\cos(fx+e)^{2}-4\left(\cos(fx+e)^{2}+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{a}-9a\cos(fx+e)+\left(a\cos(fx+e)^{2}+8a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^{3}+\cos(fx+e)^{2}+\left(\cos(fx+e)^{2}-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)-4\left(2\cos\left(fx+e\right)^{2}+\left(2\cos\left(fx+e\right)+3\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-3\right)\sqrt{a\sin\left(fx+e\right)+a}}{4\left(f\cos\left(fx+e\right)^{2}-\left(f\cos\left(fx+e\right)+f\right)\sin\left(fx+e\right)-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((a*
cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3
)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a
*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)
/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos
(f*x + e) - 1)) - 4*(2*cos(f*x + e)^2 + (2*cos(f*x + e) + 3)*sin(f*x + e) -
 cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x
 + e) + f)*sin(f*x + e) - f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\cot^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cot(e + f*x)**2, x)
```

Giac [A]
time = 3.09, size = 157, normalized size = 1.76

$$-\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{4\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1}\right)\sqrt{a}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)
)/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + 8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e) + 4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)/(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1))*sqrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^2\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/2), x)
```

3.94 $\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=163

$$\frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f}-\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}+\frac{11a\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}}-\frac{a\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 11/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f-2*a*cos(f
*x+e)/f/(a+a*sin(f*x+e))^(1/2)+11/8*a*cot(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-1
/12*a*cot(f*x+e)*csc(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-1/3*cot(f*x+e)*csc(f*x
+e)^2*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.25, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2797, 2725, 3123, 3059, 2851, 2852, 212}

$$-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}+\frac{11a\cot(e+fx)}{8f\sqrt{a\sin(e+fx)+a}}+\frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{a\cot(e+fx)\csc(e+fx)}{12f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) + (11*a*Cot[e + f*x])/(8
*f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cot[e + f*x]*Csc[e + f*x])/(12*f*Sqrt[a +
 a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/
(3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2797

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[(a + b*Sin[e + f*x])^m*((
```

```
1 - 2*Sin[e + f*x]^2)/Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \int \sqrt{a+a\sin(e+fx)}\,dx + \int \csc^4(e+fx)\sqrt{a+a\sin(e+fx)}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(}}{3f}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{a\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+}{}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + \frac{11a\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} - \frac{a\cot(e+f}{12f\sqrt{a+}}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + \frac{11a\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} - \frac{a\cot(e+f}{12f\sqrt{a+}}$$

$$= \frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 1.07, size = 309, normalized size = 1.90

$$\frac{\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(252\cos\left(\frac{1}{2}(e+fx)\right)-250\cos\left(\frac{3}{2}(e+fx)\right)-114\cos\left(\frac{5}{2}(e+fx)\right)+48\cos\left(\frac{7}{2}(e+fx)\right)-252\sin\left(\frac{1}{2}(e+fx)\right)+99\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-99\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-250\sin\left(\frac{3}{2}(e+fx)\right)+114\sin\left(\frac{5}{2}(e+fx)\right)-33\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))+33\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))+48\sin\left(\frac{7}{2}(e+fx)\right)\right)}{24f\left(1+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(\csc^2\left(\frac{1}{4}(e+fx)\right)-\sec^2\left(\frac{1}{4}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(252*Cos[(e + f*x)/2] - 250
*Cos[(3*(e + f*x))/2] - 114*Cos[(5*(e + f*x))/2] + 48*Cos[(7*(e + f*x))/2]
- 252*Sin[(e + f*x)/2] + 99*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Si
n[e + f*x] - 99*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] -
 250*Sin[(3*(e + f*x))/2] + 114*Sin[(5*(e + f*x))/2] - 33*Log[1 + Cos[(e +
f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] + 33*Log[1 - Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] + 48*Sin[(7*(e + f*x))/2]))/(24*f*(1 +
 Cot[(e + f*x)/2])*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

Maple [A]
time = 2.37, size = 170, normalized size = 1.04

method	result

default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(48\sqrt{-a(\sin(fx+e)-1)}\,a^{\frac{7}{2}}(\sin^3(fx+e))-15\sqrt{-a(\sin(fx+\right.}{24a^{\frac{7}{2}}\sin(fx+e)^3\cos(f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(48*(-a*(sin(f*x+e)-1))^(1/2
)*a^(7/2)*sin(f*x+e)^3-15*(-a*(sin(f*x+e)-1))^(1/2)*a^(7/2)+56*(-a*(sin(f*x
+e)-1))^(3/2)*a^(5/2)-33*(-a*(sin(f*x+e)-1))^(5/2)*a^(3/2)-33*arctanh((-a*(
sin(f*x+e)-1))^(1/2)/a^(1/2))*a^4*sin(f*x+e)^3)/a^(7/2)/sin(f*x+e)^3/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cot(f*x + e)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(153) = 306$.
time = 0.39, size = 417, normalized size = 2.56

$$\frac{33\left(\cos(fx+e)^4-2\cos(fx+e)^2-\left(\cos(fx+e)^3+\cos(fx+e)^2-\cos(fx+e)-1\right)\sin(fx+e)+1\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7a\cos(fx+e)^2+4\left(\cos(fx+e)^2+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin(fx+e)+a}\sqrt{a}-9a\cos(fx+e)+\left(a\cos(fx+e)^2+8a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^3+\cos(fx+e)^2+\left(\cos(fx+e)^2-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)+4\left(48\cos(fx+e)^4-33\cos(fx+e)^3-139\cos(fx+e)^2+\left(48\cos(fx+e)^3+81\cos(fx+e)^2-58\cos(fx+e)-83\right)\sin(fx+e)+25\cos(fx+e)+83\right)\sqrt{a\sin(fx+e)+a}}{96\left(f\cos(fx+e)^4-2f\cos(fx+e)^2-\left(f\cos(fx+e)^3+f\cos(fx+e)^2-f\cos(fx+e)-f\right)\sin(fx+e)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(33*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e
)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7
*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2
*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a
*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 +
 cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) +
4*(48*cos(f*x + e)^4 - 33*cos(f*x + e)^3 - 139*cos(f*x + e)^2 + (48*cos(f*x
 + e)^3 + 81*cos(f*x + e)^2 - 58*cos(f*x + e) - 83)*sin(f*x + e) + 25*cos(f
*x + e) + 83)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e
)^2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e) - f)*sin(f*x +
e) + f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\cot^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cot(e + f*x)**4, x)
```

Giac [**A**]
time = 3.62, size = 223, normalized size = 1.37

$$\frac{\sqrt{2}\left(33\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+192\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{4\left(132\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-112\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+15\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)^{3}}\right)\sqrt{a}}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*sqrt(2)*(33*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2
*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 192*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e) + 4*(132*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^5 - 112*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^3 + 15*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e))/(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3)*sqrt(a)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{cot}\left(e+f\,x\right)}^4\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/2), x)
```

3.95 $\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=167

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}f}+\frac{2a^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\frac{7a\sec(e+fx)}{}$$

```
[Out] 2/3*a^3*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^(3/2)+1/3*sec(f*x+e)^3*(a+a*sin(f*x
+e))^(3/2)/f-1/4*a^(3/2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*
x+e))^(1/2))/f*2^(1/2)-4*a^2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-7/2*a*sec(
f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.65, antiderivative size = 195, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2793, 2726, 2725, 4486, 2754, 2728, 212, 2957, 2934}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}f}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}+\frac{4\sec^3(e+fx)(a\sin(e+fx)+a)^{5/2}}{af}-\frac{23\sec^3(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}+\frac{a\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] -1/2*(a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*
x]])])/(Sqrt[2]*f) - (8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) -
(2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (a*Sec[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(2*f) - (23*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(
3*f) + (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(5/2))/(a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
```

```
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2793

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(b*f*g*(m + p + 2))), x] + Dist[1/(b*
(m + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*
(p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx &= \int (a+a\sin(e+fx))^{3/2}\,dx - \int \sec^4(e+fx)(a+a\sin(e+fx))^{3} \\
&= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{1}{3}(4a)\int \sqrt{a+a\sin(e+} \\
&= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} \\
&= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} \\
&= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} \\
&= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} \\
&= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}f} - \frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx}} \\
&= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}f} - \frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.38, size = 141, normalized size = 0.84

$$\frac{a\sec^3(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{a(1+\sin(e+fx))}\left(-45+6\cos(2(e+fx))+(3+3i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+54\sin(e+fx)+\sin(3(e+fx))\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] (a*Sec[e + f*x]^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[a*(1 + Sin[e
 + f*x])]*(-45 + 6*Cos[2*(e + f*x)] + (3 + 3*I)*(-1)^(3/4)*ArcTanh[(1/2 + I
/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^3 + 54*Sin[e + f*x] + Sin[3*(e + f*x)]))/(6*f)
```

Maple [A]
time = 1.91, size = 139, normalized size = 0.83

method	result
default	$\frac{(1+\sin(fx+e))\left(3a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)(a-a\sin(fx+e))^{\frac{3}{2}}-8a^{3}\sin(fx+e)\left(\cos^{2}(fx+e)\right)-24a^{3}\left(\right.}{12a(\sin(fx+e)-1)\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(1+sin(f*x+e))/a/(sin(f*x+e)-1)*(3*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*si
n(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(3/2)-8*a^3*sin(f*x+e)*co
s(f*x+e)^2-24*a^3*cos(f*x+e)^2-106*sin(f*x+e)*a^3+102*a^3)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.40, size = 260, normalized size = 1.56

$$\frac{3\left(\sqrt{2}\,a\cos(fx+e)\sin(fx+e)-\sqrt{2}\,a\cos(fx+e)\right)\sqrt{a}\,\log\left(-\frac{a\cos(fx+e)^{2}-2\sqrt{a\sin(fx+e)+a}\left(\sqrt{2}\cos(fx+e)-\sqrt{2}\sin(fx+e)+\sqrt{2}\right)\sqrt{a}+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^{2}-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(12a\cos(fx+e)^{2}+\left(4a\cos(fx+e)^{2}+53a\right)\sin(fx+e)-51a\right)\sqrt{a\sin(fx+e)+a}}{24\left(f\cos(fx+e)\sin(fx+e)-f\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(sqrt(2)*a*cos(f*x + e)*sin(f*x + e) - sqrt(2)*a*cos(f*x + e))*sqrt
(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a*sin(f*x + e) + a)*(sqrt(2)*cos(f*x +
e) - sqrt(2)*sin(f*x + e) + sqrt(2))*sqrt(a) + 3*a*cos(f*x + e) - (a*cos(f*
x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2)) - 4*(12*a*cos(f*x + e)^2 + (4*a*cos(f*x + e)^
2 + 53*a)*sin(f*x + e) - 51*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)*si
n(f*x + e) - f*cos(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*tan(f*x+e)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 974 vs. 2(152) = 304.
time = 154.08, size = 974, normalized size = 5.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x
+ 1/4*e)^12 - 12*a*log(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 2*tan(-1/8*pi
+ 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9 + 12*a*log(2*(tan
(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/(tan(
-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(
-1/8*pi + 1/4*f*x + 1/4*e)^9 - 78*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan
(-1/8*pi + 1/4*f*x + 1/4*e)^10 - 36*a*log(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)
^2 + 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^
2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^
7 + 36*a*log(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 2*tan(-1/8*pi + 1/4*f*x
+ 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^7 - 1089*a*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^8 - 36*a*log(2*(tan(-1/8*p
i + 1/4*f*x + 1/4*e)^2 + 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi
 + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi
 + 1/4*f*x + 1/4*e)^5 + 36*a*log(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 2*ta
n(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^5 - 996*a
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^6 - 12*
a*log(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 2*tan(-1/8*pi + 1/4*f*x + 1/4*e
) + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 + 12*a*log(2*(tan(-1/8*pi + 1/4*f*
x + 1/4*e)^2 - 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x
 + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x
 + 1/4*e)^3 - 1089*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*
f*x + 1/4*e)^4 - 78*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4
*f*x + 1/4*e)^2 + a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((tan(-1/8
*pi + 1/4*f*x + 1/4*e)^9 + 3*tan(-1/8*pi + 1/4*f*x + 1/4*e)^7 + 3*tan(-1/8*
pi + 1/4*f*x + 1/4*e)^5 + tan(-1/8*pi + 1/4*f*x + 1/4*e)^3)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(3/2),x)
[Out] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(3/2), x)
```

3.96 $\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}+\frac{7\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{3f}-\frac{2\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{3af}$$

```
[Out] 7/3*sec(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/3*sec(f*x+e)*(a+a*sin(f*x+e))^(5/
2)/a/f+11/3*a^2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2792, 2934, 2725}

$$\frac{11a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af}+\frac{7\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (11*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (7*Sec[e + f*x]*(a +
 a*Sin[e + f*x])^(3/2))/(3*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(5/2))
/(3*a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2792

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[(a + b*Sin[e + f*x])^m*((b*(m + 1) + a*Sin[e + f*x])/Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx = -\frac{2\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{3af}+\frac{2\int \sec^2(e+fx)(a+a\sin$$
$$=\frac{7\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{3f}-\frac{2\sec(e+fx)(a+a\sin(e}{3af}$$
$$=\frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}+\frac{7\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{3f}-$$

Mathematica [A]
time = 1.98, size = 46, normalized size = 0.52

$$\frac{a\sec(e+fx)(15+\cos(2(e+fx))-8\sin(e+fx))\sqrt{a(1+\sin(e+fx))}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
[Out] (a*Sec[e + f*x]*(15 + Cos[2*(e + f*x)] - 8*Sin[e + f*x])*Sqrt[a*(1 + Sin[e
+ f*x])])/(3*f)
```

Maple [A]
time = 1.05, size = 55, normalized size = 0.62

method	result	size
default	$-\frac{2a^2(1+\sin(fx+e))(\sin^2(fx+e)+4\sin(fx+e)-8)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
[Out] -2/3*a^2*(1+sin(f*x+e))*(sin(f*x+e)^2+4*sin(f*x+e)-8)/cos(f*x+e)/(a+a*sin(f
*x+e))^(1/2)/f
```

Maxima [A]
time = 0.51, size = 157, normalized size = 1.78

$$-\frac{8\left(2a^{\frac{3}{2}}-\frac{2a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{2a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{3f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -8/3*(2*a^(3/2) - 2*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^(3/2)*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 2*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(f*(sin(f*x + e)/(co
s(f*x + e) + 1) - 1)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

<u>Fricas</u> [**A**]
time = 0.36, size = 52, normalized size = 0.59

$$\frac{2\left(a\cos\left(fx+e\right)^{2}-4\,a\sin\left(fx+e\right)+7\,a\right)\sqrt{a\sin\left(fx+e\right)+a}}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(f*x + e)^2 - 4*a*sin(f*x + e) + 7*a)*sqrt(a*sin(f*x + e) + a)/(f
*cos(f*x + e))
```

<u>Sympy</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*tan(e + f*x)**2, x)
```

<u>Giac</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(82) = 164.
time = 42.48, size = 222, normalized size = 2.52

$$-\frac{\sqrt{2}\left(3\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\tan\left(-\frac{1}{8}\,\pi+\frac{1}{4}\,fx+\frac{1}{4}\,e\right)^{8}+60\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\tan\left(-\frac{1}{8}\,\pi+\frac{1}{4}\,fx+\frac{1}{4}\,e\right)^{6}+50\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\tan\left(-\frac{1}{8}\,\pi+\frac{1}{4}\,fx+\frac{1}{4}\,e\right)^{4}+60\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\tan\left(-\frac{1}{8}\,\pi+\frac{1}{4}\,fx+\frac{1}{4}\,e\right)^{2}+3\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{6\left(\tan\left(-\frac{1}{8}\,\pi+\frac{1}{4}\,fx+\frac{1}{4}\,e\right)^{7}+3\,\tan\left(-\frac{1}{8}\,\pi+\frac{1}{4}\,fx+\frac{1}{4}\,e\right)^{5}+3\,\tan\left(-\frac{1}{8}\,\pi+\frac{1}{4}\,fx+\frac{1}{4}\,e\right)^{3}+\tan\left(-\frac{1}{8}\,\pi+\frac{1}{4}\,fx+\frac{1}{4}\,e\right)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(3*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x
 + 1/4*e)^8 + 60*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*
x + 1/4*e)^6 + 50*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f
*x + 1/4*e)^4 + 60*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*
f*x + 1/4*e)^2 + 3*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((tan(-1/
```

```
8*pi + 1/4*f*x + 1/4*e)^7 + 3*tan(-1/8*pi + 1/4*f*x + 1/4*e)^5 + 3*tan(-1/8
*pi + 1/4*f*x + 1/4*e)^3 + tan(-1/8*pi + 1/4*f*x + 1/4*e))*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+fx)^2 (a+a\sin(e+fx))^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(3/2),x)
[Out] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(3/2), x)
```

3.97 $\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=121

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}+\frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}+\frac{5a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}-\frac{\cot(}{}$$

```
[Out] -3*a^(3/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f-cot(f*x+e)*
(a+a*sin(f*x+e))^(3/2)/f+11/3*a^2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+5/3*a
*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.21, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2795, 3055, 3060, 2852, 212}

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{11a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{5a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f + (
11*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (5*a*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(3*f) - (Cot[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2795

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[(a + b*Sin[e + f*x])^m*((b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x]),
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = -\frac{\cot(e+fx)(a+a\sin(e+fx))^{3/2}}{f}+\frac{\int \csc(e+fx)\left(\frac{3a}{2}-\frac{5}{2}a\sin\right.}{}$$

$$=\frac{5a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}-\frac{\cot(e+fx)(a+a\sin(e+}{f}$$

$$=\frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}+\frac{5a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}-$$

$$=\frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}+\frac{5a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}-$$

$$=-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}+\frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx}}$$

Mathematica [A]

time = 0.53, size = 233, normalized size = 1.93

$$-\frac{a \csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(14\cos\left(\frac{1}{2}(e+fx)\right)-9\cos\left(\frac{3}{2}(e+fx)\right)+\cos\left(\frac{5}{2}(e+fx)\right)-14\sin\left(\frac{1}{2}(e+fx)\right)+9\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-9\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-9\sin\left(\frac{3}{2}(e+fx)\right)-\sin\left(\frac{5}{2}(e+fx)\right)\right)}{3f\left(1+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(\csc\left(\frac{1}{4}(e+fx)\right)-\sec\left(\frac{1}{4}(e+fx)\right)\right)\left(\csc\left(\frac{1}{4}(e+fx)\right)+\sec\left(\frac{1}{4}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/3*(a*Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(14*Cos[(e + f*x)/2]
- 9*Cos[(3*(e + f*x))/2] + Cos[(5*(e + f*x))/2] - 14*Sin[(e + f*x)/2] + 9*L
og[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] - 9*Log[1 - Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 9*Sin[(3*(e + f*x))/2] - Sin[
(5*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4] - Sec[(e + f
*x)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

Maple [A]
time = 2.32, size = 144, normalized size = 1.19

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sin(fx+e)\left(2(a-a\sin(fx+e))^{\frac{3}{2}}\sqrt{a}-12\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}+9\,\mathrm{ar}\right.\right.}{3\sin(fx+e)\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx+e}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*(2*(a-a*sin(f*x+e
))^(3/2)*a^(1/2)-12*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+9*arctanh((a-a*sin(f*x+e
))^(1/2)/a^(1/2))*a^2)+3*(a-a*sin(f*x+e))^(1/2)*a^(3/2))/sin(f*x+e)/a^(1/2)
/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cot(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(113) = 226.
time = 0.37, size = 344, normalized size = 2.84

$$\frac{9\left(a\cos(fx+e)^2-(a\cos(fx+e)+a)\sin(fx+e)-a\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7a\cos(fx+e)^2-4\left(\cos(fx+e)^2+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin(fx+e)+a}\sqrt{a}-9a\cos(fx+e)+\left(a\cos(fx+e)^2+8a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^3+\cos(fx+e)^2+\left(\cos(fx+e)^2-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)+4\left(2a\cos(fx+e)^3-8a\cos(fx+e)^2+a\cos(fx+e)-\left(2a\cos(fx+e)^2+10a\cos(fx+e)+11a\right)\sin(fx+e)+11a\right)\sqrt{a\sin(fx+e)+a}}{12\left(f\cos(fx+e)^2-(f\cos(fx+e)+f)\sin(fx+e)-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(9*(a*cos(f*x + e)^2 - (a*cos(f*x + e) + a)*sin(f*x + e) - a)*sqrt(a)*
log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x +
 e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a
) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x +
e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e
) - cos(f*x + e) - 1)) + 4*(2*a*cos(f*x + e)^3 - 8*a*cos(f*x + e)^2 + a*cos
(f*x + e) - (2*a*cos(f*x + e)^2 + 10*a*cos(f*x + e) + 11*a)*sin(f*x + e) +
11*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x + e) + f)*si
n(f*x + e) - f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (e+fx)+1))^{\frac{3}{2}}\cot^2 (e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*cot(e + f*x)**2, x)
```

Giac [**A**]
time = 5.07, size = 193, normalized size = 1.60

$$\frac{\sqrt{2}\left(16\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-9\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-48\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{12\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1}\right)\sqrt{a}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*(16*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^3 - 9*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1
/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) - 48*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e) - 12*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)/(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1))*sqrt(a)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{cot}(e+f\,x)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(3/2), x)
```

3.98 $\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=197

$$\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}+\frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+fx)}}-\frac{2a\cos(e+fx)}{}$$

```
[Out] 37/8*a^(3/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f-1/3*cot(f
*x+e)*csc(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/f-8/3*a^2*cos(f*x+e)/f/(a+a*sin(f
*x+e))^(1/2)+29/24*a^2*cot(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*a*cos(f*x+e)
*(a+a*sin(f*x+e))^(1/2)/f-1/4*a*cot(f*x+e)*csc(f*x+e)*(a+a*sin(f*x+e))^(1/2
)/f
```

Rubi [A]

time = 0.34, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2797, 2726, 2725, 3123, 3054, 3059, 2852, 212}

$$\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{29a^2\cot(e+fx)}{24f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}-\frac{a\cot(e+fx)\csc(e+fx)\sqrt{a\sin(e+fx)+a}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (37*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (29*a^2*Cot[e + f*
x])/(24*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(3*f) - (a*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f)
 - (Cot[e + f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
```

```
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2797

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
```

```
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx &= \int (a+a\sin(e+fx))^{3/2}\,dx + \int \csc^4(e+fx)(a+a\sin(e+fx))^3 \\
&= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f} \\
&= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} \\
&= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e}{} \\
&= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e}{} \\
&= \frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 1.04, size = 334, normalized size = 1.70

$$
-\frac{a\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(-276\cos\left(\frac{1}{2}(e+fx)\right)+326\cos\left(\frac{3}{2}(e+fx)\right)+78\cos\left(\frac{5}{2}(e+fx)\right)-72\cos\left(\frac{7}{2}(e+fx)\right)+8\cos\left(\frac{9}{2}(e+fx)\right)+276\sin\left(\frac{1}{2}(e+fx)\right)-333\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+333\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+326\sin\left(\frac{3}{2}(e+fx)\right)-78\sin\left(\frac{5}{2}(e+fx)\right)+111\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))-111\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))-72\sin\left(\frac{7}{2}(e+fx)\right)-8\sin\left(\frac{9}{2}(e+fx)\right)\right)}{24f\left(1+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(\csc^2\left(\frac{1}{4}(e+fx)\right)-\sec^2\left(\frac{1}{4}(e+fx)\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/24*(a*Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(-276*Cos[(e + f*x)
/2] + 326*Cos[(3*(e + f*x))/2] + 78*Cos[(5*(e + f*x))/2] - 72*Cos[(7*(e + f
*x))/2] + 8*Cos[(9*(e + f*x))/2] + 276*Sin[(e + f*x)/2] - 333*Log[1 + Cos[(
e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 333*Log[1 - Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]]*Sin[e + f*x] + 326*Sin[(3*(e + f*x))/2] - 78*Sin[(5*(e
 + f*x))/2] + 111*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f
*x)] - 111*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] -
```

```
72*Sin[(7*(e + f*x))/2] - 8*Sin[(9*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2]
)*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

Maple [A]
time = 2.28, size = 196, normalized size = 0.99

method	result
default	$\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(16(-a(\sin(fx+e)-1))^{\frac{3}{2}}(\sin^3(fx+e))a^{\frac{3}{2}}-96a^{\frac{5}{2}}\sqrt{-a(\sin(fx+e)-1}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/a^(3/2)*(16*(-a*(sin(f*x+e)-1
))^(3/2)*sin(f*x+e)^3*a^(3/2)-96*a^(5/2)*(-a*(sin(f*x+e)-1))^(1/2)*sin(f*x+
e)^3+111*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+15*(-a
*(sin(f*x+e)-1))^(5/2)*a^(1/2)-8*(-a*(sin(f*x+e)-1))^(3/2)*a^(3/2)-15*(-a*(
sin(f*x+e)-1))^(1/2)*a^(5/2))/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2
)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cot(f*x + e)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(183) = 366.
time = 0.37, size = 463, normalized size = 2.35

$$\frac{111\left(a\cos(fx+e)^4-2a\cos(fx+e)^2-\left(a\cos(fx+e)^3+a\cos(fx+e)^2-a\cos(fx+e)-a\right)\sin(fx+e)+a\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7a\cos(fx+e)^2+4\left(\cos(fx+e)^2+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin(fx+e)+a}\sqrt{a}-9a\cos(fx+e)+\left(a\cos(fx+e)^2+8a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^3+\cos(fx+e)^2+\left(\cos(fx+e)^2-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)-4\left(16a\cos(fx+e)^5-64a\cos(fx+e)^4-17a\cos(fx+e)^3+165a\cos(fx+e)^2+9a\cos(fx+e)-\left(16a\cos(fx+e)^4+80a\cos(fx+e)^3+63a\cos(fx+e)^2-102a\cos(fx+e)-93a\right)\sin(fx+e)-93a\right)\sqrt{a\sin(fx+e)+a}}{96\left(f\cos(fx+e)^4-2f\cos(fx+e)^2-\left(f\cos(fx+e)^3+f\cos(fx+e)^2-f\cos(fx+e)-f\right)\sin(fx+e)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/96*(111*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 - (a*cos(f*x + e)^3 + a*co
s(f*x + e)^2 - a*cos(f*x + e) - a)*sin(f*x + e) + a)*sqrt(a)*log((a*cos(f*x
 + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f
*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*
x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f
```

```
*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x +
e) - 1)) - 4*(16*a*cos(f*x + e)^5 - 64*a*cos(f*x + e)^4 - 17*a*cos(f*x + e)
^3 + 165*a*cos(f*x + e)^2 + 9*a*cos(f*x + e) - (16*a*cos(f*x + e)^4 + 80*a*
cos(f*x + e)^3 + 63*a*cos(f*x + e)^2 - 102*a*cos(f*x + e) - 93*a)*sin(f*x +
 e) - 93*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^
2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e) - f)*sin(f*x + e)
 + f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 9.89, size = 260, normalized size = 1.32

$$-\frac{\sqrt{2}\left(128\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-111\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-384\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{4\left(60\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-16\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-15\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{3}}\right)\sqrt{a}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(128*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^3 - 111*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x
 + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) - 384*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4
*pi + 1/2*f*x + 1/2*e) - 4*(60*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^5 - 16*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^3 - 15*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(
-1/4*pi + 1/2*f*x + 1/2*e))/(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3)*sqr
t(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(3/2), x)
```

3.99 $\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=151

$$-\frac{2a^5\cos^5(e+fx)}{5f(a+a\sin(e+fx))^{5/2}}+\frac{8a^4\cos^3(e+fx)}{3f(a+a\sin(e+fx))^{3/2}}-\frac{12a^3\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\frac{8a^2\sec(e+fx)\sqrt{a+a\sin(}{f}$$

```
[Out] -2/5*a^5*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^(5/2)+8/3*a^4*cos(f*x+e)^3/f/(a+a*
sin(f*x+e))^(3/2)+2/3*a*sec(f*x+e)^3*(a+a*sin(f*x+e))^(3/2)/f-12*a^3*cos(f*
x+e)/f/(a+a*sin(f*x+e))^(1/2)-8*a^2*sec(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.67, antiderivative size = 208, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2793, 2726, 2725, 4486, 2752, 2957, 2934}

$$-\frac{64a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{46a^2\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{4\sec^3(e+fx)(a\sin(e+fx)+a)^{7/2}}{af}+\frac{26\sec^3(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}-\frac{2a\sec^3(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^4,x]
```

```
[Out] (-64*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (46*a^2*Sec[e + f*x]*Sqrt[a + a*Sin[e
 + f*x]])/(3*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) - (2*
a*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*f) + (26*Sec[e + f*x]^3*(a
+ a*Sin[e + f*x])^(5/2))/(3*f) - (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(7/
2))/(a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
```

```
- b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2793

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(b*f*g*(m + p + 2))), x] + Dist[1/(b*
(m + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*
(p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx = \int (a+a\sin(e+fx))^{5/2}\,dx - \int \sec^4(e+fx)(a+a\sin(e+fx)$$

$$= -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} + \frac{1}{5}(8a)\int (a+a\sin(e+$$

$$= -\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2a\cos(e+fx)(a+a}{5f}$$

$$= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+f}}{15f}$$

$$= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+f}}{15f}$$

$$= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+f}}{15f}$$

Mathematica [A]
time = 5.31, size = 112, normalized size = 0.74

$$\frac{a^2\sqrt{a(1+\sin(e+fx))}\,(-1225+204\cos(2(e+fx))-3\cos(4(e+fx))+1488\sin(e+fx)+16\sin(3(e+fx)))}{60f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^4,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Sin[e + f*x])]*(-1225 + 204*Cos[2*(e + f*x)] - 3*Cos[4*(e
+ f*x)] + 1488*Sin[e + f*x] + 16*Sin[3*(e + f*x)]))/(60*f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A]
time = 1.56, size = 87, normalized size = 0.58

method	result	size
default	$\frac{2a^3(1+\sin(fx+e))(3(\sin^4(fx+e))+8(\sin^3(fx+e))+48(\sin^2(fx+e))-192\sin(fx+e)+128)}{15(\sin(fx+e)-1)\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*a^3*(1+sin(f*x+e))/(sin(f*x+e)-1)*(3*sin(f*x+e)^4+8*sin(f*x+e)^3+48*si
n(f*x+e)^2-192*sin(f*x+e)+128)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(145) = 290$.
time = 0.55, size = 301, normalized size = 1.99

$$\frac{32\left(8a^{\frac{5}{2}}-\frac{24a^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{44a^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{68a^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{75a^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{68a^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{44a^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{24a^{\frac{5}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{8a^{\frac{5}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}\right)}{15f\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 32/15*(8*a^(5/2) - 24*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 44*a^(5/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 68*a^(5/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 75*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 68*a^(5/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 44*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - 24*a^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 8*a^(5/2)*sin(f*x
 + e)^8/(cos(f*x + e) + 1)^8)/(f*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)
*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
```

Fricas [A]
time = 0.36, size = 106, normalized size = 0.70

$$\frac{2\left(3a^2\cos(fx+e)^4-54a^2\cos(fx+e)^2+179a^2-8\left(a^2\cos(fx+e)^2+23a^2\right)\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}}{15\left(f\cos(fx+e)\sin(fx+e)-f\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(f*x + e)^4 - 54*a^2*cos(f*x + e)^2 + 179*a^2 - 8*(a^2*cos(f
*x + e)^2 + 23*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e)*
sin(f*x + e) - f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1580 vs. $2(145) = 290$.
time = 162.75, size = 1580, normalized size = 10.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] -1/122880*sqrt(2)*(2560*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi
 + 1/4*f*x + 1/4*e)^16 - 79560*pi*a^2*floor(-1/8*(pi - 2*f*x - 2*e)/pi + 1/
2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^13 -
79560*pi*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*f*x
+ 1/4*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^13 - 9945*(pi - 2*f*x - 2*e)*a^2*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^13 + 7956
0*a^2*arctan(1/tan(-1/8*pi + 1/4*f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^13 - 225280*a^2*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^14 - 397800*pi*a^2*floor(-1/
8*(pi - 2*f*x - 2*e)/pi + 1/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8
*pi + 1/4*f*x + 1/4*e)^11 - 397800*pi*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(tan(-1/8*pi + 1/4*f*x + 1/4*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 -
49725*(pi - 2*f*x - 2*e)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*p
i + 1/4*f*x + 1/4*e)^11 + 397800*a^2*arctan(1/tan(-1/8*pi + 1/4*f*x + 1/4*e
))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 -
4352000*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4
*e)^12 - 795600*pi*a^2*floor(-1/8*(pi - 2*f*x - 2*e)/pi + 1/2)*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9 - 795600*pi*a^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*f*x + 1/4*e))*tan(-
1/8*pi + 1/4*f*x + 1/4*e)^9 - 99450*(pi - 2*f*x - 2*e)*a^2*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9 + 795600*a^2*arctan(1/
tan(-1/8*pi + 1/4*f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/
8*pi + 1/4*f*x + 1/4*e)^9 - 10096640*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*tan(-1/8*pi + 1/4*f*x + 1/4*e)^10 - 795600*pi*a^2*floor(-1/8*(pi - 2*f*x
- 2*e)/pi + 1/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x
+ 1/4*e)^7 - 795600*pi*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8
*pi + 1/4*f*x + 1/4*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^7 - 99450*(pi - 2*f*
x - 2*e)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/
4*e)^7 + 795600*a^2*arctan(1/tan(-1/8*pi + 1/4*f*x + 1/4*e))*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^7 - 18236416*a^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^8 - 397800*pi
*a^2*floor(-1/8*(pi - 2*f*x - 2*e)/pi + 1/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^5 - 397800*pi*a^2*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*f*x + 1/4*e))*tan(-1/8*pi + 1/4*f*x +
 1/4*e)^5 - 49725*(pi - 2*f*x - 2*e)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*tan(-1/8*pi + 1/4*f*x + 1/4*e)^5 + 397800*a^2*arctan(1/tan(-1/8*pi + 1/4*
f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1
/4*e)^5 - 10096640*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/
4*f*x + 1/4*e)^6 - 79560*pi*a^2*floor(-1/8*(pi - 2*f*x - 2*e)/pi + 1/2)*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 - 79560*p
i*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*f*x + 1/4*e
```

```
))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 - 9945*(pi - 2*f*x - 2*e)*a^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 + 79560*a^2*arc
tan(1/tan(-1/8*pi + 1/4*f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*t
an(-1/8*pi + 1/4*f*x + 1/4*e)^3 - 4352000*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 - 225280*a^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 2560*a^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)))*sqrt(a)/((tan(-1/8*pi + 1/4*f*x + 1/4*e)^13 + 5*tan(-1/
8*pi + 1/4*f*x + 1/4*e)^11 + 10*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9 + 10*tan(-
1/8*pi + 1/4*f*x + 1/4*e)^7 + 5*tan(-1/8*pi + 1/4*f*x + 1/4*e)^5 + tan(-1/8
*pi + 1/4*f*x + 1/4*e)^3)*f)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^4\,(a+a\,\sin(e+f\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(5/2), x)
```

3.100 $\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=118

$$\frac{124a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}+\frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}+\frac{9\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{5f}-\frac{2\,\mathrm{se}}{}$$

```
[Out] 9/5*sec(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f-2/5*sec(f*x+e)*(a+a*sin(f*x+e))^(7/
2)/a/f+124/15*a^3*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+31/15*a^2*cos(f*x+e)*
(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2792, 2934, 2726, 2725}

$$\frac{124a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}+\frac{31a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}+\frac{9\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^2,x]
```

```
[Out] (124*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (9*Sec[e + f*x]*(a + a*Sin[e + f*x])^
(5/2))/(5*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(5*a*f)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2792

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[(a + b*Sin[e + f*x])^m*((b*(m + 1) + a*Sin[e + f*x])/Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx &= -\frac{2\sec(e+fx)(a+a\sin(e+fx))^{7/2}}{5af}+\frac{2\int \sec^2(e+fx)(a+a\sin\\ &= \frac{9\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{5f}-\frac{2\sec(e+fx)(a+a\sin(e+}{5af}\\ &= \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}+\frac{9\sec(e+fx)(a+a\sin(}{5f}\\ &= \frac{124a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}+\frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}\end{aligned}$$

Mathematica [A]
time = 5.32, size = 60, normalized size = 0.51

$$\frac{a^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,(330+22\cos(2(e+fx))-185\sin(e+fx)+3\sin(3(e+fx)))}{30f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^2,x]
```

```
[Out] (a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(330 + 22*Cos[2*(e + f*x)] - 1
85*Sin[e + f*x] + 3*Sin[3*(e + f*x)]))/(30*f)
```

Maple [A]
time = 1.36, size = 67, normalized size = 0.57

method	result	size
default	$-\frac{2a^3(1+\sin(fx+e))(3(\sin^3(fx+e))+11(\sin^2(fx+e))+44\sin(fx+e)-88)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*a^3*(1+sin(f*x+e))*(3*sin(f*x+e)^3+11*sin(f*x+e)^2+44*sin(f*x+e)-88)/
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [A]
time = 0.51, size = 207, normalized size = 1.75

$$-\frac{8\left(22\,a^{\frac{5}{2}}-\frac{22\,a^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{55\,a^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{50\,a^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{55\,a^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{22\,a^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{22\,a^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{15\,f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] -8/15*(22*a^(5/2) - 22*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 55*a^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 50*a^(5/2)*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 55*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22*a^(5/2)*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)*(sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 1)^(5/2))
```

Fricas [A]
time = 0.36, size = 75, normalized size = 0.64

$$\frac{2\left(11\,a^2\cos\left(fx+e\right)^2+77\,a^2+\left(3\,a^2\cos\left(fx+e\right)^2-47\,a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] 2/15*(11*a^2*cos(f*x + e)^2 + 77*a^2 + (3*a^2*cos(f*x + e)^2 - 47*a^2)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*tan(f*x+e)**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1502 vs. 2(110) = 220.
time = 44.08, size = 1502, normalized size = 12.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="giac")

```
[Out] -1/3840*sqrt(2)*(1080*pi*a^2*floor(-1/8*(pi - 2*f*x - 2*e)/pi + 1/2)*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 + 1080*pi*a
^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*f*x + 1/4*e))*
tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 + 135*(pi - 2*f*x - 2*e)*a^2*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 - 1080*a^2*arctan
(1/tan(-1/8*pi + 1/4*f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(
-1/8*pi + 1/4*f*x + 1/4*e)^11 + 3840*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*tan(-1/8*pi + 1/4*f*x + 1/4*e)^12 + 5400*pi*a^2*floor(-1/8*(pi - 2*f*x -
2*e)/pi + 1/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x +
1/4*e)^9 + 5400*pi*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*pi
+ 1/4*f*x + 1/4*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9 + 675*(pi - 2*f*x - 2*
e)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9
 - 5400*a^2*arctan(1/tan(-1/8*pi + 1/4*f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9 + 99840*a^2*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^10 + 10800*pi*a^2*floor(
-1/8*(pi - 2*f*x - 2*e)/pi + 1/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-
1/8*pi + 1/4*f*x + 1/4*e)^7 + 10800*pi*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sgn(tan(-1/8*pi + 1/4*f*x + 1/4*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^7 +
1350*(pi - 2*f*x - 2*e)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi
 + 1/4*f*x + 1/4*e)^7 - 10800*a^2*arctan(1/tan(-1/8*pi + 1/4*f*x + 1/4*e))*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^7 + 2009
60*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^8
 + 10800*pi*a^2*floor(-1/8*(pi - 2*f*x - 2*e)/pi + 1/2)*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^5 + 10800*pi*a^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*f*x + 1/4*e))*tan(-1/8*pi +
 1/4*f*x + 1/4*e)^5 + 1350*(pi - 2*f*x - 2*e)*a^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^5 - 10800*a^2*arctan(1/tan(-1/8*p
i + 1/4*f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4
*f*x + 1/4*e)^5 + 406528*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*p
i + 1/4*f*x + 1/4*e)^6 + 5400*pi*a^2*floor(-1/8*(pi - 2*f*x - 2*e)/pi + 1/2
)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 + 54
00*pi*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*f*x + 1
/4*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 + 675*(pi - 2*f*x - 2*e)*a^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 - 5400*a^2*a
rctan(1/tan(-1/8*pi + 1/4*f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 + 200960*a^2*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 + 1080*pi*a^2*floor(-1/8*(pi - 2*f
*x - 2*e)/pi + 1/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f
*x + 1/4*e) + 1080*pi*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*
pi + 1/4*f*x + 1/4*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 135*(pi - 2*f*x - 2
*e)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)
```

```
- 1080*a^2*arctan(1/tan(-1/8*pi + 1/4*f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 99840*a^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 3840*a^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)))*sqrt(a)/((tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 + 5*tan(
-1/8*pi + 1/4*f*x + 1/4*e)^9 + 10*tan(-1/8*pi + 1/4*f*x + 1/4*e)^7 + 10*tan
(-1/8*pi + 1/4*f*x + 1/4*e)^5 + 5*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 + tan(-1
/8*pi + 1/4*f*x + 1/4*e))*f)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(5/2), x)
```

3.101 $\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=151

$$-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}+\frac{49a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}+\frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}+$$

```
[Out] -5*a^(5/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f+7/5*a*cos(f
*x+e)*(a+a*sin(f*x+e))^(3/2)/f-cot(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f+49/15*a^
3*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+31/15*a^2*cos(f*x+e)*(a+a*sin(f*x+e))
^(1/2)/f
```

Rubi [A]

time = 0.29, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2795, 3055, 3060, 2852, 212}

$$-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{49a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}+\frac{31a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}+\frac{7a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-5*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f + (
49*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (7*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^
(3/2))/(5*f) - (Cot[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/f
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2795

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[(a + b*Sin[e + f*x])^m*((b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x]),
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = -\frac{\cot(e+fx)(a+a\sin(e+fx))^{5/2}}{f} + \frac{\int \csc(e+fx)\left(\frac{5a}{2}-\frac{7}{2}a\sin\right.}{}$$

$$= \frac{7a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} - \frac{\cot(e+fx)(a+a\sin(e+}{f}$$

$$= \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{7a\cos(e+fx)(a+a\sin}{5f}$$

$$= \frac{49a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}$$

$$= \frac{49a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}$$

$$= -\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} + \frac{49a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx}}$$

Mathematica [A]
time = 0.83, size = 261, normalized size = 1.73

$$-\frac{a^2\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(125\cos\left(\frac{1}{2}(e+fx)\right)-93\cos\left(\frac{3}{2}(e+fx)\right)+25\cos\left(\frac{5}{2}(e+fx)\right)+3\cos\left(\frac{7}{2}(e+fx)\right)-125\sin\left(\frac{1}{2}(e+fx)\right)+150\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-150\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-93\sin\left(\frac{3}{2}(e+fx)\right)-25\sin\left(\frac{5}{2}(e+fx)\right)+3\sin\left(\frac{7}{2}(e+fx)\right)\right)}{30f\left(1+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(\csc\left(\frac{1}{4}(e+fx)\right)-\sec\left(\frac{1}{4}(e+fx)\right)\right)\left(\csc\left(\frac{1}{4}(e+fx)\right)+\sec\left(\frac{1}{4}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/30*(a^2*Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(125*Cos[(e + f*x)
/2] - 93*Cos[(3*(e + f*x))/2] + 25*Cos[(5*(e + f*x))/2] + 3*Cos[(7*(e + f*x
))/2] - 125*Sin[(e + f*x)/2] + 150*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)
/2]]*Sin[e + f*x] - 150*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e
+ f*x] - 93*Sin[(3*(e + f*x))/2] - 25*Sin[(5*(e + f*x))/2] + 3*Sin[(7*(e +
f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4] - Sec[(e + f*x)/4])*
(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

Maple [A]
time = 2.16, size = 162, normalized size = 1.07

method	result

default	$\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sin(fx+e)\left(6(a-a\sin(fx+e))^{\frac{5}{2}}\sqrt{a}-40(a-a\sin(fx+e))^{\frac{3}{2}}a^{\frac{3}{2}}+90\sqrt{a-a\sin\right.\right.}{15\sin(fx+e)\sqrt{a}\cos(fx+e)\sqrt{a+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*(6*(a-a*sin(f*x+e
))^(5/2)*a^(1/2)-40*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+90*(a-a*sin(f*x+e))^(1/2
)*a^(5/2)-75*arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a^3)-15*(a-a*sin(f*x+e
))^(1/2)*a^(5/2))/sin(f*x+e)/a^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cot(f*x + e)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(141) = 282$.
time = 0.36, size = 394, normalized size = 2.61

$$\frac{75\left(a^2\cos(fx+e)^2-a^2-\left(a^2\cos(fx+e)+a^2\right)\sin(fx+e)\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7a\cos(fx+e)^2-4\left(\cos(fx+e)^2+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin(fx+e)+a}\sqrt{a}-9a\cos(fx+e)+\left(a\cos(fx+e)^2+8a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^3+\cos(fx+e)^2+\left(\cos(fx+e)^2-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)+4\left(6a^2\cos(fx+e)^4+28a^2\cos(fx+e)^3-40a^2\cos(fx+e)^2-13a^2\cos(fx+e)+49a^2+\left(6a^2\cos(fx+e)^3-22a^2\cos(fx+e)^2-62a^2\cos(fx+e)-49a^2\right)\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}}{60\left(f\cos(fx+e)^2-\left(f\cos(fx+e)+f\right)\sin(fx+e)-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/60*(75*(a^2*cos(f*x + e)^2 - a^2 - (a^2*cos(f*x + e) + a^2)*sin(f*x + e))
*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (
cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) +
a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*s
in(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*si
n(f*x + e) - cos(f*x + e) - 1)) + 4*(6*a^2*cos(f*x + e)^4 + 28*a^2*cos(f*x
+ e)^3 - 40*a^2*cos(f*x + e)^2 - 13*a^2*cos(f*x + e) + 49*a^2 + (6*a^2*cos(
f*x + e)^3 - 22*a^2*cos(f*x + e)^2 - 62*a^2*cos(f*x + e) - 49*a^2)*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x + e) + f)*si
n(f*x + e) - f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 11.80, size = 235, normalized size = 1.56

$$-\frac{\sqrt{2}\left(96\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-320\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+75\,\sqrt{2}\,a^2\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+360\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+\frac{60\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1}\right)\sqrt{a}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*(96*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^5 - 320*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e)^3 + 75*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1
/2*f*x + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 360*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*
sin(-1/4*pi + 1/2*f*x + 1/2*e) + 60*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)/(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1))*s
qrt(a)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(5/2), x)
```

3.102 $\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=227

$$\frac{55a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f}-\frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}+$$

```
[Out] 55/8*a^(5/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f-2/5*a*cos
(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-5/12*a*cot(f*x+e)*csc(f*x+e)*(a+a*sin(f*x+
e))^(3/2)/f-1/3*cot(f*x+e)*csc(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/f-9/40*a^3*c
os(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-16/15*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1
/2)/f+17/24*a^2*cot(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.42, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2797, 2726, 2725, 3123, 3054, 3060, 2852, 212}

$$\frac{55a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{9a^3\cos(e+fx)}{40f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}+\frac{17a^2\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{24f}-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}-\frac{5a\cot(e+fx)\csc(e+fx)(a\sin(e+fx)+a)^{3/2}}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (55*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (9*a^3*Cos[e + f*x])/(40*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (17*a^2*Cot[e + f*x]*Sqrt[a + a*Sin[
e + f*x]])/(24*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) - (
5*a*Cot[e + f*x]*Csc[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(12*f) - (Cot[e +
 f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(3*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
```

```
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2797

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
```

```
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx &= \int (a+a\sin(e+fx))^{5/2}\,dx + \int \csc^4(e+fx)(a+a\sin(e+fx)) \\
&= -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} - \frac{\cot(e+fx)\csc^2(e+f}{} \\
&= -\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2a\cos(e+fx)(a+a}{5f} \\
&= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+f}}{15f} \\
&= -\frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+f}}{15f} \\
&= -\frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+f}}{15f} \\
&= \frac{55a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+f}}
\end{aligned}$$

Mathematica [A]
time = 1.16, size = 360, normalized size = 1.59

$$-\frac{a^2\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(108\cos\left(\frac{1}{2}(e+fx)\right)+706\cos\left(\frac{3}{2}(e+fx)\right)-450\cos\left(\frac{5}{2}(e+fx)\right)-156\cos\left(\frac{7}{2}(e+fx)\right)+100\cos\left(\frac{9}{2}(e+fx)\right)+12\cos\left(\frac{11}{2}(e+fx)\right)-108\sin\left(\frac{1}{2}(e+fx)\right)-2475\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+2475\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+706\sin\left(\frac{3}{2}(e+fx)\right)+450\sin\left(\frac{5}{2}(e+fx)\right)+825\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))-825\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))-156\sin\left(\frac{7}{2}(e+fx)\right)-100\sin\left(\frac{9}{2}(e+fx)\right)+12\sin\left(\frac{11}{2}(e+fx)\right)\right)}{120f\left(1+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(\csc^2\left(\frac{1}{4}(e+fx)\right)-\sec^2\left(\frac{1}{4}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/120*(a^2*Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(108*Cos[(e + f*
x)/2] + 706*Cos[(3*(e + f*x))/2] - 450*Cos[(5*(e + f*x))/2] - 156*Cos[(7*(e
 + f*x))/2] + 100*Cos[(9*(e + f*x))/2] + 12*Cos[(11*(e + f*x))/2] - 108*Sin
[(e + f*x)/2] - 2475*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f
*x] + 2475*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 706*
```

```
Sin[(3*(e + f*x))/2] + 450*Sin[(5*(e + f*x))/2] + 825*Log[1 + Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 825*Log[1 - Cos[(e + f*x)/2] + S
in[(e + f*x)/2]]*Sin[3*(e + f*x)] - 156*Sin[(7*(e + f*x))/2] - 100*Sin[(9*(
e + f*x))/2] + 12*Sin[(11*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e
 + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

Maple [A]
time = 2.59, size = 222, normalized size = 0.98

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(48(-a(\sin(fx+e)-1))^{\frac{5}{2}}\left(\sin^{3}(fx+e)\right)\sqrt{a}-320(-a(\sin(fx+e)-1))^{\frac{3}{2}}\left(\sin^{3}(fx\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(48*(-a*(sin(f*x+e)-1))^(5/
2)*sin(f*x+e)^3*a^(1/2)-320*(-a*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)^3*a^(3/2)+
480*a^(5/2)*(-a*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)^3-825*arctanh((-a*(sin(f*x
+e)-1))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+135*(-a*(sin(f*x+e)-1))^(5/2)*a^(1/
2)-440*(-a*(sin(f*x+e)-1))^(3/2)*a^(3/2)+345*(-a*(sin(f*x+e)-1))^(1/2)*a^(5
/2))/sin(f*x+e)^3/a^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cot(f*x + e)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(211) = 422$.
time = 0.38, size = 526, normalized size = 2.32

$$\frac{825\left(a^{2}\cos\left(fx+e\right)^{4}-2\,a^{2}\cos\left(fx+e\right)^{2}+a^{2}-\left(a^{2}\cos\left(fx+e\right)^{3}+a^{2}\cos\left(fx+e\right)^{2}-a^{2}\cos\left(fx+e\right)-a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(fx+e)^{3}-7\,a\cos(fx+e)^{2}+4\left(\cos(fx+e)^{2}+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{a}-9\,a\cos(fx+e)+\left(a\cos(fx+e)^{2}+8\,a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^{3}+\cos(fx+e)^{2}+\left(\cos(fx+e)^{2}-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)-4\left(48\,a^{2}\cos\left(fx+e\right)^{6}+224\,a^{2}\cos\left(fx+e\right)^{5}-128\,a^{2}\cos\left(fx+e\right)^{4}-583\,a^{2}\cos\left(fx+e\right)^{3}+147\,a^{2}\cos\left(fx+e\right)^{2}+399\,a^{2}\cos\left(fx+e\right)-27\,a^{2}+\left(48\,a^{2}\cos\left(fx+e\right)^{5}-176\,a^{2}\cos\left(fx+e\right)^{4}-304\,a^{2}\cos\left(fx+e\right)^{3}+279\,a^{2}\cos\left(fx+e\right)^{2}+426\,a^{2}\cos\left(fx+e\right)+27\,a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{480\left(f\cos\left(fx+e\right)^{4}-2\,f\cos\left(fx+e\right)^{2}-\left(f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)^{2}-f\cos\left(fx+e\right)-f\right)\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(825*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2 - (a^2*cos(f*x
+ e)^3 + a^2*cos(f*x + e)^2 - a^2*cos(f*x + e) - a^2)*sin(f*x + e))*sqrt(a)
```

```
*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x
+ e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(
a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x +
 e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x +
e) - cos(f*x + e) - 1)) - 4*(48*a^2*cos(f*x + e)^6 + 224*a^2*cos(f*x + e)^5
 - 128*a^2*cos(f*x + e)^4 - 583*a^2*cos(f*x + e)^3 + 147*a^2*cos(f*x + e)^2
 + 399*a^2*cos(f*x + e) - 27*a^2 + (48*a^2*cos(f*x + e)^5 - 176*a^2*cos(f*x
 + e)^4 - 304*a^2*cos(f*x + e)^3 + 279*a^2*cos(f*x + e)^2 + 426*a^2*cos(f*x
 + e) + 27*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 -
 2*f*cos(f*x + e)^2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e)
 - f)*sin(f*x + e) + f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.16, size = 306, normalized size = 1.35

$$\frac{\sqrt{2}\left(768\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-2560\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+825\,\sqrt{2}\,a^{2}\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+1920\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\frac{20\left(108\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-176\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+69\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{3}}\right)\sqrt{a}}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*sqrt(2)*(768*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^5 - 2560*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^3 + 825*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi
+ 1/2*f*x + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 1920*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 20*(108*a^2*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 176*a^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 69*a^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/(2*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^2 - 1)^3)*sqrt(a)/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(e+f\,x)^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(5/2),x)
[Out] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(5/2), x)
```

3.103 $\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=150

$$-\frac{67\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{64\sqrt{2}\sqrt{a}\,f}-\frac{\sec(e+fx)(53+127\sin(e+fx))}{192f\sqrt{a+a\sin(e+fx)}}+\frac{a\sin(e+fx)\tan(e+fx)}{24f(a+a\sin(e+fx))^{3/2}}+$$

```
[Out] -67/128*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/f*2^
(1/2)/a^(1/2)-1/192*sec(f*x+e)*(53+127*sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)
+1/24*a*sin(f*x+e)*tan(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)+1/3*tan(f*x+e)^3/f/(
a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 241, normalized size of antiderivative = 1.61, number of steps used = 17, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2793, 2728, 212, 4486, 2766, 2760, 2729, 2956, 2934}

$$\frac{61a\cos(e+fx)}{64f(a\sin(e+fx)+a)^{3/2}}+\frac{7\sec^3(e+fx)\sqrt{a\sin(e+fx)+a}}{12af}-\frac{5\sec^3(e+fx)}{6f\sqrt{a\sin(e+fx)+a}}-\frac{61\sec(e+fx)}{48f\sqrt{a\sin(e+fx)+a}}+\frac{7a\sec(e+fx)}{24f(a\sin(e+fx)+a)^{3/2}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}+\frac{61\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{64\sqrt{2}\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (61*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(64
*Sqrt[2]*Sqrt[a]*f) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt
[a + a*Sin[e + f*x]])])/(Sqrt[a]*f) + (61*a*Cos[e + f*x])/(64*f*(a + a*Sin[
e + f*x])^(3/2)) + (7*a*Sec[e + f*x])/(24*f*(a + a*Sin[e + f*x])^(3/2)) - (
61*Sec[e + f*x])/(48*f*Sqrt[a + a*Sin[e + f*x]]) - (5*Sec[e + f*x]^3)/(6*f*
Sqrt[a + a*Sin[e + f*x]]) + (7*Sec[e + f*x]^3*Sqrt[a + a*Sin[e + f*x]])/(12
*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2793

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2956

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(
p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1))), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
```

```
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx - \int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} - \int \left(\frac{\sec^4(e+fx)}{\sqrt{a(1+\sin(e+fx))}}\right.$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + 2\int \frac{\sec^2(e+fx)\tan^2(e+fx)}{\sqrt{a(1+\sin(e+fx))}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{5\sec^3(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} + \frac{\int \sec}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{}{6f}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{}{48}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{61a\cos(e+fx)}{64f(a+a\sin(e+fx))^{3/2}} + \frac{}{24}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{61a\cos(e+fx)}{64f(a+a\sin(e+fx))^{3/2}} + \frac{}{24}$$

$$= \frac{61\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{64\sqrt{2}\sqrt{a}f} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+\ldots}\right.}{\sqrt{a}f}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.47, size = 118, normalized size = 0.79

$$\frac{(804+804i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-\sec^3(e+fx)(90+122\cos(2(e+fx))-41\sin(e+fx)+183\sin(3(e+fx)))}{768f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((804 + 804*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x
)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - Sec[e + f*x]^3*(90 + 122*Cos
[2*(e + f*x)] - 41*Sin[e + f*x] + 183*Sin[3*(e + f*x)]))/(768*f*Sqrt[a*(1 +
 Sin[e + f*x])])
```

Maple [A]
time = 2.34, size = 231, normalized size = 1.54

method	result
default	$\frac{366a^{\frac{7}{2}}\sin(fx+e)(\cos^2(fx+e))+\left(402\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)(a-a\sin(fx+e))^{\frac{3}{2}}a^2-112a^{\frac{7}{2}}\right)\sin(fx+e}{384a^{\frac{7}{2}}(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(366*a^(7/2)*sin(f*x+e)*cos(f*x+e)^2+(402*2^(1/2)*arctanh(1/2*(a-a*si
n(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(3/2)*a^2-112*a^(7/2))*si
n(f*x+e)+(-201*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
(a-a*sin(f*x+e))^(3/2)*a^2+122*a^(7/2))*cos(f*x+e)^2+402*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(3/2)*a^2-16*a^(
7/2))/a^(7/2)/(sin(f*x+e)-1)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/
2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [A]
time = 0.37, size = 250, normalized size = 1.67

$$\frac{201\sqrt{2}\left(\cos(fx+e)^3\sin(fx+e)+\cos(fx+e)^3\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{a}(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(61\cos(fx+e)^2+\left(183\cos(fx+e)^2-56\right)\sin(fx+e)-8\right)\sqrt{a\sin(fx+e)+a}}{768\left(af\cos(fx+e)^3\sin(fx+e)+af\cos(fx+e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(201*sqrt(2)*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(a)*l
og(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x
 + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f
*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
+ e) - 2)) - 4*(61*cos(f*x + e)^2 + (183*cos(f*x + e)^2 - 56)*sin(f*x + e)
- 8)*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e)^3*sin(f*x + e) + a*f*cos(f
*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e + fx)}{\sqrt{a(\sin(e + fx) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [A]
time = 42.65, size = 229, normalized size = 1.53

$$-\frac{\frac{201\sqrt{2}\log\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{201\sqrt{2}\log\left(-\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{6\sqrt{2}\left(21\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-19\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{16\sqrt{2}\left(15\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a}\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3}}{768\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/768*(201*sqrt(2)*log(sin(3/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(a)*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))) - 201*sqrt(2)*log(-sin(3/4*pi + 1/2*f*x + 1/2
*e) + 1)/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 6*sqrt(2)*(21*sin(
3/4*pi + 1/2*f*x + 1/2*e)^3 - 19*sin(3/4*pi + 1/2*f*x + 1/2*e))/((sin(3/4*p
i + 1/2*f*x + 1/2*e)^2 - 1)^2*sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
+ 16*sqrt(2)*(15*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^2 - sqrt(a))/(a*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi + 1/2*f*x + 1/2*e)^3))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e + f\,x)^4}{\sqrt{a + a\,\sin(e + f\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/2), x)
```

3.104 $\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=107

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{2}\sqrt{a}f}-\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}}+\frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af}$$

```
[Out] 5/8*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/f*2^(1/2
)/a^(1/2)-1/2*sec(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+3/4*sec(f*x+e)*(a+a*sin(f
*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2791, 2934, 2728, 212}

$$\frac{3\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{4af}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{4\sqrt{2}\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4*S
qrt[2]*Sqrt[a]*f) - Sec[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*Sec[e
+ f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*a*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[b*((a + b*Sin[e + f*x])^m/(a*f*(2*m - 1)*Cos[e + f*x])),
x] - Dist[1/(a^2*(2*m - 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*((a*m - b*(2*
```

```
m - 1)*Sin[e + f*x])/Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}} + \frac{\int \sec^2(e+fx)\sqrt{a+a\sin(e+fx)}\left(-\frac{a}{2}+2a\sin\right.}{2a^2}$$

$$= -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}} + \frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af} - \frac{5}{8}\int \frac{}{\sqrt{a+}}$$

$$= -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}} + \frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af} + \frac{5\text{Subst}\left(\int\right.}{}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{2}\sqrt{a}f} - \frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}} + \frac{3\sec(e+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 118, normalized size = 1.10

$$-\frac{\sec(e+fx)\left(-1+(5+5i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-3\sin(e+fx)\right)}{4f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -1/4*(Sec[e + f*x]*(-1 + (5 + 5*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4
)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^2 - 3*Sin[e + f*x]))/(f*Sqrt[a*(1 + Sin[e + f*x
])])
```

Maple [A]
time = 1.70, size = 130, normalized size = 1.21

method	result
default	$\frac{\sin(fx+e)\left(5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{a-a\sin(fx+e)}\,a+6a^{\frac{3}{2}}\right)+5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-}}{}\right.}{8a^{\frac{3}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(sin(f*x+e)*(5*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*(a-a*sin(f*x+e))^(1/2)*a+6*a^(3/2))+5*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+
e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(1/2)*a+2*a^(3/2))/a^(3/2)/cos(
f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(94) = 188.
time = 0.36, size = 219, normalized size = 2.05

$$\frac{5\sqrt{2}\left(\cos(fx+e)\sin(fx+e)+\cos(fx+e)\right)\sqrt{a}\,\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{2}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{a}\,(\cos(fx+e)-\sin(fx+e)+1)+3\,a\cos(fx+e)-(a\cos(fx+e)-2\,a)\sin(fx+e)+2\,a}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\,\sqrt{a\sin(fx+e)+a}\,(3\,\sin(fx+e)+1)}{16\,(af\cos(fx+e)\sin(fx+e)+af\cos(fx+e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(5*sqrt(2)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*sqrt(a)*log(-(a*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
 sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
 + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
2)) + 4*sqrt(a*sin(f*x + e) + a)*(3*sin(f*x + e) + 1))/(a*f*cos(f*x + e)*si
n(f*x + e) + a*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [A]
time = 20.31, size = 164, normalized size = 1.53

$$\frac{\frac{5\sqrt{2}\log\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{5\sqrt{2}\log\left(-\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2\sqrt{2}\left(3\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-2\right)}{\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sqrt{a}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(5*sqrt(2)*log(sin(3/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(a)*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))) - 5*sqrt(2)*log(-sin(3/4*pi + 1/2*f*x + 1/2*e) +
1)/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*sqrt(2)*(3*sin(3/4*pi
+ 1/2*f*x + 1/2*e)^2 - 2)/((sin(3/4*pi + 1/2*f*x + 1/2*e)^3 - sin(3/4*pi +
1/2*f*x + 1/2*e))*sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^2}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/2), x)
```

3.105 $\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

[Out] arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f/a^(1/2)-cot(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2795, 21, 2852, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]

[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(Sqrt[a]*f) - Cot[e + f*x]/(f*Sqrt[a + a*Sin[e + f*x]])

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2795

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[(a + b*Sin[e + f*x])^m*((b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x]),
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
```

```
2] &&  !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx &= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}+\frac{\int \frac{\csc(e+fx)\left(-\frac{a}{2}-\frac{1}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}\\
&= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\,dx}{2a}\\
&= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 138 vs. 2(62) = 124.
time = 0.23, size = 138, normalized size = 2.23

$$\frac{\csc\left(\frac{1}{4}(e+fx)\right)\sec\left(\frac{1}{4}(e+fx)\right)\left(-2\cos\left(\frac{1}{2}(e+fx)\right)+2\sin\left(\frac{1}{2}(e+fx)\right)+\left(\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)}{8f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/4]*Sec[(e + f*x)/4]*(-2*Cos[(e + f*x)/2] + 2*Sin[(e + f*x)/2
] + (Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[1 - Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]])*Sin[e + f*x])*(1 + Tan[(e + f*x)/2]))/(8*f*Sqrt[a*(1
 + Sin[e + f*x])])
```

Maple [A]
time = 2.48, size = 103, normalized size = 1.66

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}}{\sqrt{a}}\right)a\sin(fx+e)+\sqrt{a-a\sin\left(fx\right.}\right.}{a^{\frac{3}{2}}\sin(fx+e)\cos(fx+e)\sqrt{a+a\sin\left(fx+e\right)}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-arctanh((a-a*sin(f*x+e))^(1/2)/
a^(1/2))*a*sin(f*x+e)+(a-a*sin(f*x+e))^(1/2)*a^(1/2))/a^(3/2)/sin(f*x+e)/co
s(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(58) = 116$.
time = 0.38, size = 288, normalized size = 4.65

$$\frac{\left(\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(fx+e)^{3}-7\,a\cos(fx+e)^{2}+4\left(\cos(fx+e)^{2}+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{a}-9\,a\cos(fx+e)+\left(a\cos(fx+e)^{2}+8\,a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^{3}+\cos(fx+e)^{2}+\left(\cos(fx+e)^{2}-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)+4\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{4\left(af\cos\left(fx+e\right)^{2}-af-\left(af\cos\left(fx+e\right)+af\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((a*
cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3
)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a
*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)
/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos
(f*x + e) - 1)) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) +
 1))/(a*f*cos(f*x + e)^2 - a*f - (a*f*cos(f*x + e) + a*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(58) = 116.
time = 10.74, size = 138, normalized size = 2.23

$$\frac{\sqrt{2}\,\sqrt{a}\left(\frac{\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(a)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x +
 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))) - 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)/((2*sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^2 - 1)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(e+f\,x)^2}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/2), x)
```

3.106 $\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=135

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{a}\,f}+\frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}}+\frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -7/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f/a^(1/2)+9/8*cot(f
*x+e)/f/(a+a*sin(f*x+e))^(1/2)+1/12*cot(f*x+e)*csc(f*x+e)/f/(a+a*sin(f*x+e)
)^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2797, 2728, 212, 3123, 3063, 3064, 2852}

$$\frac{9\cot(e+fx)}{8f\sqrt{a\sin(e+fx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8\sqrt{a}\,f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*Sqrt[a]*f)
 + (9*Cot[e + f*x])/(8*f*Sqrt[a + a*Sin[e + f*x]]) + (Cot[e + f*x]*Csc[e +
f*x])/(12*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*f*
Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2797

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[(a + b*Sin[e + f*x])^m*((
```

```
1 - 2*Sin[e + f*x]^2)/Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx + \int \frac{\csc^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{3a} - \frac{2\text{Subst}\left(\int \frac{1}{2a-x^2}\right.}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e}{3f\sqrt{}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e}{12f\sqrt{}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e}{12f\sqrt{}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e}{12f\sqrt{}}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\,c}{12f\sqrt{a+a\,s}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(135) = 270.
time = 0.41, size = 292, normalized size = 2.16

$$\frac{\csc^9\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(36\cos\left(\frac{1}{2}(e+fx)\right)-46\cos\left(\frac{3}{2}(e+fx)\right)-54\cos\left(\frac{5}{2}(e+fx)\right)-36\sin\left(\frac{1}{2}(e+fx)\right)-63\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+63\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-46\sin\left(\frac{3}{2}(e+fx)\right)+54\sin\left(\frac{5}{2}(e+fx)\right)+21\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))-21\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))\right)}{24f\left(\csc^2\left(\frac{1}{4}(e+fx)\right)-\sec^2\left(\frac{1}{4}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(36*Cos[(e + f*x)
/2] - 46*Cos[(3*(e + f*x))/2] - 54*Cos[(5*(e + f*x))/2] - 36*Sin[(e + f*x)/
2] - 63*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 63*Log[
1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 46*Sin[(3*(e + f*x)
)/2] + 54*Sin[(5*(e + f*x))/2] + 21*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]]*Sin[3*(e + f*x)] - 21*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Si
n[3*(e + f*x)]))/(24*f*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3*Sqrt[a*(
1 + Sin[e + f*x])])
```

Maple [A]
time = 2.27, size = 144, normalized size = 1.07

method	result
default	$\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-21\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}}{\sqrt{a}}\right)(\sin^3(fx+e))a^3+27(-a(\sin(}{24\sin(fx+e)^3 a^{\frac{7}{2}}\cos(fx+e)\sqrt{a+a\sin(fx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-21*arctanh((-a*(sin(f*x+e)-
1))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+27*(-a*(sin(f*x+e)-1))^(5/2)*a^(1/2)-56
*(-a*(sin(f*x+e)-1))^(3/2)*a^(3/2)+21*(-a*(sin(f*x+e)-1))^(1/2)*a^(5/2))/si
n(f*x+e)^3/a^(7/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(125) = 250.
time = 0.38, size = 404, normalized size = 2.99

$$\frac{21\left(\cos(fx+e)^4-2\cos(fx+e)^2-\left(\cos(fx+e)^3+\cos(fx+e)^2-\cos(fx+e)-1\right)\sin(fx+e)+1\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7a\cos(fx+e)^2-4\left(\cos(fx+e)^2+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin(fx+e)+a}\sqrt{a}-9a\cos(fx+e)+\left(a\cos(fx+e)^2+8a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^3+\cos(fx+e)^2+\left(\cos(fx+e)^2-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)-4\left(27\cos(fx+e)^3+25\cos(fx+e)^2-\left(27\cos(fx+e)^2+2\cos(fx+e)-17\right)\sin(fx+e)-19\cos(fx+e)-17\right)\sqrt{a\sin(fx+e)+a}}{96\left(af\cos(fx+e)^4-2af\cos(fx+e)^2+af-\left(af\cos(fx+e)^3+af\cos(fx+e)^2-af\cos(fx+e)-af\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(21*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e
)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7
*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2
*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a
*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 +
 cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) -
4*(27*cos(f*x + e)^3 + 25*cos(f*x + e)^2 - (27*cos(f*x + e)^2 + 2*cos(f*x +
 e) - 17)*sin(f*x + e) - 19*cos(f*x + e) - 17)*sqrt(a*sin(f*x + e) + a))/(a
*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f - (a*f*cos(f*x + e)^3 + a*f*
cos(f*x + e)^2 - a*f*cos(f*x + e) - a*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^4}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/2), x)
```

3.107 $\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{256\sqrt{2}\,a^{3/2}f}+\frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}}-\frac{\sec(e+fx)(65+87\sin(e+fx))}{192f(a+a\sin(e+fx))^{3/2}}+\frac{a\,\mathrm{s}}{12}$$

```
[Out] 7/256*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/192*sec(f*x+e)*(65+87*sin(f*x+e
))/f/(a+a*sin(f*x+e))^(3/2)+7/512*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a
+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/12*a*sin(f*x+e)*tan(f*x+e)/f/(a+a
*sin(f*x+e))^(5/2)+1/3*tan(f*x+e)^3/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A]
time = 0.80, antiderivative size = 195, normalized size of antiderivative = 1.10, number of steps used = 20, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2793, 2729, 2728, 212, 4486, 2760, 2766, 2956, 2934}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{256\sqrt{2}\,a^{3/2}f}+\frac{7\cos(e+fx)}{256f(a\sin(e+fx)+a)^{3/2}}+\frac{\sec^3(e+fx)}{4af\sqrt{a\sin(e+fx)+a}}-\frac{\sec^3(e+fx)}{6f(a\sin(e+fx)+a)^{3/2}}-\frac{45\sec(e+fx)}{64af\sqrt{a\sin(e+fx)+a}}+\frac{9\sec(e+fx)}{32f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(256
*Sqrt[2]*a^(3/2)*f) + (7*Cos[e + f*x])/(256*f*(a + a*Sin[e + f*x])^(3/2)) +
 (9*Sec[e + f*x])/(32*f*(a + a*Sin[e + f*x])^(3/2)) - Sec[e + f*x]^3/(6*f*(
a + a*Sin[e + f*x])^(3/2)) - (45*Sec[e + f*x])/(64*a*f*Sqrt[a + a*Sin[e + f
*x]]) + Sec[e + f*x]^3/(4*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2793

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2956

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(
p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1))), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx - \int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a} - \int \left(\frac{\sec^4(e \cdots}{(a(1+\sin(e \cdots}\right. \cdots$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + 2\int \frac{\sec^2(e+fx)\tan^2(e+fx)}{(a(1+\sin(e+fx)))^{3/2}}\,dx - \frac{\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\mathrm{s}\cdots}{6f(a+\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\mathrm{s}\cdots}{6f(a+\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{9\cdots}{32f(a\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{9\cdots}{32f(a\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{\cdots}{32f(\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{\cdots}{32f(\cdots}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{256\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{\cdots}{32f(\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 334, normalized size = 1.89

$$\frac{124+\frac{64\sin\left(\frac{1}{2}(e+fx)\right)}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-\frac{32}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}-\frac{248\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}+342\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-171\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-(21+21i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+\frac{32\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-\frac{192\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)}}{768f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (124 + (64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 32/(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (248*Sin[(e + f*x)/2])/(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) + 342*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]) - 171*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (21 + 21*I)*(-
1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + (32*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3
)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (192*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^3)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(768*f*(a*(1 + Sin[e
+ f*x]))^(3/2))
```

Maple [A]
time = 2.43, size = 289, normalized size = 1.63

method	result
default	$-\frac{\left(-1080 a^{\frac{9}{2}}-21(a-a\sin(fx+e))^{\frac{3}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{3}\right)\sin(fx+e)\left(\cos^{2}(fx+e)\right)+\left(384 a^{\frac{9}{2}}+8\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1536/a^(11/2)*((-1080*a^(9/2)-21*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)*cos(f*x+e)^2+(3
84*a^(9/2)+84*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)+42*a^(9/2)*cos(f*x+e)^4+(-648*a^(9/2)
-63*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^3)*cos(f*x+e)^2+128*a^(9/2)+84*(a-a*sin(f*x+e))^(3/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(sin(f*x+e)-1)/(1
+sin(f*x+e))^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.41, size = 294, normalized size = 1.66

$$\frac{21\sqrt{2}\left(\cos\left(fx+e\right)^{5}-2\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-2\cos\left(fx+e\right)^{3}\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^{2}+2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos(fx+e)-\sin(fx+e)+1\right)+3a\cos(fx+e)-\left(a\cos(fx+e)-2a\right)\sin(fx+e)+2a}{\cos(fx+e)^{2}-\left(\cos(fx+e)+2\right)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(21\cos\left(fx+e\right)^{4}-324\cos\left(fx+e\right)^{2}-12\left(45\cos\left(fx+e\right)^{2}-16\right)\sin\left(fx+e\right)+64\right)\sqrt{a\sin\left(fx+e\right)+a}}{3072\left(a^{2}f\cos\left(fx+e\right)^{5}-2a^{2}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-2a^{2}f\cos\left(fx+e\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(21*sqrt(2)*(cos(f*x + e)^5 - 2*cos(f*x + e)^3*sin(f*x + e) - 2*cos(
f*x + e)^3)*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e)
+ a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(
f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*si
n(f*x + e) - cos(f*x + e) - 2)) - 4*(21*cos(f*x + e)^4 - 324*cos(f*x + e)^2
 - 12*(45*cos(f*x + e)^2 - 16)*sin(f*x + e) + 64)*sqrt(a*sin(f*x + e) + a))
/(a^2*f*cos(f*x + e)^5 - 2*a^2*f*cos(f*x + e)^3*sin(f*x + e) - 2*a^2*f*cos(
f*x + e)^3)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [A]

time = 46.83, size = 231, normalized size = 1.31

$$\frac{\frac{21\sqrt{2}\log\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{21\sqrt{2}\log\left(-\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{2\sqrt{2}\left(21\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{8}-312\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6}+507\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-240\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+16\sqrt{a}\right)}{\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)^{3}a^{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{3072f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3072*(21*sqrt(2)*log(sin(3/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^(3/2)*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))) - 21*sqrt(2)*log(-sin(3/4*pi + 1/2*f*x + 1/2*e
) + 1)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*sqrt(2)*(21*sqrt(a
)*sin(3/4*pi + 1/2*f*x + 1/2*e)^8 - 312*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*
e)^6 + 507*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^4 - 240*sqrt(a)*sin(3/4*pi
 + 1/2*f*x + 1/2*e)^2 + 16*sqrt(a))/((sin(3/4*pi + 1/2*f*x + 1/2*e)^3 - sin
(3/4*pi + 1/2*f*x + 1/2*e))^3*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e + f\,x)^4}{(a + a\,\sin(e + f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(3/2), x)
```

3.108 $\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{32\sqrt{2}\,a^{3/2}f}+\frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}}-\frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}}+\frac{5\sec(e+\ldots}{8af\sqrt{a+a\sin(\ldots}}$$

```
[Out] 1/32*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*sec(f*x+e)/f/(a+a*sin(f*x+e))^
(3/2)+1/64*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a
^(3/2)/f*2^(1/2)+5/8*sec(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2791, 2934, 2729, 2728, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{32\sqrt{2}\,a^{3/2}f}+\frac{\cos(e+fx)}{32f(a\sin(e+fx)+a)^{3/2}}+\frac{5\sec(e+fx)}{8af\sqrt{a\sin(e+fx)+a}}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(32*Sqrt
[2]*a^(3/2)*f) + Cos[e + f*x]/(32*f*(a + a*Sin[e + f*x])^(3/2)) - Sec[e + f
*x]/(4*f*(a + a*Sin[e + f*x])^(3/2)) + (5*Sec[e + f*x])/(8*a*f*Sqrt[a + a*S
in[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[b*((a + b*Sin[e + f*x])^m/(a*f*(2*m - 1)*Cos[e + f*x])),
x] - Dist[1/(a^2*(2*m - 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*((a*m - b*(2*
m - 1)*Sin[e + f*x])/Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\sec^2(e+fx)\left(-\frac{3a}{2}+4a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a^2}$$

$$= -\frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} - \frac{1}{16}\int \frac{1}{(a+a\sin(e$$

$$= \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+}}$$

$$= \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{32\sqrt{2}\,a^{3/2}f} + \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\mathrm{s}}{4f(a+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 128, normalized size = 0.96

$$-\frac{\sec(e+fx)\left(-25-\cos(2(e+fx))+(2+2i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-40\sin(e+fx)\right)}{64f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/64*(Sec[e + f*x]*(-25 - Cos[2*(e + f*x)] + (2 + 2*I)*(-1)^(3/4)*ArcTanh[
(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 40*Sin[e + f*x]))/(f*(
a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [A]
time = 2.05, size = 202, normalized size = 1.51

method	result
default	$\frac{\sin(fx+e)\left(2\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}+40a^{\frac{5}{2}}\right)+\left(-\sqrt{a-a\sin\left(f\right.}\right.}{64a^{\frac{7}{2}}\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/a^(7/2)*(sin(f*x+e)*(2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+40*a^(5/2))+(-(a-a*sin(f*x+e))^(1/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+2*a^(5/2)
)*cos(f*x+e)^2+2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*a^2+24*a^(5/2))/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin
(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(119) = 238$.
time = 0.36, size = 259, normalized size = 1.93

$$\frac{\sqrt{2}\left(\cos\left(fx+e\right)^{3}-2\,\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,\cos\left(fx+e\right)\right)\sqrt{a}\,\log\left(-\frac{a\cos(fx+e)^{2}+2\,\sqrt{2}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{a}\,(\cos(fx+e)-\sin(fx+e)+1)+3\,a\cos(fx+e)-(a\cos(fx+e)-2\,a)\sin(fx+e)+2\,a}{\cos(fx+e)^{2}-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(\cos\left(fx+e\right)^{2}+20\,\sin\left(fx+e\right)+12\right)\sqrt{a\sin\left(fx+e\right)+a}}{128\left(a^{2}f\cos\left(fx+e\right)^{3}-2\,a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*(cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x +
e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqr
t(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e)
 - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) - 4*(cos(f*x + e)^2 + 20*sin(f*x + e) + 12)*sqrt(a*
sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^3 - 2*a^2*f*cos(f*x + e)*sin(f*x + e
) - 2*a^2*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [A]
time = 12.39, size = 127, normalized size = 0.95

$$\frac{\frac{8\sqrt{2}}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}-\frac{\sqrt{2}\left(9\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-7\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(8*sqrt(2)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi + 1
/2*f*x + 1/2*e)) - sqrt(2)*(9*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^3 - 7*s
qrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e))/((sin(3/4*pi + 1/2*f*x + 1/2*e)^2 - 1
)^2*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^2}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(3/2), x)
```

3.109 $\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 3*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f-2*arctanh(1/
2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)-cot(
f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2794, 3064, 2728, 212, 2852}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(a^(3/2)*f) -
(2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(a^(3/2)*f) - Cot[e + f*x]/(a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2794

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Di
st[1/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*((b*m - a*(m + 1)*Sin[e + f*x])/
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
```

```
egerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}+\frac{\int \frac{\csc(e+fx)\left(-\frac{3a}{2}+\frac{1}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}\\ &= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}-\frac{3\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\,dx}{2a^2}+\frac{2\int -}{\sqrt{}}\\ &= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}+\frac{3\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{af}-\\ &= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right.}{a^{3/2}f}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.51, size = 206, normalized size = 1.82

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left((16+16i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)-\cot\left(\frac{1}{4}(e+fx)\right)+2\left(3\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-3\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sec\left(\frac{1}{2}(e+fx)\right)+\csc(e+fx)\sin^2\left(\frac{1}{4}(e+fx)\right)-\csc(e+fx)\sin\left(\frac{1}{4}(e+fx)\right)\sin\left(\frac{3}{4}(e+fx)\right)\right)\right)}{4f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*((16 + 16*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - Cot[(e + f*x)/4] + 2*(3*Log
[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 3*Log[1 - Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]] + Sec[(e + f*x)/2] + Csc[e + f*x]*Sin[(e + f*x)/4]^2 - Csc[
e + f*x]*Sin[(e + f*x)/4]*Sin[(3*(e + f*x))/4])))/(4*f*(a*(1 + Sin[e + f*x]
))^(3/2))
```

Maple [A]
time = 2.47, size = 135, normalized size = 1.19

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(-\sin(fx+e)a^{2}\left(-2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)+3\,\mathrm{ar}\right.}{a^{\frac{7}{2}}\sin(fx+e)\cos(fx+e)\sqrt{a+a\sin\left(fx+e\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^(7/2)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-sin(f*x+e)*a^2*(-2*2^
(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))+3*arctanh((a-a*si
n(f*x+e))^(1/2)/a^(1/2)))+(a-a*sin(f*x+e))^(1/2)*a^(3/2))/sin(f*x+e)/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(102) = 204.
time = 0.39, size = 460, normalized size = 4.07

$$\frac{3\left(\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)-1\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^{3}-7a\cos(fx+e)^{2}+4\left(\cos(fx+e)^{2}+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{a}-9a\cos(fx+e)+\left(a\cos(fx+e)^{2}+8a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^{3}+\cos(fx+e)^{2}+\left(\cos(fx+e)^{2}-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)+\frac{4\sqrt{2}\left(a\cos(fx+e)^{2}-(a\cos(fx+e)+a)\sin(fx+e)-a\right)\log\left(-\frac{\cos(fx+e)^{2}-(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\,(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^{2}-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}+4\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{4\left(a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f-\left(a^{2}f\cos\left(fx+e\right)+a^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((
a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) +
 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9
```

```
*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) -
a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - c
os(f*x + e) - 1)) + 4*sqrt(2)*(a*cos(f*x + e)^2 - (a*cos(f*x + e) + a)*sin(
f*x + e) - a)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sq
rt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) +
3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos
(f*x + e) - 2))/sqrt(a) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*
x + e) + 1))/(a^2*f*cos(f*x + e)^2 - a^2*f - (a^2*f*cos(f*x + e) + a^2*f)*s
in(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(102) = 204.
time = 5.31, size = 215, normalized size = 1.90

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{3\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{4\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(a)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x
 + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))) + 4*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a
^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*log(-sin(-1/4*pi + 1/2*f*x + 1/
2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*sin(-1/4*pi + 1/2*f
*x + 1/2*e)/((2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^2}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(3/2),x)
[Out] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(3/2), x)
```

3.110 $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{3/2}f}-\frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}}+\frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}}-\frac{\cot(e+fx)\csc$$

```
[Out] -1/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f-1/8*cot(f
*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)+11/12*cot(f*x+e)*csc(f*x+e)/a/f/(a+a*sin(f
*x+e))^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A]
time = 0.36, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2796, 2851, 2852, 212, 3123, 3059}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{3/2}f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3a^2f}-\frac{\cot(e+fx)}{8af\sqrt{a\sin(e+fx)+a}}+\frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/8*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(a^(3/2)*f) -
 Cot[e + f*x]/(8*a*f*Sqrt[a + a*Sin[e + f*x]]) + (11*Cot[e + f*x]*Csc[e + f
*x])/(12*a*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[
a + a*Sin[e + f*x]])/(3*a^2*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2796

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
, x], x] + Dist[1/a^2, Int[(a + b*Sin[e + f*x])^(m + 2)*((1 + Sin[e + f*x]^
2)/Sin[e + f*x]^4), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
```

```
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx &= \frac{\int \csc^4(e+fx)\sqrt{a+a\sin(e+fx)}\,(1+\sin^2(e+fx))\,dx}{a^2} - \frac{2\int \csc^3(e+fx)}{} \\ &= \frac{\cot(e+fx)\csc(e+fx)}{af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3a^2f} + \\ &= \frac{3\cot(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e}{} \\ &= -\frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2}{} \\ &= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+}{12af\sqrt{a+}} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{3/2}f} - \frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+}{12af\sqrt{a+}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 294 vs. 2(144) = 288.
time = 0.52, size = 294, normalized size = 2.04

$$\frac{\csc^9\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(-132\cos\left(\frac{1}{2}(e+fx)\right)+62\cos\left(\frac{3}{2}(e+fx)\right)+6\cos\left(\frac{5}{2}(e+fx)\right)+132\sin\left(\frac{1}{2}(e+fx)\right)-9\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+9\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+62\sin\left(\frac{3}{2}(e+fx)\right)-6\sin\left(\frac{5}{2}(e+fx)\right)+3\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))-3\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))\right)}{24f\left(\csc^2\left(\frac{1}{4}(e+fx)\right)-\sec^2\left(\frac{1}{4}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Csc[(e + f*x)/2]^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-132*Cos[(e +
f*x)/2] + 62*Cos[(3*(e + f*x))/2] + 6*Cos[(5*(e + f*x))/2] + 132*Sin[(e + f
*x)/2] - 9*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 9*Lo
g[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 62*Sin[(3*(e + f*
x))/2] - 6*Sin[(5*(e + f*x))/2] + 3*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]]*Sin[3*(e + f*x)] - 3*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin
[3*(e + f*x)]))/(24*f*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3*(a*(1 + S
in[e + f*x]))^(3/2))
```

Maple [A]
time = 2.24, size = 144, normalized size = 1.00

method	result

default	$-\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(3(-a(\sin(fx+e)-1))^{\frac{5}{2}}a^{\frac{3}{2}}+3\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}}{\sqrt{a}}\right)a^{4}(\right.}{24a^{\frac{11}{2}}\sin(fx+e)^{3}\cos(fx+e)\sqrt{a+a\sin\left(fx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/a^(11/2)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*(-a*(sin(f*x+e)-
1))^(5/2)*a^(3/2)+3*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*a^4*sin(f*x+
e)^3+8*(-a*(sin(f*x+e)-1))^(3/2)*a^(5/2)-3*(-a*(sin(f*x+e)-1))^(1/2)*a^(7/2
))/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(134) = 268$.
time = 0.37, size = 418, normalized size = 2.90

$$\frac{3\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^{3}-7a\cos(fx+e)^{2}-4\left(\cos(fx+e)^{2}+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}-9a\cos(fx+e)+\left(a\cos(fx+e)^{2}+8a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^{3}+\cos(fx+e)^{2}+\left(\cos(fx+e)^{2}-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)+4\left(3\cos\left(fx+e\right)^{3}+17\cos\left(fx+e\right)^{2}-\left(3\cos\left(fx+e\right)^{2}-14\cos\left(fx+e\right)-25\right)\sin\left(fx+e\right)-11\cos\left(fx+e\right)-25\right)\sqrt{a\sin\left(fx+e\right)+a}}{96\left(a^{2}f\cos\left(fx+e\right)^{4}-2a^{2}f\cos\left(fx+e\right)^{2}+a^{2}f-\left(a^{2}f\cos\left(fx+e\right)^{3}+a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f\cos\left(fx+e\right)-a^{2}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e)
^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*
a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*
cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*
cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 +
cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 4
*(3*cos(f*x + e)^3 + 17*cos(f*x + e)^2 - (3*cos(f*x + e)^2 - 14*cos(f*x + e
) - 25)*sin(f*x + e) - 11*cos(f*x + e) - 25)*sqrt(a*sin(f*x + e) + a))/(a^2
*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f - (a^2*f*cos(f*x + e)^3
+ a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - a^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^{4}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [A]
time = 9.80, size = 176, normalized size = 1.22

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{3\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{4\left(12\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+16\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*sqrt(a)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f
*x + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) + 4*(12*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 1
6*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 3*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((2*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^4}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(3/2), x)
```

3.111 $\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=207

$$\frac{317\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{4096\sqrt{2}\,a^{5/2}f}+\frac{317\cos(e+fx)}{3072f(a+a\sin(e+fx))^{5/2}}-\frac{\sec(e+fx)(115+129\sin(e+fx))}{384f(a+a\sin(e+fx))^{5/2}}$$

```
[Out] 317/3072*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/384*sec(f*x+e)*(115+129*sin(
f*x+e))/f/(a+a*sin(f*x+e))^(5/2)+317/4096*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(
3/2)+317/8192*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)
)/a^(5/2)/f*2^(1/2)+5/48*a*sin(f*x+e)*tan(f*x+e)/f/(a+a*sin(f*x+e))^(7/2)+1
/3*tan(f*x+e)^3/f/(a+a*sin(f*x+e))^(5/2)
```

Rubi [A]
time = 0.97, antiderivative size = 260, normalized size of antiderivative = 1.26, number of steps used = 23, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2793, 2729, 2728, 212, 4486, 2760, 2766, 2956, 2938}

$$\frac{317\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{4096\sqrt{2}\,a^{5/2}f}-\frac{31\sec^3(e+fx)}{192a^2f\sqrt{a\sin(e+fx)+a}}-\frac{1085\sec(e+fx)}{3072a^2f\sqrt{a\sin(e+fx)+a}}+\frac{317\cos(e+fx)}{4096af(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}+\frac{53\sec^3(e+fx)}{96af(a\sin(e+fx)+a)^{3/2}}-\frac{\sec^3(e+fx)}{8f(a\sin(e+fx)+a)^{5/2}}+\frac{217\sec(e+fx)}{1536af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (317*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4
096*Sqrt[2]*a^(5/2)*f) - Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)) - Se
c[e + f*x]^3/(8*f*(a + a*Sin[e + f*x])^(5/2)) + (317*Cos[e + f*x])/(4096*a*
f*(a + a*Sin[e + f*x])^(3/2)) + (217*Sec[e + f*x])/(1536*a*f*(a + a*Sin[e +
 f*x])^(3/2)) + (53*Sec[e + f*x]^3)/(96*a*f*(a + a*Sin[e + f*x])^(3/2)) - (
1085*Sec[e + f*x])/(3072*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (31*Sec[e + f*x]
^3)/(192*a^2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2793

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2956

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(
p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1))), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
```

```
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{1}{(a+a\sin(e+fx))^{5/2}}\,dx - \int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{(a+a\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{3\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a} - \int \Bigg(\frac{\sec^4(e+fx}{(a(1+\sin(e+f}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + 2\int \frac{\sec^2(e+fx}{(a(1+\sin}$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx}{16af(a+a\sin(e+}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{}{8f(a}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{}{8f(a}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{}{8f(a}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{}{8f(a}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{}{8f(a}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{}{8f(a}$$

$$= \frac{317\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{4096\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{}{8f(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.37, size = 394, normalized size = 1.90

$$\frac{1312 + \frac{768\sin\left(\frac{1}{2}(e+fx)\right)}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} - \frac{384}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2} - \frac{2624\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)} + 2584\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right) - 1292\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2 + 402\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3 - 201\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4 - (951+951i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5 + \frac{256\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} - \frac{1152\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)}}{12288f(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]

[Out] (1312 + (768*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 384/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (2624*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2584*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 1292*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 402*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 201*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (951 + 951*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + (256*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (1152*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(12288*f*(a*(1 + Sin[e + f*x]))^(5/2))

Maple [A]
time = 2.74, size = 353, normalized size = 1.71

method	result
default	$-\frac{1902 a^{\frac{11}{2}} \sin(fx+e)(\cos^4(fx+e))+\left(-13888 a^{\frac{11}{2}}-3804(a-a\sin(fx+e))^{\frac{3}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/24576/a^(15/2)*(1902*a^(11/2)*sin(f*x+e)*cos(f*x+e)^4+(-13888*a^(11/2)-3804*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(f*x+e)^2*sin(f*x+e)+(5632*a^(11/2)+7608*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)*sin(f*x+e)+(4438*a^(11/2)+951*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(f*x+e)^4+(-9920*a^(11/2)-7608*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(f*x+e)^2+2560*a^(11/2)+7608*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)/(sin(f*x+e)-1)/(1+sin(f*x+e))^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [A]
time = 0.39, size = 334, normalized size = 1.61

$$\frac{951\,\sqrt{2}\left(3\,\cos\left(fx+e\right)^{5}-4\,\cos\left(fx+e\right)^{3}+\left(\cos\left(fx+e\right)^{5}-4\,\cos\left(fx+e\right)^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a}\,\log\left(-\frac{a\cos(fx+e)^{2}+2\,\sqrt{2}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{a}\,(\cos(fx+e)-\sin(fx+e)+1)+3\,a\cos(fx+e)-(a\cos(fx+e)-2\,a)\sin(fx+e)+2\,a}{\cos(fx+e)^{2}-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)-4\left(2219\,\cos\left(fx+e\right)^{4}-4960\,\cos\left(fx+e\right)^{2}+\left(951\,\cos\left(fx+e\right)^{4}-6944\,\cos\left(fx+e\right)^{2}+2816\right)\sin\left(fx+e\right)+1280\right)\sqrt{a\sin\left(fx+e\right)+a}}{49152\left(3\,a^{3}f\cos\left(fx+e\right)^{5}-4\,a^{3}f\cos\left(fx+e\right)^{3}+\left(a^{3}f\cos\left(fx+e\right)^{5}-4\,a^{3}f\cos\left(fx+e\right)^{3}\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/49152*(951*sqrt(2)*(3*cos(f*x + e)^5 - 4*cos(f*x + e)^3 + (cos(f*x + e)^5
 - 4*cos(f*x + e)^3)*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(
2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a
*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2
- (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(2219*cos(f*x +
e)^4 - 4960*cos(f*x + e)^2 + (951*cos(f*x + e)^4 - 6944*cos(f*x + e)^2 + 28
16)*sin(f*x + e) + 1280)*sqrt(a*sin(f*x + e) + a))/(3*a^3*f*cos(f*x + e)^5
- 4*a^3*f*cos(f*x + e)^3 + (a^3*f*cos(f*x + e)^5 - 4*a^3*f*cos(f*x + e)^3)*
sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(5/2), x)
```

Giac [A]
time = 39.98, size = 186, normalized size = 0.90

$$-\frac{\frac{128\,\sqrt{2}\left(9\,\sin\left(\frac{3}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(\frac{3}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}}-\frac{\sqrt{2}\left(201\,\sqrt{a}\,\sin\left(\frac{3}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}-1249\,\sqrt{a}\,\sin\left(\frac{3}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+1567\,\sqrt{a}\,\sin\left(\frac{3}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-567\,\sqrt{a}\,\sin\left(\frac{3}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\sin\left(\frac{3}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{4}a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}}{24576\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*(128*sqrt(2)*(9*sin(3/4*pi + 1/2*f*x + 1/2*e)^2 - 1)/(a^(5/2)*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi + 1/2*f*x + 1/2*e)^3) - sqrt(2)*
(201*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^7 - 1249*sqrt(a)*sin(3/4*pi + 1/
2*f*x + 1/2*e)^5 + 1567*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^3 - 567*sqrt(
a)*sin(3/4*pi + 1/2*f*x + 1/2*e))/((sin(3/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^4*
a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(e+f\,x)^4}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(5/2),x)
[Out] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(5/2), x)
```

3.112 $\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{128\sqrt{2}\,a^{5/2}f}-\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}}-\frac{11\cos(e+fx)}{128af(a+a\sin(e+fx))^{3/2}}+\frac{1\cdots}{48af(a\cdots}$$

```
[Out] -1/6*sec(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-11/128*cos(f*x+e)/a/f/(a+a*sin(f*x
+e))^(3/2)+17/48*sec(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-11/256*arctanh(1/2*c
os(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+11/96*s
ec(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2791, 2938, 2766, 2729, 2728, 212}

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{128\sqrt{2}\,a^{5/2}f}+\frac{11\sec(e+fx)}{96a^2f\sqrt{a\sin(e+fx)+a}}-\frac{11\cos(e+fx)}{128af(a\sin(e+fx)+a)^{3/2}}+\frac{17\sec(e+fx)}{48af(a\sin(e+fx)+a)^{3/2}}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(1
28*Sqrt[2]*a^(5/2)*f) - Sec[e + f*x]/(6*f*(a + a*Sin[e + f*x])^(5/2)) - (11
*Cos[e + f*x])/(128*a*f*(a + a*Sin[e + f*x])^(3/2)) + (17*Sec[e + f*x])/(48
*a*f*(a + a*Sin[e + f*x])^(3/2)) + (11*Sec[e + f*x])/(96*a^2*f*Sqrt[a + a*S
in[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[b*((a + b*Sin[e + f*x])^m/(a*f*(2*m - 1)*Cos[e + f*x])),
x] - Dist[1/(a^2*(2*m - 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*((a*m - b*(2*
m - 1)*Sin[e + f*x])/Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{\sec^2(e+fx)\left(-\frac{5a}{2}+6a\sin(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx}{6a^2}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} + \frac{17\sec(e+fx)}{48af(a+a\sin(e+fx))^{3/2}} + \frac{11\int \frac{\sec^2(e\cdots}{\sqrt{a+a\sin\cdots}}}{96a^2\cdots}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} + \frac{17\sec(e+fx)}{48af(a+a\sin(e+fx))^{3/2}} + \frac{11\sec(e+\cdots}{96a^2f\sqrt{a+a\sin\cdots}}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} - \frac{11\cos(e+fx)}{128af(a+a\sin(e+fx))^{3/2}} + \frac{17\sec(e\cdots}{48af(a+a\sin\cdots}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} - \frac{11\cos(e+fx)}{128af(a+a\sin(e+fx))^{3/2}} + \frac{17\sec(e\cdots}{48af(a+a\sin\cdots}$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{128\sqrt{2}\,a^{5/2}f} - \frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} - \frac{\cdots}{128\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.27, size = 284, normalized size = 1.70

$$\frac{-32+\frac{64\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}-104\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+52\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-30\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+15\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4+(33+33i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+\frac{48\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)}}{384f(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-32 + (64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 104*Si
n[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 52*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^2 - 30*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^3 + 15*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (33 + 33*I)*(-1)^(
3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^5 + (48*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(C
os[(e + f*x)/2] - Sin[(e + f*x)/2]))/(384*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A]
time = 2.65, size = 266, normalized size = 1.59

method	result

default	$-\frac{\left(66a^{\frac{7}{2}}-33\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{3}\right)\sin(fx+e)\left(\cos^{2}(fx+e)\right)+\left(-4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/768/a^(11/2)*((66*a^(7/2)-33*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)*cos(f*x+e)^2+(-448*
a^(7/2)+132*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)+(154*a^(7/2)-99*(a-a*sin(f*x+e))^(1/2)*
2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*cos(f*x+e)
^2-320*a^(7/2)+132*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+
e))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(1+sin(f*x+e))^2/cos(f*x+e)/(a+a*sin(f*x+e)
)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(150) = 300$.
time = 0.37, size = 304, normalized size = 1.82

$$\frac{33\sqrt{2}\left(3\cos\left(fx+e\right)^{3}+\left(\cos\left(fx+e\right)^{3}-4\cos\left(fx+e\right)\right)\sin\left(fx+e\right)-4\cos\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^{2}-2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{a}\,(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(fx+e)+2a}{\cos(fx+e)^{2}-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)+4\left(77\cos\left(fx+e\right)^{2}+\left(33\cos\left(fx+e\right)^{2}-224\right)\sin\left(fx+e\right)-160\right)\sqrt{a\sin\left(fx+e\right)+a}}{1536\left(3a^{3}f\cos\left(fx+e\right)^{3}-4a^{3}f\cos\left(fx+e\right)+\left(a^{3}f\cos\left(fx+e\right)^{3}-4a^{3}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(33*sqrt(2)*(3*cos(f*x + e)^3 + (cos(f*x + e)^3 - 4*cos(f*x + e))*si
n(f*x + e) - 4*cos(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqr
t(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f
*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos
(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(77*cos(f*x + e)^2 + (
33*cos(f*x + e)^2 - 224)*sin(f*x + e) - 160)*sqrt(a*sin(f*x + e) + a))/(3*a
^3*f*cos(f*x + e)^3 - 4*a^3*f*cos(f*x + e) + (a^3*f*cos(f*x + e)^3 - 4*a^3*
f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(5/2), x)
```

Giac [A]
time = 23.15, size = 147, normalized size = 0.88

$$\frac{\frac{48\sqrt{2}}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}-\frac{\sqrt{2}\left(15\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^5-56\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3+33\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{768f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/768*(48*sqrt(2)/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi +
 1/2*f*x + 1/2*e)) - sqrt(2)*(15*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^5 -
56*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^3 + 33*sqrt(a)*sin(3/4*pi + 1/2*f*
x + 1/2*e))/((sin(3/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3*a^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^2}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(5/2), x)
```

3.113 $\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=141

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\,a^{5/2}f}-\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}}-\frac{}{af(a}$$

[Out] 5*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f-2*cos(f*x+e)
/a/f/(a+a*sin(f*x+e))^(3/2)-cot(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-7/2*arcta
nh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)

Rubi [A]
time = 0.23, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$,
Rules used = {2794, 3057, 3064, 2728, 212, 2852}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}\,a^{5/2}f}-\frac{2\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]

[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(a^(5/2)*f) -
(7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqr
t[2]*a^(5/2)*f) - (2*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)) - Cot[e
 + f*x]/(a*f*(a + a*Sin[e + f*x])^(3/2))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2794

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Di
st[1/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*((b*m - a*(m + 1)*Sin[e + f*x])/
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\csc(e+fx)\left(-\frac{5a}{2}+\frac{3}{2}a\sin(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx}{a^2}$$

$$= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\csc(e+fx)\left(-5a^2+2a^2 \sin\ldots\right)}{\sqrt{a+a\sin(e+\ldots}}}{2a^4}$$

$$= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{5\int \csc(e+fx)\sqrt{a\ldots}}{2a\ldots}$$

$$= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{5\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, \ldots\right.}{\ldots}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}\,a^{5/2}f} - \ldots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.50, size = 451, normalized size = 3.20

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(8*Sin[(e + f*x)/2] - 4*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) + 2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (
28 + 28*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4]
)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - Cot[(e + f*x)/4]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^2 + 10*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 10*Log[1 - Cos[(e + f*x)/2] + S
in[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (2*Sin[(e + f*x)
/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/4] - Sin[(e + f
*x)/4]) - (2*Sin[(e + f*x)/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos
[(e + f*x)/4] + Sin[(e + f*x)/4]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2
*Tan[(e + f*x)/4]))/(4*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A]
time = 2.23, size = 219, normalized size = 1.55

method	result

default	$-\frac{\left(7\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\sqrt{2}}{2\sqrt{a}}\right)\left(\sin^{2}\left(fx+e\right)\right)a-10\,\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}}{\sqrt{a}}\right)\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^(7/2)*(7*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/
2))*sin(f*x+e)^2*a-10*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*sin(f*x+e)
^2*a+7*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*sin
(f*x+e)+4*(-a*(sin(f*x+e)-1))^(1/2)*a^(1/2)*sin(f*x+e)-10*arctanh((-a*(sin(
f*x+e)-1))^(1/2)/a^(1/2))*a*sin(f*x+e)+2*(-a*(sin(f*x+e)-1))^(1/2)*a^(1/2))
*(-a*(sin(f*x+e)-1))^(1/2)/sin(f*x+e)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(130) = 260$.
time = 0.39, size = 589, normalized size = 4.18

$$\frac{5\left(\cos\left(fx+e\right)^{3}+2\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2\right)\sqrt{a}\,\log\left(\frac{a\cos\left(fx+e\right)^{3}-7a\cos\left(fx+e\right)^{2}+4\left(\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+3\right)\sin\left(fx+e\right)-2\cos\left(fx+e\right)-3\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{a}-9a\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^{2}+8a\cos\left(fx+e\right)-a\right)\sin\left(fx+e\right)-a}{\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-1}\right)+\frac{7\sqrt{2}\left(a\cos\left(fx+e\right)^{3}+2a\cos\left(fx+e\right)^{2}-a\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^{2}-a\cos\left(fx+e\right)-2a\right)\sin\left(fx+e\right)-2a\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{\sqrt{a}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{\sqrt{a}}+4\left(2\cos\left(fx+e\right)^{2}+\left(2\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)-1\right)\sqrt{a\sin\left(fx+e\right)+a}}{4\left(a^{3}f\cos\left(fx+e\right)^{3}+2a^{3}f\cos\left(fx+e\right)^{2}-a^{3}f\cos\left(fx+e\right)-2a^{3}f+\left(a^{3}f\cos\left(fx+e\right)^{2}-a^{3}f\cos\left(fx+e\right)-2a^{3}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*(cos(f*x + e)^3 + 2*cos(f*x + e)^2 + (cos(f*x + e)^2 - cos(f*x + e)
- 2)*sin(f*x + e) - cos(f*x + e) - 2)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*c
os(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos
(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos
(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos
(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 7*sq
rt(2)*(a*cos(f*x + e)^3 + 2*a*cos(f*x + e)^2 - a*cos(f*x + e) + (a*cos(f*x
+ e)^2 - a*cos(f*x + e) - 2*a)*sin(f*x + e) - 2*a)*log(-(cos(f*x + e)^2 - (
cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*
x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 -
```

```
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(2*cos(f*x
 + e)^2 + (2*cos(f*x + e) + 1)*sin(f*x + e) + cos(f*x + e) - 1)*sqrt(a*sin(
f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 2*a^3*f*cos(f*x + e)^2 - a^3*f*cos(f
*x + e) - 2*a^3*f + (a^3*f*cos(f*x + e)^2 - a^3*f*cos(f*x + e) - 2*a^3*f)*s
in(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(130) = 260.
time = 9.94, size = 275, normalized size = 1.95

$$\frac{\sqrt{a}\left(\frac{7\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{7\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{10\log\left(\left|\sqrt{2}+2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{10\log\left(\left|-\sqrt{2}+2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{2\left(4\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-3\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4-3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a)*(7*sqrt(2)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))) - 7*sqrt(2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 10*log(abs(sqrt(2) + 2*
sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
+ 10*log(abs(-sqrt(2) + 2*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^3*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))) - 2*(4*sqrt(2)*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 -
 3*sqrt(2)*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((2*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^4 - 3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)*a^3*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^2}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(5/2), x)
```

3.114 $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{5/2}f}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f}-\frac{19\cot(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 45/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f-4*arctanh
(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)-1
9/8*cot(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)+13/12*cot(f*x+e)*csc(f*x+e)/a^2
/f/(a+a*sin(f*x+e))^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2/a^2/f/(a+a*sin(f*x+e)
)^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2796, 2858, 3063, 3064, 2728, 212, 2852, 3123}

$$\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{5/2}f}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f}-\frac{19\cot(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3a^2f\sqrt{a\sin(e+fx)+a}}+\frac{13\cot(e+fx)\csc(e+fx)}{12a^2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (45*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*a^(5/2)*f)
 - (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*
x]])])/(a^(5/2)*f) - (19*Cot[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]) +
 (13*Cot[e + f*x]*Csc[e + f*x])/(12*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[
e + f*x]*Csc[e + f*x]^2)/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2796

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
```

```
, x], x] + Dist[1/a^2, Int[(a + b*Sin[e + f*x])^(m + 2)*((1 + Sin[e + f*x]^
2)/Sin[e + f*x]^4), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\csc^4(e+fx)\left(1+\sin^2(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^3(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= \frac{\cot(e+fx)\csc(e+fx)}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^2f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}+\frac{11}{2}a\,\mathrm{s}\right.}{\sqrt{a+a\sin(e+}}}{3a^3}$$

$$= -\frac{\cot(e+fx)}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{13\cot(e+fx)\csc(e+fx)}{12a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc}{3a^2f\sqrt{a+a\sin}}$$

$$= -\frac{19\cot(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}} + \frac{13\cot(e+fx)\csc(e+fx)}{12a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc}{3a^2f\sqrt{a+a\sin}}$$

$$= -\frac{19\cot(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}} + \frac{13\cot(e+fx)\csc(e+fx)}{12a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc}{3a^2f\sqrt{a+a\sin}}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2a^{5/2}f} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right.}{a^{5/2}f}$$

$$= \frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{5/2}f} - \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx}}\right.}{a^{5/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.58, size = 332, normalized size = 1.74

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left((1536+1536i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)-\frac{8\csc^9\left(\frac{1}{2}(e+fx)\right)\left(396\cos\left(\frac{1}{2}(e+fx)\right)-218\cos\left(\frac{3}{2}(e+fx)\right)-114\cos\left(\frac{5}{2}(e+fx)\right)-396\sin\left(\frac{1}{2}(e+fx)\right)-405\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+405\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-218\sin\left(\frac{3}{2}(e+fx)\right)+114\sin\left(\frac{5}{2}(e+fx)\right)+135\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))-135\log\left(1-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))\right)}{\left(\csc^2\left(\frac{1}{4}(e+fx)\right)-\sec^2\left(\frac{1}{4}(e+fx)\right)\right)^3}\right)}{192f\left(a(1+\sin(e+fx))\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*((1536 + 1536*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - (8*Csc[(e + f*x)/2]^9*(396*Cos[(e + f*x)/2] - 218*Cos[(3*(e + f*x))/2] - 114*Cos[(5*(e + f*x))/2] - 396*Sin[(e + f*x)/2] - 405*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 405*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 218*Sin[(3*(e + f*x))/2] + 114*Sin[(5*(e + f*x))/2] + 135*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 135*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)]))/(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3))/(192*f*(a*(1 + Sin[e + f*x]))^(5/2))

Maple [A]
time = 2.59, size = 182, normalized size = 0.95

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\left(-135a^{5}\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}}{\sqrt{a}}\right)(\sin^{3}(fx+e))+57(-a(\sin}{24a^{\frac{15}{2}}\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/24/a^(15/2)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-135*a^5*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*sin(f*x+e)^3+57*(-a*(sin(f*x+e)-1))^(5/2)*a^(5/2)+96*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^5*sin(f*x+e)^3-88*(-a*(sin(f*x+e)-1))^(3/2)*a^(7/2)+39*(-a*(sin(f*x+e)-1))^(1/2)*a^(9/2))/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(174) = 348.
time = 0.42, size = 616, normalized size = 3.23

$$\frac{135\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^{3}-7a\cos(fx+e)^{2}+4\left(\cos(fx+e)^{2}+(\cos(fx+e)+3)\sin(fx+e)-2\cos(fx+e)-3\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}-9a\cos(fx+e)+\left(a\cos(fx+e)^{2}+8a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^{3}+\cos(fx+e)^{2}+\left(\cos(fx+e)^{2}-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)+\frac{192\sqrt{2}\left(a\cos(fx+e)^{4}-2a\cos(fx+e)^{2}-\left(a\cos(fx+e)^{3}+a\cos(fx+e)^{2}-a\cos(fx+e)-a\right)\sin(fx+e)+a\right)\log\left(-\frac{\cos(fx+e)^{2}-(\cos(fx+e)-2)\sin(fx+e)-\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^{2}-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}+4\left(57\cos\left(fx+e\right)^{3}+83\cos\left(fx+e\right)^{2}-\left(57\cos\left(fx+e\right)^{2}-26\cos\left(fx+e\right)-91\right)\sin\left(fx+e\right)-65\cos\left(fx+e\right)-91\right)\sqrt{a\sin\left(fx+e\right)+a}}{96\left(a^{3}f\cos\left(fx+e\right)^{4}-2a^{3}f\cos\left(fx+e\right)^{2}+a^{3}f-\left(a^{3}f\cos\left(fx+e\right)^{3}+a^{3}f\cos\left(fx+e\right)^{2}-a^{3}f\cos\left(fx+e\right)-a^{3}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(135*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x +
e)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 -
7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) -
2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (
a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3
+ cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) +
 192*sqrt(2)*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 - (a*cos(f*x + e)^3 + a
*cos(f*x + e)^2 - a*cos(f*x + e) - a)*sin(f*x + e) + a)*log(-(cos(f*x + e)^
2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(c
os(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)
^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(57*c
os(f*x + e)^3 + 83*cos(f*x + e)^2 - (57*cos(f*x + e)^2 - 26*cos(f*x + e) -
91)*sin(f*x + e) - 65*cos(f*x + e) - 91)*sqrt(a*sin(f*x + e) + a))/(a^3*f*c
os(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f - (a^3*f*cos(f*x + e)^3 + a^
3*f*cos(f*x + e)^2 - a^3*f*cos(f*x + e) - a^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(5/2), x)
```

Giac [A]
time = 50.44, size = 252, normalized size = 1.32

$$\frac{\sqrt{2}\,\sqrt{a}\left(\frac{135\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|}\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{192\,\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{192\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{4\left(228\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-176\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+39\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/96*sqrt(2)*sqrt(a)*(135*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*
f*x + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^3*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))) + 192*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) +
1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 192*log(-sin(-1/4*pi + 1/2*f
*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*(228*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^5 - 176*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 39*sin(-
```

```
1/4*pi + 1/2*f*x + 1/2*e))/((2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3*a^3*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^4}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(5/2),x)
[Out] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(5/2), x)
```

3.115 $\int \sqrt[3]{a+a\sin(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=982

$$-\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f}+\frac{361\sec(e+fx)(1-\sin(e+fx))\sqrt[3]{a+a\sin(e+fx)}}{63f}-\frac{\sec(e+}{42f(a-a}$$

```
[Out] -361/126*sec(f*x+e)*(a+a*sin(f*x+e))^(1/3)/f+361/63*sec(f*x+e)*(1-sin(f*x+e
))*(a+a*sin(f*x+e))^(1/3)/f-1/42*sec(f*x+e)*(65*a^2-142*a^2*sin(f*x+e))/f/(
a-a*sin(f*x+e))/(a+a*sin(f*x+e))^(2/3)+361/63*sec(f*x+e)*(1-sin(f*x+e))*(a+
a*sin(f*x+e))^(2/3)*(1+3^(1/2))/f/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(
1+3^(1/2)))-361/63*2^(1/3)*((2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1
/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1
/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1/2)))*(2^(1/3)*a^(1/3)-(a+a*sin(f
*x+e))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1
/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*sec(f*x+e)*(a+a*sin(f*x+e))^(2/3)*(2^(1/3)*a
^(1/3)-(a+a*sin(f*x+e))^(1/3))*((2^(2/3)*a^(2/3)+2^(1/3)*a^(1/3)*(a+a*sin(f
*x+e))^(1/3)+(a+a*sin(f*x+e))^(2/3))/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3
)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/f/(-(a+a*sin(f*x+e))^(1/3)*(2^(1/3)
*a^(1/3)-(a+a*sin(f*x+e))^(1/3))/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1
+3^(1/2)))^2)^(1/2)-361/378*((2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(
1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(
1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1/2)))*(2^(1/3)*a^(1/3)-(a+a*sin(
f*x+e))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(
1/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*sec(f*x+e)*(a+a*sin(f*x+e))^(2/3)*(2^(1/3)*
a^(1/3)-(a+a*sin(f*x+e))^(1/3))*(1-3^(1/2))*((2^(2/3)*a^(2/3)+2^(1/3)*a^(1/
3)*(a+a*sin(f*x+e))^(1/3)+(a+a*sin(f*x+e))^(2/3))/(2^(1/3)*a^(1/3)-(a+a*sin
(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)*3^(3/4)/a^(2/3)/f/(-(a+a*sin(f
*x+e))^(1/3)*(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3))/(2^(1/3)*a^(1/3)-(a+a
*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)+3/2*a^2*sin(f*x+e)*tan(f*x+e)/f/(a
-a*sin(f*x+e))/(a+a*sin(f*x+e))^(2/3)-3*a^2*sin(f*x+e)^2*tan(f*x+e)/f/(a-a*
sin(f*x+e))/(a+a*sin(f*x+e))^(2/3)
```

Rubi [A]

time = 0.89, antiderivative size = 982, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2798, 102, 158, 149, 53, 65, 314, 231, 1895}

$$-\frac{3\sin^2(e+fx)\tan(e+fx)a^2}{f(a-a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}}+\frac{3\sin(e+fx)\tan(e+fx)a^2}{2f(a-a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}}-\frac{\sec(e+fx)\left(65a^2-142a^2\sin(e+fx)\right)}{42f(a-a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}}-\frac{361\sec(e+fx)\sqrt[3]{\sin(e+fx)a+a}}{126f}+\frac{361\sec(e+fx)(1-\sin(e+fx))\sqrt[3]{\sin(e+fx)a+a}}{63f}+\frac{361\left(1+\sqrt{3}\right)\sec(e+fx)(1-\sin(e+fx))(\sin(e+fx)a+a)^{2/3}}{63f\left(\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\sin(e+fx)a+a}\right)}-\frac{361\sqrt[3]{2}E\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{\sin(e+fx)a+a}}{\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\sin(e+fx)a+a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sec(e+fx)(\sin(e+fx)a+a)^{2/3}\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{\sin(e+fx)a+a}\right)\sqrt{\frac{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{\sin(e+fx)a+a}\sqrt[3]{a}+(\sin(e+fx)a+a)^{2/3}}{\left(\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\sin(e+fx)a+a}\right)^2}}}{21\,3^{3/4}f\sqrt{-\frac{\sqrt[3]{\sin(e+fx)a+a}\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{\sin(e+fx)a+a}\right)}{\left(\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\sin(e+fx)a+a}\right)^2}}\,a^{2/3}}-\frac{361\left(1-\sqrt{3}\right)F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{\sin(e+fx)a+a}}{\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\sin(e+fx)a+a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sec(e+fx)(\sin(e+fx)a+a)^{2/3}\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{\sin(e+fx)a+a}\right)\sqrt{\frac{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{\sin(e+fx)a+a}\sqrt[3]{a}+(\sin(e+fx)a+a)^{2/3}}{\left(\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\sin(e+fx)a+a}\right)^2}}}{63\,2^{2/3}\sqrt[4]{3}f\sqrt{-\frac{\sqrt[3]{\sin(e+fx)a+a}\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{\sin(e+fx)a+a}\right)}{\left(\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\sin(e+fx)a+a}\right)^2}}\,a^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] (-361*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1/3))/(126*f) + (361*Sec[e + f*x]*
(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(1/3))/(63*f) - (Sec[e + f*x]*(65*a
^2 - 142*a^2*Sin[e + f*x]))/(42*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])
^(2/3)) + (361*(1 + Sqrt[3])*Sec[e + f*x]*(1 - Sin[e + f*x])*(a + a*Sin[e +
 f*x])^(2/3))/(63*f*(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(
1/3))) - (361*2^(1/3)*EllipticE[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a
+ a*Sin[e + f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*
x])^(1/3))], (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1
/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a
^(1/3)*(a + a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^
(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(21*3^(3/4)*a^(2/3)*f
*Sqrt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e + f*x])
^(1/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2)])
- (361*(1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a +
 a*Sin[e + f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x
])^(1/3))], (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1/
3)*a^(1/3) - (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^
(1/3)*(a + a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^(
1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(63*2^(2/3)*3^(1/4)*a^
(2/3)*f*Sqrt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e
+ f*x])^(1/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3)
)^2)]) + (3*a^2*Sin[e + f*x]*Tan[e + f*x])/(2*f*(a - a*Sin[e + f*x])*(a + a
*Sin[e + f*x])^(2/3)) - (3*a^2*Sin[e + f*x]^2*Tan[e + f*x])/(f*(a - a*Sin[e
 + f*x])*(a + a*Sin[e + f*x])^(2/3))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*
h*(n + 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n
 + 2)) + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*
(m + 1) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*
d)^2*(m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Dist[(a^2*
d^2*f*h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
 + n + 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n
+ 3) + d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m
+ 1)*(n + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2798

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x])), Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2
)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \sqrt[3]{a+a\sin(e+fx)}\,\tan^4(e+fx)\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\Big(}{af}$$

$$= -\frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}} - \frac{\Big(3\sec(e+fx)\sqrt{}}{}$$

$$= \frac{3a^2\sin(e+fx)\tan(e+fx)}{2f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}} - \frac{3a^2\sin^2(e}{f(a-a\sin(e+f}$$

$$= -\frac{\sec(e+fx)\left(65a^2-142a^2\sin(e+fx)\right)}{42f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}} + \frac{3a^2\sin}{2f(a-a\sin(e}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} - \frac{\sec(e+fx)\left(65a^2-\right.}{42f(a-a\sin(e+fx))}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin}{}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin}{}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin}{}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.63, size = 318, normalized size = 0.32

$$\frac{\sqrt[3]{a(1+\sin(e+fx))}\left(\frac{\left(\frac{1083}{10}+\frac{1083i}{10}\right)(-1)^{3/4}e^{-i(e+fx)}\left(20e^{i(e+fx)}\sqrt{\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-ie^{-i(e+fx)}\right)-2\left(1+ie^{-i(e+fx)}\right)^{2/3}\left(1+e^{2i(e+fx)}\right){}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)+5i\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-ie^{-i(e+fx)}\right)\sqrt{2-2\sin(e+fx)}\right)}{\sqrt{2}\left(1+ie^{-i(e+fx)}\right)^{2/3}\sqrt{ie^{-i(e+fx)}\left(-i+e^{i(e+fx)}\right)^2}}+3(361+86\sec(e+fx)-3\sec^3(e+fx)-172\tan(e+fx)+24\sec^2(e+fx)\tan(e+fx))\right)}{189f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1/3)*(((1083/10 + (1083*I)/10)*(-1)^(3/4)*(20*E^(I
*(e + f*x))*Sqrt[Cos[(2*e + Pi + 2*f*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3,
2/3, (-I)/E^(I*(e + f*x))] - 2*(1 + I/E^(I*(e + f*x)))^(2/3)*(1 + E^((2*I)*
(e + f*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*e + Pi + 2*f*x)/4]^2]
+ (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(e + f*x))]*Sqrt[2 - 2*S
in[e + f*x]]))/(Sqrt[2]*E^(I*(e + f*x))*(1 + I/E^(I*(e + f*x)))^(2/3)*Sqrt[
(I*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]) + 3*(361 + 86*Sec[e + f*x] -
 3*Sec[e + f*x]^3 - 172*Tan[e + f*x] + 24*Sec[e + f*x]^2*Tan[e + f*x])))/(1
89*f)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(a + a\sin\left(fx+e\right)\right)^{\frac{1}{3}} \left(\tan^{4}\left(fx+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)} \tan^{4}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/3)*tan(f*x+e)**4,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(e+f\,x)^4\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/3),x)
```

```
[Out] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/3), x)
```

3.116 $\int \sqrt[3]{a+a\sin(e+fx)}\,\tan^2(e+fx)\,dx$

Optimal. Leaf size=123

$$-\frac{5a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sqrt[6]{1+\sin(e+fx)}}{3\sqrt[6]{2}\,f(a+a\sin(e+fx))^{2/3}}+\frac{7\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{f}-\frac{3\text{s}}{}$$

```
[Out] -5/6*a*cos(f*x+e)*hypergeom([1/2, 7/6],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x
+e))^(1/6)*2^(5/6)/f/(a+a*sin(f*x+e))^(2/3)+7*sec(f*x+e)*(a+a*sin(f*x+e))^(
1/3)/f-3*sec(f*x+e)*(a+a*sin(f*x+e))^(4/3)/a/f
```

Rubi [A]
time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2792, 2934, 2731, 2730}

$$-\frac{5a\sqrt[6]{\sin(e+fx)+1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3\sqrt[6]{2}\,f(a\sin(e+fx)+a)^{2/3}}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}+\frac{7\sec(e+fx)\sqrt[3]{a\sin(e+fx)+a}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

```
[Out] (-5*a*Cos[e + f*x]*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - Sin[e + f*x])/2]*(
1 + Sin[e + f*x])^(1/6))/(3*2^(1/6)*f*(a + a*Sin[e + f*x])^(2/3)) + (7*Sec[
e + f*x]*(a + a*Sin[e + f*x])^(1/3))/f - (3*Sec[e + f*x]*(a + a*Sin[e + f*x
])^(4/3))/(a*f)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2792

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[(a + b*Sin[e + f*x])^m*((b*(m + 1) + a*Sin[e + f*x])/Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
```

```
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{a+a\sin(e+fx)}\,\tan^2(e+fx)\,dx &= -\frac{3\sec(e+fx)(a+a\sin(e+fx))^{4/3}}{af}+\frac{3\int\sec^2(e+fx)\sqrt[3]{a+a}\\ &= \frac{7\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{f}-\frac{3\sec(e+fx)(a+a\sin(e+}{af}\\ &= \frac{7\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{f}-\frac{3\sec(e+fx)(a+a\sin(e+}{af}\\ &= -\frac{5a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sqrt[6]{1+\sin(e+f}{3\sqrt[6]{2}\,f(a+a\sin(e+fx))^{2/3}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.37, size = 290, normalized size = 2.36

$$\frac{\sqrt[3]{a(1+\sin(e+fx))}\left(\frac{\left(\frac{3}{2}+\frac{3i}{2}\right)(-1)^{3/4}e^{-i(e+fx)}\left(-20e^{i(e+fx)}\sqrt{\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-ie^{-i(e+fx)}\right)+2\left(1+ie^{-i(e+fx)}\right)^{2/3}\left(1+e^{2i(e+fx)}\right){}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)-5i\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-ie^{-i(e+fx)}\right)\sqrt{2-2\sin(e+fx)}\right)}{\sqrt{2}\left(1+ie^{-i(e+fx)}\right)^{2/3}\sqrt{ie^{-i(e+fx)}\left(-i+e^{i(e+fx)}\right)^2}}-3(5+\sec(e+fx)-2\tan(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1/3)*(((3/2 + (3*I)/2)*(-1)^(3/4)*(-20*E^(I*(e + f
*x))*Sqrt[Cos[(2*e + Pi + 2*f*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3, 2/3, (-
I)/E^(I*(e + f*x))] + 2*(1 + I/E^(I*(e + f*x)))^(2/3)*(1 + E^((2*I)*(e + f*
x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*e + Pi + 2*f*x)/4]^2] - (5*I)
*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(e + f*x))]*Sqrt[2 - 2*Sin[e +
f*x]]))/(Sqrt[2]*E^(I*(e + f*x))*(1 + I/E^(I*(e + f*x)))^(2/3)*Sqrt[(I*(-I
+ E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]) - 3*(5 + Sec[e + f*x] - 2*Tan[e + f
*x]))/(3*f)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (a + a\sin(fx + e))^{\frac{1}{3}} \left(\tan^2(fx + e)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a(\sin(e + fx) + 1)} \tan^2(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/3)*tan(f*x+e)**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/3),x)
```

```
[Out] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/3), x)
```

3.117 $\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=80

$$\frac{6\sqrt{2}\,F_1\left(\frac{11}{6};-\frac{1}{2},2;\frac{17}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}\,(a+a\sin(e+fx))}{11a^2f}$$

```
[Out] 6/11*AppellF1(11/6,2,-1/2,17/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)*
(a+a*sin(f*x+e))^(7/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2798, 142, 141}

$$\frac{6\sqrt{2}\,\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{7/3}F_1\left(\frac{11}{6};-\frac{1}{2},2;\frac{17}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{11a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*Sqrt[2]*AppellF1[11/6, -1/2, 2, 17/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(7/3))/(11*a
^2*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2798

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
```

```
f*Cos[e + f*x])), Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2
)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\mathrm{Subst}\Big(\int}{af}$$

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\mathrm{Subst}}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{6\sqrt{2}F_1\left(\frac{11}{6};-\frac{1}{2},2;\frac{17}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e}{11a^2f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 19.00, size = 2692, normalized size = 33.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] ((15/2 + (15*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f
*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3))/
(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2
]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])] + (AppellF1[5/3, 1/3, 4/3, 8/3, (1/
2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])] + I*Ap
pellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*
(1 + Cot[(e + f*x)/2])])*(1 + Cot[(e + f*x)/2]))) + ((-4 - Cot[e + f*x])*(a
*(1 + Sin[e + f*x]))^(1/3))/f + ((5/2 + (5*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/
3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]*
(a*(1 + Sin[e + f*x]))^(1/3))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1
/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])] + (Ap
pellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*
(1 + Tan[(e + f*x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + T
an[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])])*(1 + Tan[(e + f*x)/2
]))) + (Cos[(3*(e + f*x))/2]*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(a*(1 + Sin[
```

```
e + f*x]))^(1/3)*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8
 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2])
)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2
])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/
3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/
2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e +
 f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])*(1 + Tan[(e + f*x)/2])*((-3*Sec[(e + f*x)/2]^2*((1 + Tan[(e + f*x)/
2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot
[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5
/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan
[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x
)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e +
f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2
])))/(4*(1 + Tan[(e + f*x)/2])^2) + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Co
t[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3,
5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Ta
n[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*
x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e +
 f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/
2]))*(Sqrt[Sec[(e + f*x)/2]^2]/2 - (Tan[(e + f*x)/2]*(1 + Tan[(e + f*x)/2])
)/(2*Sqrt[Sec[(e + f*x)/2]^2])))/((1 + Tan[(e + f*x)/2])*((1 + Tan[(e + f*x
)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(1/3)) + (3*((1 + Tan[(e + f*x)/2])/Sqrt[Se
c[(e + f*x)/2]^2])^(2/3)*(-1/2*(AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1
 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 -
 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Sec[(
e + f*x)/2]^2) + ((1 + I)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f
*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e +
 f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*Sec[(e + f*x)/2]^2)/2^(1/3) + ((1/3 + I
/3)*2^(2/3)*(((1/2 - I/2)*(I + Cot[(e + f*x)/2])*Csc[(e + f*x)/2]^2)/(1 + C
ot[(e + f*x)/2])^2 - ((1/2 - I/2)*Csc[(e + f*x)/2]^2)/(1 + Cot[(e + f*x)/2]
))*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2
 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]))/(((1 - I)*(I + Cot[(e + f*x
)/2]))/(1 + Cot[(e + f*x)/2]))^(2/3) - ((1/6 + I/6)*AppellF1[2/3, 1/3, 1/3,
 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2]
)]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*Csc[(e + f*x)/2]^2*(1 + T
an[(e + f*x)/2]))/((-1 - I)*(I + Cot[(e + f*x)/2]))^(2/3) - ((1/3 - I/3)*Ap
pellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*
(1 + Cot[(e + f*x)/2])]*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Csc[(e + f*
x)/2]^2*(1 + Tan[(e + f*x)/2]))/((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(2
/3) - ((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e
+ f*x)/2]))^(1/3)*((-1/30 + I/30)*AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*
(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2
]^2 - (1/30 + I/30)*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e +
f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2]^2)*(1 + Tan[
```

```
(e + f*x)/2]) + ((2/3 + (2*I)/3)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/
(1 + Cot[(e + f*x)/2]))^(1/3)*(I + Tan[(e + f*x)/2])*(2 + 2*Tan[(e + f*x)/2
])*(-((Sec[(e + f*x)/2]^2*((1 + I) + (1 - I)*Tan[(e + f*x)/2]))/(2 + 2*Tan[
(e + f*x)/2])^2) + ((1/2 - I/2)*Sec[(e + f*x)/2]^2)/(2 + 2*Tan[(e + f*x)/2]
))*(-Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/
(2 + 2*Tan[(e + f*x)/2])] + (1 - ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 +
2*Tan[(e + f*x)/2]))^(-1/3)))/((1 + I) + (1 - I...
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\cot^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x)
[Out] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}\,\cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/3),x)
[Out] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/3), x)
```

3.118 $\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=80

$$\frac{12\sqrt{2}\,F_1\left(\frac{17}{6};-\frac{3}{2},4;\frac{23}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}\,(a+a\sin(e+fx)}{17a^3f}$$

```
[Out] 12/17*AppellF1(17/6,4,-3/2,23/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)
*(a+a*sin(f*x+e))^(10/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^3/f
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2798, 142, 141}

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{10/3}F_1\left(\frac{17}{6};-\frac{3}{2},4;\frac{23}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{17a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (12*Sqrt[2]*AppellF1[17/6, -3/2, 4, 23/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
 f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(10/3))/(17
*a^3*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2798

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
```

```
f*Cos[e + f*x])), Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2
)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int\right.}{af}$$

$$= \frac{\left(2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{12\sqrt{2}\,F_1\left(\frac{17}{6};-\frac{3}{2},4;\frac{23}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e}{17a^3f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.90, size = 2796, normalized size = 34.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(1/3),x]
[Out] ((239/54 + (77*Cot[e + f*x])/54 - (Cot[e + f*x]*Csc[e + f*x])/18 - (Cot[e +
 f*x]*Csc[e + f*x]^2)/3)*(a*(1 + Sin[e + f*x]))^(1/3))/f - ((70/9 + (70*I)/
9)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 -
I/2)*(1 + Cot[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3)*(1 + Tan[(e + f*x
)/2]))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e +
 f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Sec[(e + f*x)/2] + AppellF1[
5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Co
t[(e + f*x)/2])]*(Csc[(e + f*x)/2] + Sec[(e + f*x)/2]) + I*AppellF1[5/3, 4/
3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e +
f*x)/2])]*(Csc[(e + f*x)/2] + Sec[(e + f*x)/2]))*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])) - ((355/108 + (355*I)/108)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 +
 I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]*(a*(1 + S
in[e + f*x]))^(1/3))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)
*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])] + (AppellF1[5/
3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[
(e + f*x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f
*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])])*(1 + Tan[(e + f*x)/2]))) - (2
```

```
39*Cos[(3*(e + f*x))/2]*Csc[e + f*x]*(a*(1 + Sin[e + f*x]))^(1/3)*((1 + Tan
[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 -
I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[
1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2]
)]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + C
ot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I
)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[
(e + f*x)/2])))/(216*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(1 + Tan[(e +
f*x)/2])*((-3*Sec[(e + f*x)/2]^2*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)
/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + C
ot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)
*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - Appel
lF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1
+ Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 -
I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2])))/(8*(1 + Tan[(e +
f*x)/2])^2) + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 +
Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I
)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - Appe
llF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1
 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 -
 I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2]))*(Sqrt[Sec[(e + f*
x)/2]^2]/2 - (Tan[(e + f*x)/2]*(1 + Tan[(e + f*x)/2]))/(2*Sqrt[Sec[(e + f*x
)/2]^2])))/(2*(1 + Tan[(e + f*x)/2])*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e +
f*x)/2]^2])^(1/3)) + (3*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(
2/3)*(-1/2*(AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2])
, (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/
2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Sec[(e + f*x)/2]^2) + ((1
 + I)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hyper
geometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan
[(e + f*x)/2])]*Sec[(e + f*x)/2]^2)/2^(1/3) + ((1/3 + I/3)*2^(2/3)*(((1/2 -
 I/2)*(I + Cot[(e + f*x)/2])*Csc[(e + f*x)/2]^2)/(1 + Cot[(e + f*x)/2])^2 -
 ((1/2 - I/2)*Csc[(e + f*x)/2]^2)/(1 + Cot[(e + f*x)/2]))*Hypergeometric2F1
[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2
])]*(I + Tan[(e + f*x)/2]))/(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e +
 f*x)/2]))^(2/3) - ((1/6 + I/6)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1
 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 -
 2*I)*Cot[(e + f*x)/2])^(1/3)*Csc[(e + f*x)/2]^2*(1 + Tan[(e + f*x)/2]))/((
-1 - I)*(I + Cot[(e + f*x)/2]))^(2/3) - ((1/3 - I/3)*AppellF1[2/3, 1/3, 1/3
, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2
])]*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Csc[(e + f*x)/2]^2*(1 + Tan[(e
+ f*x)/2]))/((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(2/3) - ((2 + 2*I) - (
2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*((
-1/30 + I/30)*AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2
]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2]^2 - (1/30 + I/30)*
AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2
```

```
)*(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]) + ((2/
3 + (2*I)/3)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2
]))^(1/3)*(I + Tan[(e + f*x)/2])*(2 + 2*Tan[(e + f*x)/2])*(-((Sec[(e + f*x)
/2]^2*((1 + I) + (1 - I)*Tan[(e + f*x)/2]))/(2 + 2*Tan[(e + f*x)/2])^2) + (
(1/2 - I/2)*Sec[(e + f*x)/2]^2)/(2 + 2*Tan[(e +...
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(\cot^{4}\left(f x +e \right)\right) \left(a +a \sin \left(f x +e \right)\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x)
[Out] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^4, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a \left(\sin{\left(e + f x \right)} + 1\right)} \cot^{4}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(1/3),x)
[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*cot(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/3),x)
[Out] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/3), x)
```

3.119 $\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=551

$$\frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}}-\frac{973\sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}}-\frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}}-$$

```
[Out] 973/396*sec(f*x+e)/f/(a+a*sin(f*x+e))^(1/3)-973/495*sec(f*x+e)*(1-sin(f*x+e
))/f/(a+a*sin(f*x+e))^(1/3)-1/132*sec(f*x+e)*(95*a+356*a*sin(f*x+e))/f/(1-s
in(f*x+e))/(a+a*sin(f*x+e))^(4/3)+973/2970*((2^(1/3)*a^(1/3)-(a+a*sin(f*x+e
))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2))
)^2)^(1/2)/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1/2)))*(2^(1/3)*a^
(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)*a^(1/3)-(a+
a*sin(f*x+e))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*
(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*sec(f*x+e)*(a+a*sin(f*x+e))^
(2/3)*(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3))*((2^(2/3)*a^(2/3)+2^(1/3)*a^
(1/3)*(a+a*sin(f*x+e))^(1/3)+(a+a*sin(f*x+e))^(2/3))/(2^(1/3)*a^(1/3)-(a+a*
sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)*3^(3/4)/a^(4/3)/f/(-(a+a*si
n(f*x+e))^(1/3)*(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3))/(2^(1/3)*a^(1/3)-(
a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)+3/4*a^2*sin(f*x+e)*tan(f*x+e)/f
/(a-a*sin(f*x+e))/(a+a*sin(f*x+e))^(4/3)+3*a^2*sin(f*x+e)^2*tan(f*x+e)/f/(a
-a*sin(f*x+e))/(a+a*sin(f*x+e))^(4/3)
```

Rubi [A]
time = 0.38, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2798, 102, 158, 149, 53, 65, 231}

$$\frac{973\sec(e+fx)(a\sin(e+fx)+a)^{2/3}\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a\sin(e+fx)+a}\right)\sqrt{\frac{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a\sin(e+fx)+a}+(a\sin(e+fx)+a)^{2/3}}{\left(\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a\sin(e+fx)+a}\right)^2}}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{\sin(e+fx)a+a}}{\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\sin(e+fx)a+a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{495\sqrt[3]{2}\sqrt[4]{3}a^{4/3}f\sqrt{-\frac{\sqrt[3]{a\sin(e+fx)+a}\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a\sin(e+fx)+a}\right)}{\left(\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a\sin(e+fx)+a}\right)^2}}}+\frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a\sin(e+fx)+a)^{4/3}}+\frac{3a^2\sin(e+fx)\tan(e+fx)}{4f(a-a\sin(e+fx))(a\sin(e+fx)+a)^{4/3}}-\frac{\sec(e+fx)(356a\sin(e+fx)+95a)}{132f(1-\sin(e+fx))(a\sin(e+fx)+a)^{4/3}}+\frac{973\sec(e+fx)}{396f\sqrt[3]{a\sin(e+fx)+a}}-\frac{973(1-\sin(e+fx))\sec(e+fx)}{495f\sqrt[3]{a\sin(e+fx)+a}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (973*Sec[e + f*x])/(396*f*(a + a*Sin[e + f*x])^(1/3)) - (973*Sec[e + f*x]*(
1 - Sin[e + f*x]))/(495*f*(a + a*Sin[e + f*x])^(1/3)) - (Sec[e + f*x]*(95*a
 + 356*a*Sin[e + f*x]))/(132*f*(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(4/3
)) + (973*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a + a*Sin[e +
```

```
f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))],
 (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1/3)*a^(1/3)
- (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a +
a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^(1/3) - (1 +
 Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(495*2^(1/3)*3^(1/4)*a^(4/3)*f*Sq
rt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1
/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2)]) + (
3*a^2*Sin[e + f*x]*Tan[e + f*x])/(4*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f
*x])^(4/3)) + (3*a^2*Sin[e + f*x]^2*Tan[e + f*x])/(f*(a - a*Sin[e + f*x])*(
a + a*Sin[e + f*x])^(4/3))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*
h*(n + 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n
 + 2)) + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*
(m + 1) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*
d)^2*(m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Dist[(a^2*
d^2*f*h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
```

```
 + n + 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n
+ 3) + d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m
+ 1)*(n + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 2798

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x])), Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2
)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\Big(\int \frac{x^4}{(a-x)^{5/2}(a+x)}}{af}$$

$$= \frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{\Big(3\sec(e+fx)\sqrt{a-a\sin(e+}}{}$$

$$= \frac{3a^2\sin(e+fx)\tan(e+fx)}{4f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2\sin^2(e+fx)\tan(e}{f(a-a\sin(e+fx))(a+a\sin}$$

$$= -\frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2\sin(e+fx)\tan(}{4f(a-a\sin(e+fx))(a+a}$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}} +$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973\sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+}{132f(1-\text{si}}$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973\sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+}{132f(1-\text{si}}$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973\sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+}{132f(1-\text{si}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.53, size = 128, normalized size = 0.23

$$\frac{973\sqrt{2}\cos(e+fx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)+\sec^3(e+fx)\sqrt{1-\sin(e+fx)}\,(-49-64\cos(2(e+fx))+22\sin(e+fx)-128\sin(3(e+fx)))}{495f\sqrt{1-\sin(e+fx)}\,\sqrt[3]{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (973*Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*e + Pi +
2*f*x)/4]^2] + Sec[e + f*x]^3*Sqrt[1 - Sin[e + f*x]]*(-49 - 64*Cos[2*(e + f
```

```
*x)] + 22*Sin[e + f*x] - 128*Sin[3*(e + f*x)]))/(495*f*Sqrt[1 - Sin[e + f*x
]]*(a*(1 + Sin[e + f*x]))^(1/3))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(fx+e)}{(a+a\sin(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
[Out] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(1/3),x)
[Out] Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(e+f\,x)^4}{(a+a\,\sin(e+f\,x))^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/3), x)
```

3.120 $\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}}+\frac{11\sqrt[6]{2}\,\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{15f\sqrt[6]{1+\sin(e+fx)}\,\sqrt[3]{a+a\sin(e+fx)}}+\frac{4\sec(e+fx)(a+a\sin(e+\ldots}{5af}$$

```
[Out] -3/5*sec(f*x+e)/f/(a+a*sin(f*x+e))^(1/3)+11/15*2^(1/6)*cos(f*x+e)*hypergeom
([1/2, 5/6],[3/2],1/2-1/2*sin(f*x+e))/f/(1+sin(f*x+e))^(1/6)/(a+a*sin(f*x+e
))^(1/3)+4/5*sec(f*x+e)*(a+a*sin(f*x+e))^(2/3)/a/f
```

Rubi [A]
time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2791, 2934, 2731, 2730}

$$\frac{11\sqrt[6]{2}\,\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{15f\sqrt[6]{\sin(e+fx)+1}\,\sqrt[3]{a\sin(e+fx)+a}}+\frac{4\sec(e+fx)(a\sin(e+fx)+a)^{2/3}}{5af}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-3*Sec[e + f*x])/(5*f*(a + a*Sin[e + f*x])^(1/3)) + (11*2^(1/6)*Cos[e + f*
x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[e + f*x])/2])/(15*f*(1 + Sin[e
 + f*x])^(1/6)*(a + a*Sin[e + f*x])^(1/3)) + (4*Sec[e + f*x]*(a + a*Sin[e +
 f*x])^(2/3))/(5*a*f)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[b*((a + b*Sin[e + f*x])^m/(a*f*(2*m - 1)*Cos[e + f*x])),
```

```
x] - Dist[1/(a^2*(2*m - 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*((a*m - b*(2*
m - 1)*Sin[e + f*x])/Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{3\int \sec^2(e+fx)(a+a\sin(e+fx))^{2/3}\left(-\frac{a}{3}+\frac{5}{3}a\right.}{5a^2}$$

$$= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{4\sec(e+fx)(a+a\sin(e+fx))^{2/3}}{5af} - \frac{11}{15}\int \frac{}{\sqrt[3]{a}}$$

$$= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{4\sec(e+fx)(a+a\sin(e+fx))^{2/3}}{5af} - \frac{\left(11\sqrt[3]{1+\right.}{}$$

$$= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{11\sqrt[6]{2}\,\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)\right.}{15f\sqrt[6]{1+\sin(e+fx)}\,\sqrt[3]{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 0.34, size = 100, normalized size = 0.79

$$\frac{-22\cos(e+fx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)+\sqrt{2-2\sin(e+fx)}\,(\sec(e+fx)+4\tan(e+fx))}{5f\sqrt{2-2\sin(e+fx)}\,\sqrt[3]{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-22*Cos[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*e + Pi + 2*f*x)/4
]^2] + Sqrt[2 - 2*Sin[e + f*x]]*(Sec[e + f*x] + 4*Tan[e + f*x]))/(5*f*Sqrt[
2 - 2*Sin[e + f*x]]*(a*(1 + Sin[e + f*x]))^(1/3))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(fx+e)}{(a+a\sin(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

```
[Out] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2{(e+fx)}}{\sqrt[3]{a(\sin{(e+fx)}+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(e+f\,x)^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/3), x)
```

3.121 $\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=80

$$\frac{6\sqrt{2}\,F_1\left(\frac{7}{6};-\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}\,(a+a\sin(e+fx))^{5}}{7a^2f}$$

```
[Out] 6/7*AppellF1(7/6,2,-1/2,13/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(a
+a*sin(f*x+e))^(5/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2798, 142, 141}

$$\frac{6\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{5/3}F_1\left(\frac{7}{6};-\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{7a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*Sqrt[2]*AppellF1[7/6, -1/2, 2, 13/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f
*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(5/3))/(7*a^2
*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2798

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x])), Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2
)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a-x}\sqrt[6]{}}{x^2}\right.}{af}$$

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a+x}}{}\right.}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{6\sqrt{2}\,F_1\left(\frac{7}{6};-\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-}}{7a^2f}$$

Mathematica [F]
time = 7.34, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
[Out] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3), x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(fx+e)}{(a+a\sin(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
[Out] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}\left(e+fx\right)}{\sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(1/3),x)
[Out] Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cot}\left(e+f\,x\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/3),x)
[Out] int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/3), x)
```

3.122 $\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=80

$$\frac{12\sqrt{2}\,F_1\left(\frac{13}{6};-\frac{3}{2},4;\frac{19}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}\,(a+a\sin(e+fx$$
$$13a^3f$$

```
[Out] 12/13*AppellF1(13/6,4,-3/2,19/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)
*(a+a*sin(f*x+e))^(8/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^3/f
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2798, 142, 141}

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{8/3}F_1\left(\frac{13}{6};-\frac{3}{2},4;\frac{19}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{13a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (12*Sqrt[2]*AppellF1[13/6, -3/2, 4, 19/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
 f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(8/3))/(13*
a^3*f)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 2798

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x])), Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2
)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a-x)^{3/2}(a+x)^{7/}}{x^4}\right.}{af}$$

$$= \frac{\left(2\sqrt{2}\,\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)^{7/6}(}{x^4}\right.}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{12\sqrt{2}\,F_1\left(\frac{13}{6};-\frac{3}{2},4;\frac{19}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-}}{13a^3f}$$

Mathematica [**F**]
time = 5.07, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
[Out] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3), x]
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(fx+e)}{(a+a\sin(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
[Out] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(e+f\,x)^4}{(a+a\,\sin(e+f\,x))^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/3),x)
```

```
[Out] int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/3), x)
```

3.123 $\int (a+a\sin(e+fx))^3(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=269

$$\frac{a^3\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)}+\frac{3a^3\cos^2(e+fx)^{\frac{1+p}{2}}\,{}_2F_1\left(\frac{1+p}{2},\frac{2+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)}{fg(2+p)}$$

```
[Out] a^3*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p
)/f/g/(1+p)+3*a^3*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p]
,[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)+a^3*(cos
(f*x+e)^2)^(1/2+1/2*p)*hypergeom([2+1/2*p, 1/2+1/2*p],[3+1/2*p],sin(f*x+e)^
2)*sin(f*x+e)^3*(g*tan(f*x+e))^(1+p)/f/g/(4+p)+3*a^3*hypergeom([2, 3/2+1/2*
p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/g^3/(3+p)
```

Rubi [A]
time = 0.25, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2789, 3557, 371, 2682, 2657, 2671}

$$\frac{3a^3(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^3(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{3a^3\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{fg(p+2)}+\frac{a^3\sin^3(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+4}{2};\frac{p+6}{2};\sin^2(e+fx)\right)}{fg(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^3*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^3*(Cos[e + f*x]^2)^((1 + p)/2)*H
ypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e + f
*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a^3*Hypergeometric2F1[2,
 (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3
 + p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
```

```
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(g\tan(e+fx))^p\,dx = \int \left(a^3(g\tan(e+fx))^p+3a^3\sin(e+fx)(g\tan(e+fx))^p+3a\right.$$

$$= a^3\int (g\tan(e+fx))^p\,dx + a^3\int \sin^3(e+fx)(g\tan(e+fx))^p$$

$$= \frac{(a^3g)\,\mathrm{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{(3a^3g)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{a^3\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{3a^3\,\mathrm{c}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 22.88, size = 4715, normalized size = 17.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

```
[Out] (4*(3 + p)*AppellF1[(1 + p)/2, p, 4, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Cos[(e + f*x)/2]^9*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(g
*Tan[e + f*x])^p)/(f*(1 + p)*(2*(4*AppellF1[(3 + p)/2, p, 5, (5 + p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - p*AppellF1[(3 + p)/2, 1 + p, 4, (5
+ p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (3
+ p)*AppellF1[(1 + p)/2, p, 4, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*(1 + Cos[e + f*x]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (24*
(4 + p)*AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Cos[(e + f*x)/2]^8*Sin[(e + f*x)/2]^2*(a + a*Sin[e + f*x])^3*(g*Tan[
e + f*x])^p)/(f*(2 + p)*(2*(4*AppellF1[2 + p/2, p, 5, 3 + p/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] - p*AppellF1[2 + p/2, 1 + p, 4, 3 + p/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (4 + p)*AppellF1
[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[
e + f*x]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (60*(AppellF1[(1 + p)
/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[(1
 + p)/2, p, 4, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e
+ f*x)/2]^5*Sin[(e + f*x)/2]^3*(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p)/(
f*(1 + p)*(2*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] - ((-2*(3*AppellF1[(3 + p)/2, p, 4, (5 + p)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[(3 + p)/2, p, 5, (5 + p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*(-AppellF1[(3 + p)/2, 1 + p, 3, (5
+ p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[(3 + p)/2, 1 +
p, 4, (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*(-1 + Cos[e + f
*x]) + (3 + p)*AppellF1[(1 + p)/2, p, 4, (3 + p)/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*(1 + Cos[e + f*x]))*Sec[(e + f*x)/2]^2)/(3 + p))*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^6) + (80*(4 + p)*(AppellF1[1 + p/2, p, 3, 2 +
 p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2
+ p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^6*Sin[(e
+ f*x)/2]^4*(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p)/(f*(2 + p)*(-2*(4 +
p)*AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Cos[(e + f*x)/2]^2 + 2*(3*AppellF1[2 + p/2, p, 4, 3 + p/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[2 + p/2, p, 5, 3 + p/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + p*(-AppellF1[2 + p/2, 1 + p, 3, 3 + p/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[2 + p/2, 1 + p, 4, 3 + p/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*(-1 + Cos[e + f*x]) + (4 + p)
*AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
(1 + Cos[e + f*x]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(AppellF1
[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*Appel
```

```
lF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + Appe
llF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[
e + f*x]^3*(a + a*Sin[e + f*x])^3*Tan[(e + f*x)/2]^2*(g*Tan[e + f*x])^p)/(4
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*((AppellF1[1 + p/2, p, 2, 2 + p/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*AppellF1[1 + p/2, p, 3, 2 +
 p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[1 + p/2, p, 4, 2
+ p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Csc[(e + f*x)/2]*Sec[(e +
f*x)/2] + p*(AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - 2*AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*Csc[e + f*x]*Sec[e + f*x] + ((2 + p)*(-2*AppellF1[2 + p/2,
 p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 6*AppellF1[2 + p
/2, p, 4, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[2
+ p/2, p, 5, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1
[2 + p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*p
*AppellF1[2 + p/2, 1 + p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + p*AppellF1[2 + p/2, 1 + p, 4, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(4 + p) - p*(AppellF1[1 +
p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*AppellF1[1
 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[
1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[e + f
*x]*Tan[(e + f*x)/2])) + (2*(AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 3*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 3*AppellF1[(1 + p)/2, p, 3, (3 + p
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[(1 + p)/2, p, 4, (
3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^6*(a +
 a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p)/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^6*((AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - 3*AppellF1[(1 + p)/2, p, 2, (3 + ...
```

Maple [F]
time = 1.22, size = 0, normalized size = 0.00

$$\int (a + a \sin (fx + e))^3 (g \tan (fx + e))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
[Out] int((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*(g*tan(f*x + e))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \left(g\tan{\left(e+fx \right)}\right)^{p}\, dx + \int 3\left(g\tan{\left(e+fx \right)}\right)^{p}\sin{\left(e+fx \right)}\, dx + \int 3\left(g\tan{\left(e+fx \right)}\right)^{p}\sin^{2}{\left(e+fx \right)}\, dx + \int \left(g\tan{\left(e+fx \right)}\right)^{p}\sin^{3}{\left(e+fx \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(g*tan(f*x+e))**p,x)
```

```
[Out] a**3*(Integral((g*tan(e + f*x))**p, x) + Integral(3*(g*tan(e + f*x))**p*sin
(e + f*x), x) + Integral(3*(g*tan(e + f*x))**p*sin(e + f*x)**2, x) + Integr
al((g*tan(e + f*x))**p*sin(e + f*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\mathrm{tan}\left(e+f\,x\right)\right)}^p\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^3, x)
```

3.124 $\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=187

$$\frac{a^2\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)}+\frac{2a^2\cos^2(e+fx)^{\frac{1+p}{2}}\,{}_2F_1\left(\frac{1+p}{2},\frac{2+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right.}{fg(2+p)}$$

```
[Out] a^2*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p
)/f/g/(1+p)+2*a^2*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p]
,[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)+a^2*hype
rgeom([2, 3/2+1/2*p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/g^3/
(3+p)
```

Rubi [A]

time = 0.17, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2789, 3557, 371, 2682, 2657, 2671}

$$\frac{a^2(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^2(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2a^2\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{fg(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (2*a^2*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^2*Hypergeometric2F1[2, (3 + p)/2
, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx = \int \left(a^2(g\tan(e+fx))^p + 2a^2\sin(e+fx)(g\tan(e+fx))^p + a^2\,\mathrm{s}\right.$$

$$= a^2\int (g\tan(e+fx))^p\,dx + a^2\int \sin^2(e+fx)(g\tan(e+fx))^p\,d$$

$$= \frac{(a^2g)\,\mathrm{Subst}\left(\int \frac{x^{2+p}}{(g^2+x^2)^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{(a^2g)\,\mathrm{Subst}\left(\int \right.}{}$$

$$= \frac{a^2\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{2a^2\cos}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 13.24, size = 1452, normalized size = 7.76

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
[Out] (2*a^2*(1 + Sin[e + f*x])^2*Tan[(e + f*x)/2]*((2 + p)*AppellF1[(1 + p)/2, p
, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(2 + p)*Appell
F1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4
*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + 4*(1 + p)*AppellF1[(2 + p)/2, p, 2, (4 + p)/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])*(g*Tan[e + f*x])^p)/(f*(Sec[(e
 + f*x)/2]^2*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p,
3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1
[(2 + p)/2, p, 2, (4 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(
e + f*x)/2]) - 16*p*Cos[(e + f*x)/2]*Csc[e + f*x]^3*Sec[e + f*x]*Sin[(e + f
*x)/2]^5*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p, 3, (
3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1[(2
+ p)/2, p, 2, (4 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e +
f*x)/2]) + 2*p*Csc[e + f*x]*Sec[e + f*x]*Tan[(e + f*x)/2]*((2 + p)*AppellF1
[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(
2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1[(2 + p)/2, p, 2, (4 + p)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]) + 2*(1 + p)*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2]*(2*AppellF1[(2 + p)/2, p, 2, (4 + p)/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + ((2 + p)*(-AppellF1[(3 + p)/2, p, 2,
 (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[(3 + p)/2
, 1 + p, 1, (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2])/(3 + p) + (4*(2 + p)*(-2*AppellF1[(3 + p)/2, p, 3, (5 + p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[(3 + p)/2, 1 + p, 2, (5 +
p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2])/(3 + p) -
 (4*(2 + p)*(-3*AppellF1[(3 + p)/2, p, 4, (5 + p)/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + p*AppellF1[(3 + p)/2, 1 + p, 3, (5 + p)/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2])/(3 + p) + (4*(2 + p)*(-2*A
ppellF1[(4 + p)/2, p, 3, (6 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + p*AppellF1[(4 + p)/2, 1 + p, 2, (6 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)/(4 + p))))
```

Maple [F]

time = 0.97, size = 0, normalized size = 0.00

$$\int (a + a\sin(fx + e))^2 (g\tan(fx + e))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(g*tan(f*x + e)
)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \left(g\tan\left(e+fx\right)\right)^{p}\, dx+\int 2\left(g\tan\left(e+fx\right)\right)^{p}\sin\left(e+fx\right)dx+\int \left(g\tan\left(e+fx\right)\right)^{p}\sin^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(g*tan(f*x+e))**p,x)
```

```
[Out] a**2*(Integral((g*tan(e + f*x))**p, x) + Integral(2*(g*tan(e + f*x))**p*sin
(e + f*x), x) + Integral((g*tan(e + f*x))**p*sin(e + f*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\tan(e+f\,x))^p\,(a+a\,\sin\left(e+f\,x\right))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^2,x)
[Out] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^2, x)
```

3.125 $\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=129

$$\frac{a\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)}+\frac{a\cos^2(e+fx)^{\frac{1+p}{2}}\,{}_2F_1\left(\frac{1+p}{2},\frac{2+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sin}{fg(2+p)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p)/
f/g/(1+p)+a*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p],[2+1/
2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2789, 3557, 371, 2682, 2657}

$$\frac{a(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{a\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{fg(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f
*x])^(1 + p))/(f*g*(1 + p)) + (a*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometri
c2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e
 + f*x])^(1 + p))/(f*g*(2 + p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
```

```
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx &= \int \left(a(g\tan(e+fx))^p + a\sin(e+fx)(g\tan(e+fx))^p\right)\,dx\\ &= a\int (g\tan(e+fx))^p\,dx + a\int \sin(e+fx)(g\tan(e+fx))^p\,dx\\ &= \frac{(ag)\text{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{(a\cos^{1+p}(e+fx)\,\text{si}}{}\\ &= \frac{a\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{a\cos^2(}{}\end{aligned}$$

Mathematica [F]
time = 1.10, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
[Out] Integrate[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p, x]
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))\,(g\tan(fx+e))^p\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\left(g\tan\left(e+fx\right)\right)^{p}\,dx+\int\left(g\tan\left(e+fx\right)\right)^{p}\sin\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))**p,x)
```

```
[Out] a*(Integral((g*tan(e + f*x))**p, x) + Integral((g*tan(e + f*x))**p*sin(e +
f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\tan(e + f\,x))^p\,(a + a\,\sin(e + f\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x)),x)
[Out] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x)), x)
```

3.126 $\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=108

$$\frac{(g\tan(e+fx))^{1+p}}{afg(1+p)}-\frac{\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{3+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec(e+fx)(g\tan(e+fx))^{2+p}}{afg^2(2+p)}$$

```
[Out] (g*tan(f*x+e))^(1+p)/a/f/g/(1+p)-(cos(f*x+e)^2)^(3/2+1/2*p)*hypergeom([1+1/
2*p, 3/2+1/2*p],[2+1/2*p],sin(f*x+e)^2)*sec(f*x+e)*(g*tan(f*x+e))^(2+p)/a/f
/g^2/(2+p)
```

Rubi [A]
time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2785, 2687, 32, 2697}

$$\frac{(g\tan(e+fx))^{p+1}}{afg(p+1)}-\frac{\sec(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+3}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{afg^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x]),x]
```

```
[Out] (g*Tan[e + f*x])^(1 + p)/(a*f*g*(1 + p)) - ((Cos[e + f*x]^2)^((3 + p)/2)*Hy
pergeometric2F1[(2 + p)/2, (3 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e + f*
x]*(g*Tan[e + f*x])^(2 + p))/(a*f*g^2*(2 + p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(g\tan(e+fx))^p\,dx}{a} - \frac{\int \sec(e+fx)(g\tan(e+fx))^{1+p}\,dx}{ag}$$

$$= -\frac{\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{3+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec(e+fx)(g\tan(e+fx))^{2+p}}{afg^2(2+p)}$$

$$= \frac{(g\tan(e+fx))^{1+p}}{afg(1+p)} - \frac{\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{3+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec(e+f}{afg^2(2+p)}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(108) = 216.
time = 1.74, size = 232, normalized size = 2.15

$$\frac{2\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^p\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\tan\left(\frac{1}{2}(e+fx)\right)\left((6+5p+p^2)\,{}_2F_1\left(\frac{1+p}{2},2+p;\frac{3+p}{2};\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-(1+p)\tan\left(\frac{1}{2}(e+fx)\right)\left(2(3+p)\,{}_2F_1\left(\frac{2+p}{2},2+p;\frac{4+p}{2};\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-(2+p)\,{}_2F_1\left(2+p,\frac{3+p}{2};\frac{5+p}{2};\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)(g\tan(e+fx))^p}{f(1+p)(2+p)(3+p)(a+a\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x]),x]
```

```
[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2*Tan[(e + f*x)/2]*((6 + 5*p + p^2)*Hypergeometric2F1[(1 + p)/2, 2 + p, (
3 + p)/2, Tan[(e + f*x)/2]^2] - (1 + p)*Tan[(e + f*x)/2]*(2*(3 + p)*Hyperge
ometric2F1[(2 + p)/2, 2 + p, (4 + p)/2, Tan[(e + f*x)/2]^2] - (2 + p)*Hyper
geometric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)
/2]))*(g*Tan[e + f*x])^p)/(f*(1 + p)*(2 + p)*(3 + p)*(a + a*Sin[e + f*x]))
```

Maple [**F**]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(g\tan(fx+e))^p}{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x)
```

```
[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(g\tan(e+fx))^p}{\sin(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e)),x)
[Out] Integral((g*tan(e + f*x))**p/(sin(e + f*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\tan(e+f\,x))^p}{a+a\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x)),x)
[Out] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x)), x)
```

3.127 $\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{(g\tan(e+fx))^{1+p}}{a^2fg(1+p)}-\frac{2\cos^2(e+fx)^{\frac{5+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{5+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^3(e+fx)(g\tan(e+fx))^{2+p}}{a^2fg^2(2+p)}+\frac{2(g\ldots$$

```
[Out] (g*tan(f*x+e))^(1+p)/a^2/f/g/(1+p)-2*(cos(f*x+e)^2)^(5/2+1/2*p)*hypergeom([
1+1/2*p, 5/2+1/2*p],[2+1/2*p],sin(f*x+e)^2)*sec(f*x+e)^3*(g*tan(f*x+e))^(2+
p)/a^2/f/g^2/(2+p)+2*(g*tan(f*x+e))^(3+p)/a^2/f/g^3/(3+p)
```

Rubi [A]
time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2790, 2687, 14, 16, 2697, 32}

$$\frac{2(g\tan(e+fx))^{p+3}}{a^2fg^3(p+3)}-\frac{2\sec^3(e+fx)\cos^2(e+fx)^{\frac{p+5}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+5}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{a^2fg^2(p+2)}+\frac{(g\tan(e+fx))^{p+1}}{a^2fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (g*Tan[e + f*x])^(1 + p)/(a^2*f*g*(1 + p)) - (2*(Cos[e + f*x]^2)^((5 + p)/2
)*Hypergeometric2F1[(2 + p)/2, (5 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e
+ f*x]^3*(g*Tan[e + f*x])^(2 + p))/(a^2*f*g^2*(2 + p)) + (2*(g*Tan[e + f*x]
)^(3 + p))/(a^2*f*g^3*(3 + p))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx &= \frac{\int \left(a^2\sec^4(e+fx)(g\tan(e+fx))^p - 2a^2\sec^3(e+fx)\tan(e+fx)(g\tan(e+f
}{a^4} \\
&= \frac{\int \sec^4(e+fx)(g\tan(e+fx))^p\,dx}{a^2} + \frac{\int \sec^2(e+fx)\tan^2(e+fx)(g\tan(e+fx
}{a^2} \\
&= \frac{\operatorname{Subst}\left(\int (gx)^p\left(1+x^2\right)\,dx, x, \tan(e+fx)\right)}{a^2 f} + \frac{\int \sec^2(e+fx)(g\tan(e+fx))^{2+p}
}{a^2g^2} \\
&= -\frac{2\cos^2(e+fx)^{\frac{5+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{5+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^3(e+fx)(g\tan(e+fx)
}{a^2fg^2(2+p)} \\
&= \frac{(g\tan(e+fx))^{1+p}}{a^2fg(1+p)} - \frac{2\cos^2(e+fx)^{\frac{5+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{5+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^3(e
}{a^2fg^2(2+p)}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 3.04, size = 710, normalized size = 5.14

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((2 + p)*(AppellF1[1 + p, p, 2 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)
/2]] - 2*AppellF1[1 + p, p, 3 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/
2]] + 2*AppellF1[1 + p, p, 4 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2
]])*Sin[e + f*x]*(g*Tan[e + f*x])^p)/(a^2*f*(1 + p)*(1 + Sin[e + f*x])^2*((
2 + p)*AppellF1[1 + p, p, 2 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]
] - 2*(2 + p)*AppellF1[1 + p, p, 3 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e +
f*x)/2]] + 4*AppellF1[1 + p, p, 4 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f
*x)/2]] + 2*p*AppellF1[1 + p, p, 4 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e +
f*x)/2]] - 2*AppellF1[2 + p, p, 3 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f
*x)/2]]*Tan[(e + f*x)/2] - p*AppellF1[2 + p, p, 3 + p, 3 + p, Tan[(e + f*x)
/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2] + 6*AppellF1[2 + p, p, 4 + p, 3 +
p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2] + 2*p*AppellF1[2 +
 p, p, 4 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2]
- 8*AppellF1[2 + p, p, 5 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]*T
an[(e + f*x)/2] - 2*p*AppellF1[2 + p, p, 5 + p, 3 + p, Tan[(e + f*x)/2], -T
an[(e + f*x)/2]]*Tan[(e + f*x)/2] + p*AppellF1[2 + p, 1 + p, 2 + p, 3 + p,
Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2] - 2*p*AppellF1[2 + p,
 1 + p, 3 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2]
 + 2*p*AppellF1[2 + p, 1 + p, 4 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f*x
)/2]]*Tan[(e + f*x)/2]))
```

Maple [F]
time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{(g \tan (f x+e))^{p}}{(a+a \sin (f x+e))^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(g*tan(f*x + e))^p/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a
^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(g\tan{(e+fx)})^p}{\sin^2{(e+fx)}+2\sin{(e+fx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral((g*tan(e + f*x))**p/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x)/a**
2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\tan(e+f\,x))^p}{(a+a\,\sin(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^2, x)
```

3.128 $\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=248

$$\frac{(g\tan(e+fx))^{1+p}}{a^3fg(1+p)}-\frac{3\cos^2(e+fx)^{\frac{7+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{7+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^5(e+fx)(g\tan(e+fx))^{2+p}}{a^3fg^2(2+p)}+\frac{5(g$$

```
[Out] (g*tan(f*x+e))^(1+p)/a^3/f/g/(1+p)-3*(cos(f*x+e)^2)^(7/2+1/2*p)*hypergeom([
1+1/2*p, 7/2+1/2*p],[2+1/2*p],sin(f*x+e)^2)*sec(f*x+e)^5*(g*tan(f*x+e))^(2+
p)/a^3/f/g^2/(2+p)+5*(g*tan(f*x+e))^(3+p)/a^3/f/g^3/(3+p)-(cos(f*x+e)^2)^(7
/2+1/2*p)*hypergeom([2+1/2*p, 7/2+1/2*p],[3+1/2*p],sin(f*x+e)^2)*sec(f*x+e)
^3*(g*tan(f*x+e))^(4+p)/a^3/f/g^4/(4+p)+4*(g*tan(f*x+e))^(5+p)/a^3/f/g^5/(5
+p)
```

Rubi [A]

time = 0.29, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2790, 2687, 276, 16, 2697, 14}

$$\frac{4(g\tan(e+fx))^{p+5}}{a^3fg^5(p+5)}-\frac{\sec^3(e+fx)\cos^2(e+fx)^{\frac{p+7}{2}}(g\tan(e+fx))^{p+4}\,{}_2F_1\left(\frac{p+4}{2},\frac{p+7}{2};\frac{p+6}{2};\sin^2(e+fx)\right)}{a^3fg^4(p+4)}+\frac{5(g\tan(e+fx))^{p+3}}{a^3fg^3(p+3)}-\frac{3\sec^5(e+fx)\cos^2(e+fx)^{\frac{p+7}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+7}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{a^3fg^2(p+2)}+\frac{(g\tan(e+fx))^{p+1}}{a^3fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (g*Tan[e + f*x])^(1 + p)/(a^3*f*g*(1 + p)) - (3*(Cos[e + f*x]^2)^((7 + p)/2
)*Hypergeometric2F1[(2 + p)/2, (7 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e
+ f*x]^5*(g*Tan[e + f*x])^(2 + p))/(a^3*f*g^2*(2 + p)) + (5*(g*Tan[e + f*x]
)^(3 + p))/(a^3*f*g^3*(3 + p)) - ((Cos[e + f*x]^2)^((7 + p)/2)*Hypergeometr
ic2F1[(4 + p)/2, (7 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(g*Ta
n[e + f*x])^(4 + p))/(a^3*f*g^4*(4 + p)) + (4*(g*Tan[e + f*x])^(5 + p))/(a^
3*f*g^5*(5 + p))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx = \frac{\int (a^3\sec^6(e+fx)(g\tan(e+fx))^p - 3a^3\sec^5(e+fx)\tan(e+fx)(g\tan(e+f$$

$$= \frac{\int \sec^6(e+fx)(g\tan(e+fx))^p\,dx}{a^3} - \frac{\int \sec^3(e+fx)\tan^3(e+fx)(g\tan(e+fx}{a^3}$$

$$= \frac{\mathrm{Subst}\left(\int (gx)^p\left(1+x^2\right)^2\,dx, x, \tan(e+fx)\right)}{a^3 f} - \frac{\int \sec^3(e+fx)(g\tan(e+fx))^{3+}}{a^3g^3}$$

$$= -\frac{3\cos^2(e+fx)^{\frac{7+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{7+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^5(e+fx)(g\tan(e+fx)}{a^3fg^2(2+p)}$$

$$= \frac{(g\tan(e+fx))^{1+p}}{a^3fg(1+p)} - \frac{3\cos^2(e+fx)^{\frac{7+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{7+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^5(e}{a^3fg^2(2+p)}$$

$$= \frac{(g\tan(e+fx))^{1+p}}{a^3fg(1+p)} - \frac{3\cos^2(e+fx)^{\frac{7+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{7+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^5(e}{a^3fg^2(2+p)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 5.44, size = 1200, normalized size = 4.84

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((2 + p)*(AppellF1[1 + p, p, 2 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)
/2]] - 4*AppellF1[1 + p, p, 3 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/
2]] + 8*AppellF1[1 + p, p, 4 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2
]] - 8*AppellF1[1 + p, p, 5 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]
] + 4*AppellF1[1 + p, p, 6 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]
)*Sin[e + f*x]*(g*Tan[e + f*x])^p)/(a^3*f*(1 + p)*(1 + Sin[e + f*x])^3*((2
+ p)*AppellF1[1 + p, p, 2 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]
- 4*(2 + p)*AppellF1[1 + p, p, 3 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*
x)/2]] + 16*AppellF1[1 + p, p, 4 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f*
x)/2]] + 8*p*AppellF1[1 + p, p, 4 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f
*x)/2]] - 16*AppellF1[1 + p, p, 5 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f
*x)/2]] - 8*p*AppellF1[1 + p, p, 5 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e +
f*x)/2]] + 8*AppellF1[1 + p, p, 6 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e + f
*x)/2]] + 4*p*AppellF1[1 + p, p, 6 + p, 2 + p, Tan[(e + f*x)/2], -Tan[(e +
f*x)/2]] - 2*AppellF1[2 + p, p, 3 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f
*x)/2]]*Tan[(e + f*x)/2] - p*AppellF1[2 + p, p, 3 + p, 3 + p, Tan[(e + f*x)
/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2] + 12*AppellF1[2 + p, p, 4 + p, 3 +
 p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2] + 4*p*AppellF1[2
+ p, p, 4 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2]
 - 32*AppellF1[2 + p, p, 5 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]
*Tan[(e + f*x)/2] - 8*p*AppellF1[2 + p, p, 5 + p, 3 + p, Tan[(e + f*x)/2],
-Tan[(e + f*x)/2]]*Tan[(e + f*x)/2] + 40*AppellF1[2 + p, p, 6 + p, 3 + p, T
an[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2] + 8*p*AppellF1[2 + p,
p, 6 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2] - 24
*AppellF1[2 + p, p, 7 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[
(e + f*x)/2] - 4*p*AppellF1[2 + p, p, 7 + p, 3 + p, Tan[(e + f*x)/2], -Tan[
(e + f*x)/2]]*Tan[(e + f*x)/2] + p*AppellF1[2 + p, 1 + p, 2 + p, 3 + p, Tan
[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2] - 4*p*AppellF1[2 + p, 1
+ p, 3 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2] +
8*p*AppellF1[2 + p, 1 + p, 4 + p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2
]]*Tan[(e + f*x)/2] - 8*p*AppellF1[2 + p, 1 + p, 5 + p, 3 + p, Tan[(e + f*x
)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2] + 4*p*AppellF1[2 + p, 1 + p, 6 +
p, 3 + p, Tan[(e + f*x)/2], -Tan[(e + f*x)/2]]*Tan[(e + f*x)/2]))
```

Maple [F]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{(g \tan (fx+e))^p}{(a+a \sin (fx+e))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(g*tan(f*x + e))^p/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x +
 e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(g\tan(e+fx))^p}{\sin^3(e+fx)+3\sin^2(e+fx)+3\sin(e+fx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Integral((g*tan(e + f*x))**p/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e
 + f*x) + 1), x)/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\tan(e+f\,x))^p}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^3,x)
[Out] int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^3, x)
```

3.129 $\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=111

$$\frac{F_1\left(1+p;\frac{1+p}{2},\frac{1}{2}(1-2m+p);2+p;\sin(e+fx),-\sin(e+fx)\right)(1-\sin(e+fx))^{\frac{1+p}{2}}(1+\sin(e+fx))^{\frac{1}{2}(1-2m}}{fg(1+p)}$$

```
[Out] AppellF1(1+p,1/2-m+1/2*p,1/2+1/2*p,2+p,-sin(f*x+e),sin(f*x+e))*(1-sin(f*x+e
))^(1/2+1/2*p)*(1+sin(f*x+e))^(1/2-m+1/2*p)*(a+a*sin(f*x+e))^m*(g*tan(f*x+e
))^(1+p)/f/g/(1+p)
```

Rubi [A]
time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2799, 140, 138}

$$\frac{(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^m(g\tan(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m+p+1)}F_1\left(p+1;\frac{p+1}{2},\frac{1}{2}(-2m+p+1);p+2;\sin(e+fx),-\sin(e+fx)\right)}{fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] (AppellF1[1 + p, (1 + p)/2, (1 - 2*m + p)/2, 2 + p, Sin[e + f*x], -Sin[e +
f*x]]*(1 - Sin[e + f*x])^((1 + p)/2)*(1 + Sin[e + f*x])^((1 - 2*m + p)/2)*(
a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2799

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_), x_Symbol] :> Dist[(g*Tan[e + f*x])^(p + 1)*(a - b*Sin[e + f*x])^
((p + 1)/2)*((a + b*Sin[e + f*x])^((p + 1)/2)/(f*g*(b*Sin[e + f*x])^(p + 1)
)), Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2)), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&
```

```
!IntegerQ[m] &&  !IntegerQ[p]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m (g\tan(e+fx))^p\,dx = \frac{\left((a\sin(e+fx))^{-1-p}(a-a\sin(e+fx))^{\frac{1+p}{2}}(a+a\sin(e+fx)\right.}{}$$

$$= \frac{\left((1-\sin(e+fx))^{\frac{1}{2}+\frac{p}{2}}(a\sin(e+fx))^{-1-p}(a-a\sin(e+fx))\right.}{}$$

$$= \frac{\left((1-\sin(e+fx))^{\frac{1}{2}+\frac{p}{2}}(a\sin(e+fx))^{-1-p}(1+\sin(e+fx))^{\frac{1}{2}}\right.}{}$$

$$= \frac{F_1\left(1+p;\frac{1+p}{2},\frac{1}{2}(1-2m+p);2+p;\sin(e+fx),-\sin(e+fx\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 367 vs. 2(111) = 222.
time = 1.42, size = 367, normalized size = 3.31

$$-\frac{2(-3+p)F_1\left(\frac{1-p}{2};-p,1+m;\frac{3-p}{2};\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\cos^3\left(\frac{1}{4}(2e-\pi+2fx)\right)(a(1+\sin(e+fx)))^m\sin\left(\frac{1}{4}(2e-\pi+2fx)\right)(g\tan(e+fx))^p}{f(-1+p)\left((-3+p)F_1\left(\frac{1-p}{2};-p,1+m;\frac{3-p}{2};\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\cos^2\left(\frac{1}{4}(2e-\pi+2fx)\right)+2\left(pF_1\left(\frac{3-p}{2};1-p,1+m;\frac{5-p}{2};\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)+(1+m)F_1\left(\frac{3-p}{2};-p,2+m;\frac{5-p}{2};\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] (-2*(-3 + p)*AppellF1[(1 - p)/2, -p, 1 + m, (3 - p)/2, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[(2*e - Pi + 2*f*x)/4]^3*(a*(1 +
Sin[e + f*x]))^m*Sin[(2*e - Pi + 2*f*x)/4]*(g*Tan[e + f*x])^p)/(f*(-1 + p)*
((-3 + p)*AppellF1[(1 - p)/2, -p, 1 + m, (3 - p)/2, Cot[(2*e + Pi + 2*f*x)/
4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[(2*e - Pi + 2*f*x)/4]^2 + 2*(p*Appe
llF1[(3 - p)/2, 1 - p, 1 + m, (5 - p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[
(2*e - Pi + 2*f*x)/4]^2] + (1 + m)*AppellF1[(3 - p)/2, -p, 2 + m, (5 - p)/2
, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Sin[(2*e - Pi
 + 2*f*x)/4]^2))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m (g\tan(fx+e))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (g\tan(e+fx))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(g*tan(f*x+e))**p,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(g*tan(e + f*x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\tan(e+f\,x))^p\,(a+a\,\sin(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^m,x)
[Out] int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^m, x)
```

3.130 $\int (a+a\sin(e+fx))^m \tan^3(e+fx)\,dx$

Optimal. Leaf size=163

$$\frac{a(4+m)\,{}_2F_1\left(1,-1+m;m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{-1+m}}{4f(1-m)}-\frac{a^2\sin^2(e+fx)(a+a\sin(e+fx}{fm(a-a\sin(e+fx))}$$

```
[Out] 1/4*a*(4+m)*hypergeom([1, -1+m],[m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))^(-
1+m)/f/(1-m)-a^2*sin(f*x+e)^2*(a+a*sin(f*x+e))^(-1+m)/f/m/(a-a*sin(f*x+e))+
1/2*(a+a*sin(f*x+e))^(-1+m)*(a*(-m^2-3*m+2)+2*a*m*sin(f*x+e))/f/(1-m)/m/(1-
sin(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2786, 102, 151, 70}

$$-\frac{a^2\sin^2(e+fx)(a\sin(e+fx)+a)^{m-1}}{fm(a-a\sin(e+fx))}+\frac{a(m+4)(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)}{4f(1-m)}+\frac{\left(2am\sin(e+fx)+a\left(-m^2-3m+2\right)\right)(a\sin(e+fx)+a)^{m-1}}{2f(1-m)m(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] (a*(4 + m)*Hypergeometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]*(a + a*Sin
[e + f*x])^(-1 + m))/(4*f*(1 - m)) - (a^2*Sin[e + f*x]^2*(a + a*Sin[e + f*x
])^(-1 + m))/(f*m*(a - a*Sin[e + f*x])) + ((a + a*Sin[e + f*x])^(-1 + m)*(a
*(2 - 3*m - m^2) + 2*a*m*Sin[e + f*x]))/(2*f*(1 - m)*m*(1 - Sin[e + f*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\tan^3(e+fx)\,dx=\frac{\text{Subst}\left(\int\frac{x^3(a+x)^{-2+m}}{(a-x)^2}\,dx,x,a\sin(e+fx)\right)}{f}$$
$$=-\frac{a^2\sin^2(e+fx)(a+a\sin(e+fx))^{-1+m}}{fm(a-a\sin(e+fx))}-\frac{\text{Subst}\left(\int\frac{x(a+x)^{-2+m}(-}{(a-x)}\right.}{}$$
$$=-\frac{a^2\sin^2(e+fx)(a+a\sin(e+fx))^{-1+m}}{fm(a-a\sin(e+fx))}+\frac{(a+a\sin(e+fx))^{-1}}{2f(1}$$
$$=\frac{a(4+m)\,{}_2F_1\left(1,-1+m;m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx}{4f(1-m)}$$

Mathematica [A]
time = 0.17, size = 105, normalized size = 0.64

$$\frac{a(a(1+\sin(e+fx)))^{-1+m}\left(-2(-2+3m+m^2)-m(4+m)\,{}_2F_1\left(1,-1+m;m;\frac{1}{2}(1+\sin(e+fx))\right)(-1+\sin(e+fx))+4m\sin(e+fx)+4(-1+m)\sin^2(e+fx)\right)}{4f(-1+m)m(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] (a*(a*(1 + Sin[e + f*x]))^(-1 + m)*(-2*(-2 + 3*m + m^2) - m*(4 + m)*Hyperge
ometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]*(-1 + Sin[e + f*x]) + 4*m*Si
```

```
n[e + f*x] + 4*(-1 + m)*Sin[e + f*x]^2))/(4*f*(-1 + m)*m*(-1 + Sin[e + f*x]
))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (a + a\sin(fx + e))^m \left(\tan^3(fx + e)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x)
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e + fx) + 1))^m \tan^3(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**3,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^3\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)^3*(a + a*sin(e + f*x))^m, x)
```

3.131 $\int (a+a\sin(e+fx))^m \tan(e+fx)\,dx$

Optimal. Leaf size=72

$$-\frac{(a+a\sin(e+fx))^m}{2fm}+\frac{{}_2F_1\left(1,1+m;2+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{1+m}}{4af(1+m)}$$

```
[Out] -1/2*(a+a*sin(f*x+e))^m/f/m+1/4*hypergeom([1, 1+m],[2+m],1/2+1/2*sin(f*x+e)
)*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2786, 80, 70}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)}{4af(m+1)}-\frac{(a\sin(e+fx)+a)^m}{2fm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] -1/2*(a + a*Sin[e + f*x])^m/(f*m) + (Hypergeometric2F1[1, 1 + m, 2 + m, (1
+ Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^(1 + m))/(4*a*f*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^{-1+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{f}$$
$$= -\frac{(a+a\sin(e+fx))^m}{2fm} + \frac{\text{Subst}\left(\int \frac{(a+x)^m}{a-x}\,dx, x, a\sin(e+fx)\right)}{2f}$$
$$= -\frac{(a+a\sin(e+fx))^m}{2fm} + \frac{{}_2F_1\left(1,1+m;2+m;\frac{1}{2}(1+\sin(e+fx))\right)}{4af(1+m)}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 0.88

$$\frac{(a(1+\sin(e+fx)))^m\left(-2(1+m)+m\,{}_2F_1\left(1,1+m;2+m;\frac{1}{2}(1+\sin(e+fx))\right)(1+\sin(e+fx))\right)}{4fm(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x],x]
[Out] ((a*(1 + Sin[e + f*x]))^m*(-2*(1 + m) + m*Hypergeometric2F1[1, 1 + m, 2 + m
, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))/(4*f*m*(1 + m))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m\tan(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e),x)
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e + fx) + 1))^m \tan(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e + f\,x)\,\left(a + a\,\sin(e + f\,x)\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + a*sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)*(a + a*sin(e + f*x))^m, x)
```

3.132 $\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=43

$$-\frac{{}_2F_1(1,1+m;2+m;1+\sin(e+fx))(a+a\sin(e+fx))^{1+m}}{af(1+m)}$$

```
[Out] -hypergeom([1, 1+m],[2+m],1+sin(f*x+e))*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 67}

$$-\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(e+fx)+1)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x]
)^(1 + m))/(a*f*(1 + m)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{x}\,dx,x,a\sin(e+fx)\right)}{f}\\ &= -\frac{{}_2F_1(1,1+m;2+m;1+\sin(e+fx))(a+a\sin(e+fx))^{1+m}}{af(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 1.00

$$-\frac{{}_2F_1(1,1+m;2+m;1+\sin(e+fx))(a+a\sin(e+fx))^{1+m}}{af(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x]
)^(1 + m))/(a*f*(1 + m)))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \cot(fx+e)(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m\cot(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e + f\,x)\,\left(a + a\,\sin\left(e + f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)*(a + a*sin(e + f*x))^m, x)
```

3.133 $\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{\csc^2(e+fx)(a+a\sin(e+fx))^{2+m}}{2a^2f}-\frac{(2-m)\,{}_2F_1(2,2+m;3+m;1+\sin(e+fx))(a+a\sin(e+fx))^{2+m}}{2a^2f(2+m)}$$

```
[Out] -1/2*csc(f*x+e)^2*(a+a*sin(f*x+e))^(2+m)/a^2/f-1/2*(2-m)*hypergeom([2, 2+m]
,[3+m],1+sin(f*x+e))*(a+a*sin(f*x+e))^(2+m)/a^2/f/(2+m)
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2786, 79, 67}

$$-\frac{(2-m)(a\sin(e+fx)+a)^{m+2}\,{}_2F_1(2,m+2;m+3;\sin(e+fx)+1)}{2a^2f(m+2)}-\frac{\csc^2(e+fx)(a\sin(e+fx)+a)^{m+2}}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -1/2*(Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(2 + m))/(a^2*f) - ((2 - m)*Hyper
geometric2F1[2, 2 + m, 3 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(2 + m
))/(2*a^2*f*(2 + m))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
```

```
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^{1+m}}{x^3}\,dx, x, a\sin(e+fx)\right)}{f}$$
$$= -\frac{\csc^2(e+fx)(a+a\sin(e+fx))^{2+m}}{2a^2f} - \frac{(2-m)\text{Subst}\left(\int \frac{(a+x)^{1+m}}{x^2}\right.}{2f}$$
$$= -\frac{\csc^2(e+fx)(a+a\sin(e+fx))^{2+m}}{2a^2f} - \frac{(2-m)\,{}_2F_1(2,2+m;3+}{}$$

Mathematica [A]
time = 0.13, size = 68, normalized size = 0.82

$$-\frac{((2+m)\csc^2(e+fx)-(-2+m)\,{}_2F_1(2,2+m;3+m;1+\sin(e+fx)))\,(1+\sin(e+fx))^2(a(1+\sin(e+fx)))^m}{2f(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + a*Sin[e + f*x])^m,x]
[Out] -1/2*(((2 + m)*Csc[e + f*x]^2 - (-2 + m)*Hypergeometric2F1[2, 2 + m, 3 + m,
 1 + Sin[e + f*x]])*(1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^m)/(f*(2 +
m))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(\cot^3(fx+e)\right)(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x)
[Out] int(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (e+fx)+1))^m \cot^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+a*sin(f*x+e))**m,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^3\,(a+a\,\sin(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + a*sin(e + f*x))^m,x)
[Out] int(cot(e + f*x)^3*(a + a*sin(e + f*x))^m, x)
```

3.134 $\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=123

$$\frac{(9-m)\csc^3(e+fx)(a+a\sin(e+fx))^{3+m}}{12a^3f}-\frac{\csc^4(e+fx)(a+a\sin(e+fx))^{3+m}}{4a^3f}-\frac{(12-9m+m^2)\ {}_2F_1(3,3}{}$$

[Out] 1/12*(9-m)*csc(f*x+e)^3*(a+a*sin(f*x+e))^(3+m)/a^3/f-1/4*csc(f*x+e)^4*(a+a*sin(f*x+e))^(3+m)/a^3/f-1/12*(m^2-9*m+12)*hypergeom([3, 3+m],[4+m],1+sin(f*x+e))*(a+a*sin(f*x+e))^(3+m)/a^3/f/(3+m)

Rubi [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2786, 91, 79, 67}

$$-\frac{(m^2-9m+12)\,(a\sin(e+fx)+a)^{m+3}\,{}_2F_1(3,m+3;m+4;\sin(e+fx)+1)}{12a^3f(m+3)}-\frac{\csc^4(e+fx)(a\sin(e+fx)+a)^{m+3}}{4a^3f}+\frac{(9-m)\csc^3(e+fx)(a\sin(e+fx)+a)^{m+3}}{12a^3f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^5*(a + a*Sin[e + f*x])^m,x]

[Out] ((9 - m)*Csc[e + f*x]^3*(a + a*Sin[e + f*x])^(3 + m))/(12*a^3*f) - (Csc[e + f*x]^4*(a + a*Sin[e + f*x])^(3 + m))/(4*a^3*f) - ((12 - 9*m + m^2)*Hypergeometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m))/(12*a^3*f*(3 + m))

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
```

```
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^{2+m}}{x^5}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= -\frac{\csc^4(e+fx)(a+a\sin(e+fx))^{3+m}}{4a^3f} + \frac{\text{Subst}\left(\int \frac{(a+x)^{2+m}(-a^2(9-}{x^4}\right.}{}$$

$$= \frac{(9-m)\csc^3(e+fx)(a+a\sin(e+fx))^{3+m}}{12a^3f} - \frac{\csc^4(e+fx)(a+}{4a}$$

$$= \frac{(9-m)\csc^3(e+fx)(a+a\sin(e+fx))^{3+m}}{12a^3f} - \frac{\csc^4(e+fx)(a+}{4a}$$

Mathematica [A]
time = 0.18, size = 83, normalized size = 0.67

$$-\frac{((3+m)\csc^3(e+fx)(-9+m+3\csc(e+fx))+(12-9m+m^2)\,{}_2F_1(3,3+m;4+m;1+\sin(e+fx)))\,(1+\sin(e+fx))^3(a(1+\sin(e+fx)))^m}{12f(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -1/12*(((3 + m)*Csc[e + f*x]^3*(-9 + m + 3*Csc[e + f*x]) + (12 - 9*m + m^2)
*Hypergeometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]])*(1 + Sin[e + f*x])^3
*(a*(1 + Sin[e + f*x]))^m)/(f*(3 + m))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(\cot^5(fx+e)\right)(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (e+fx)+1))^m \cot^5 (e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^5\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + a*sin(e + f*x))^m,x)
[Out] int(cot(e + f*x)^5*(a + a*sin(e + f*x))^m, x)
```

3.135 $\int (a+a\sin(e+fx))^m \tan^4(e+fx)\,dx$

Optimal. Leaf size=311

$$\frac{2^{-\frac{3}{2}+m}(9-12m-7m^2+6m^3+m^4)\ {}_2F_1\left(\frac{1}{2},\frac{5}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1-\sin(e+fx))(1+}{3f(1-m)m}$$

```
[Out] 1/3*2^(-3/2+m)*(m^4+6*m^3-7*m^2-12*m+9)*hypergeom([1/2, 5/2-m],[3/2],1/2-1/
2*sin(f*x+e))*sec(f*x+e)*(1-sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x
+e))^m/f/(1-m)/m-1/3*sec(f*x+e)*(a+a*sin(f*x+e))^(-1+m)*(a*(-m^3-7*m^2-m+6)
-a*(-m^3-8*m^2-6*m+9)*sin(f*x+e))/f/(1-m)/m/(1-sin(f*x+e))+a^2*sin(f*x+e)*(
a+a*sin(f*x+e))^(-1+m)*tan(f*x+e)/f/(1-m)/(a-a*sin(f*x+e))-a^2*sin(f*x+e)^2
*(a+a*sin(f*x+e))^(-1+m)*tan(f*x+e)/f/m/(a-a*sin(f*x+e))
```

Rubi [A]

time = 0.25, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2798, 102, 158, 150, 72, 71}

$$-\frac{a^2\sin^2(e+fx)\tan(e+fx)(a\sin(e+fx)+a)^{m-1}}{fm(a-a\sin(e+fx))}+\frac{a^2\sin(e+fx)\tan(e+fx)(a\sin(e+fx)+a)^{m-1}}{f(1-m)(a-a\sin(e+fx))}+\frac{2^{m-\frac{3}{2}}(m^4+6m^3-7m^2-12m+9)(1-\sin(e+fx))\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\ {}_2F_1\left(\frac{1}{2},\frac{5}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f(1-m)m}-\frac{\sec(e+fx)(a\sin(e+fx)+a)^{m-1}\left(a(-m^3-7m^2-m+6)-a(-m^3-8m^2-6m+9)\sin(e+fx)\right)}{3f(1-m)m(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] (2^(-3/2 + m)*(9 - 12*m - 7*m^2 + 6*m^3 + m^4)*Hypergeometric2F1[1/2, 5/2 -
 m, 3/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 - Sin[e + f*x])*(1 + Sin[e +
 f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(3*f*(1 - m)*m) - (Sec[e + f*x]*(a
 + a*Sin[e + f*x])^(-1 + m)*(a*(6 - m - 7*m^2 - m^3) - a*(9 - 6*m - 8*m^2 -
 m^3)*Sin[e + f*x]))/(3*f*(1 - m)*m*(1 - Sin[e + f*x])) + (a^2*Sin[e + f*x]
*(a + a*Sin[e + f*x])^(-1 + m)*Tan[e + f*x])/(f*(1 - m)*(a - a*Sin[e + f*x]
)) - (a^2*Sin[e + f*x]^2*(a + a*Sin[e + f*x])^(-1 + m)*Tan[e + f*x])/(f*m*(
a - a*Sin[e + f*x]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
```

```
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 2798

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x])), Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2
)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m \tan^4(e+fx)\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\,\right)\mathrm{Subst}\left(\int\right.}{af}$$

$$= -\frac{a^2\sin^2(e+fx)(a+a\sin(e+fx))^{-1+m}\tan(e+fx)}{fm(a-a\sin(e+fx))} - \frac{\left(\sec(e+\right.}{}$$

$$= \frac{a^2\sin(e+fx)(a+a\sin(e+fx))^{-1+m}\tan(e+fx)}{f(1-m)(a-a\sin(e+fx))} - \frac{a^2\sin^2(e+}{}$$

$$= -\frac{\sec(e+fx)(a+a\sin(e+fx))^{-1+m}\left(a(6-m-7m^2-m^3)-a\right.}{3f(1-m)m(1-\sin(e+fx)}$$

$$= -\frac{\sec(e+fx)(a+a\sin(e+fx))^{-1+m}\left(a(6-m-7m^2-m^3)-a\right.}{3f(1-m)m(1-\sin(e+fx)}$$

$$= \frac{2^{-\frac{3}{2}+m}(9-12m-7m^2+6m^3+m^4)\,{}_2F_1\left(\frac{1}{2},\frac{5}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e\right.}{}$$

Mathematica [F]
time = 0.76, size = 0, normalized size = 0.00

$$\int (a+a\sin(e+fx))^m\tan^4(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
[Out] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4, x]
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(\tan^4\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x)
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m \tan^4(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**4,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(e+f\,x)^4\,(a+a\,\sin(e+f\,x))^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)^4*(a + a*sin(e + f*x))^m, x)
```

3.136 $\int (a+a\sin(e+fx))^m \tan^2(e+fx)\,dx$

Optimal. Leaf size=157

$$\frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m}+\frac{2^{-\frac{1}{2}+m}(1-m-m^2)\ {}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1}{f(1-m)m}$$

```
[Out] sec(f*x+e)*(a+a*sin(f*x+e))^m/f/(1-m)/m+2^(-1/2+m)*(-m^2-m+1)*hypergeom([-1
/2, 3/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a
*sin(f*x+e))^m/f/(1-m)/m-sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/m
```

Rubi [A]
time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2792, 2939, 2768, 72, 71}

$$\frac{2^{m-\frac{1}{2}}(-m^2-m+1)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\ {}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(1-m)m}+\frac{\sec(e+fx)(a\sin(e+fx)+a)^m}{f(1-m)m}-\frac{\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{afm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] (Sec[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) + (2^(-1/2 + m)*(1 - m
- m^2)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Sec[e +
f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) - (
Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2792

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[(a + b*Sin[e + f*x])^m*((b*(m + 1) + a*Sin[e + f*x])/Cos[
e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^m\tan^2(e+fx)\,dx &= -\frac{\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{afm}+\frac{\int \sec^2(e+fx)(a+a\sin(\\ &= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m}-\frac{\sec(e+fx)(a+a\sin(e+fx}{afm}\\ &= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m}-\frac{\sec(e+fx)(a+a\sin(e+fx}{afm}\\ &= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m}-\frac{\sec(e+fx)(a+a\sin(e+fx}{afm}\\ &= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m}+\frac{2^{-\frac{1}{2}+m}(1-m-m^2)\ {}_2F_1\big(-\frac{1}{2}}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.26, size = 4043, normalized size = 25.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] (-2*Cos[(-e + Pi/2 - f*x)/2]*Hypergeometric2F1[1/2, (1 + 2*m)/2, (3 + 2*m)/
2, Cos[(-e + Pi/2 - f*x)/2]^2]*Sin[(-e + Pi/2 - f*x)/2]*(a + a*Sin[e + f*x]
)^m)/(f*(1 + 2*m)*Sqrt[Sin[(-e + Pi/2 - f*x)/2]^2]) - ((Cos[(-e + Pi/2 - f*
x)/4]^2)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]*(a + a*Sin[e + f*x])^m*(-(AppellF1[
-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f
*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m
, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(Appe
llF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2,
-Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)))/(4*f*(Cos[Pi/4
+ (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^2*(-1/2*(m*(Cos[(-e
 + Pi/2 - f*x)/4]^2)^(2*m)*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*
m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/
4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -
Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 +
 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-
e + Pi/2 - f*x)/4]^2))) - ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Csc[(-e + Pi/
2 - f*x)/4]^2*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2,
 -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (3*Appe
llF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/
(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x
)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*
x)/4]^2)))/8 + ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]
*(-(m*AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)
/4]) - (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(m*AppellF1[1/2, 1 - 2*m, 2*m, 3/
2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2
- f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4] + m*AppellF1[1/2, -2*m, 1 + 2*m, 3/2,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f
*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(
-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4
]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(2*(3*
AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
```

```
 f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4
]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(
-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/
4]^2)) + (3*Tan[(-e + Pi/2 - f*x)/4]^2*(-1/3*(m*AppellF1[3/2, 1 - 2*m, 2*m,
 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi
/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (m*AppellF1[3/2, -2*m, 1 + 2*m,
5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/
2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/3)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)
^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m,
 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + P
i/2 - f*x)/4]^2) - (3*m*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi
/2 - f*x)/4]^3*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/(3*AppellF1[1/2
, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
- 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2) - (3*A
ppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m
)*(-2*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2
- f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(
-e + Pi/2 - f*x)/4] + 3*(-1/3*(m*AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e +
 Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]...
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (a + a\sin(fx + e))^m \left(\tan^2(fx + e)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x)
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m \tan^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**2,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^2\,(a+a\,\sin(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)^2*(a + a*sin(e + f*x))^m, x)
```

3.137 $\int (a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=74

$$-\frac{2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^m}{f}$$

```
[Out] -2^(1/2+m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+s
in(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2731, 2730}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^m\,dx &= \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^m\,dx\\ &= -\frac{2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-}}{f}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 90, normalized size = 1.22

$$\frac{\sqrt{2}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(a(1+\sin(e+fx)))^m}{(f+2fm)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (Cos[e + f*x
]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)/((f + 2*f*m)*
Sqrt[1 - Sin[e + f*x]])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m,x)
```

```
[Out] int((a+a*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(e+fx)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \sin(e + f x))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m,x)
```

```
[Out] int((a + a*sin(e + f*x))^m, x)
```

3.138 $\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{2}\,F_1\left(\frac{3}{2}+m;-\frac{1}{2},2;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}\,(a+a\sin(}{a^2f(3+2m)}$$

```
[Out] 2*AppellF1(3/2+m,2,-1/2,5/2+m,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(
a+a*sin(f*x+e))^(2+m)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^2/f/(3+2*m)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2798, 142, 141}

$$\frac{2\sqrt{2}\,\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{m+2}F_1\left(m+\frac{3}{2};-\frac{1}{2},2;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{a^2f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*Sqrt[2]*AppellF1[3/2 + m, -1/2, 2, 5/2 + m, (1 + Sin[e + f*x])/2, 1 + Si
n[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(2 + m
))/(a^2*f*(3 + 2*m))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2798

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
```

```
f*Cos[e + f*x])), Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2
)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\Big(\int$$
$$af$$

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}}{af\sqrt{\frac{a-a\sin(e+fx}{a}}}$$

$$= \frac{2\sqrt{2}F_1\left(\frac{3}{2}+m;-\frac{1}{2},2;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx\right.}{a^2f(3+2m}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.88, size = 5048, normalized size = 56.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
[Out] Result too large to show
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\cot^2(fx+e)\right)(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
[Out] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m \cot^2(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^2\,(a+a\,\sin(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)^2*(a + a*sin(e + f*x))^m, x)
```

3.139 $\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=89

$$\frac{4\sqrt{2}\,F_1\left(\frac{5}{2}+m;-\frac{3}{2},4;\frac{7}{2}+m;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}\,(a+a\sin$$
$$a^3f(5+2m)$$

```
[Out] 4*AppellF1(5/2+m,4,-3/2,7/2+m,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(
a+a*sin(f*x+e))^(3+m)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^3/f/(5+2*m)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2798, 142, 141}

$$\frac{4\sqrt{2}\,\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{m+3}F_1\left(m+\frac{5}{2};-\frac{3}{2},4;m+\frac{7}{2};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{a^3f(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (4*Sqrt[2]*AppellF1[5/2 + m, -3/2, 4, 7/2 + m, (1 + Sin[e + f*x])/2, 1 + Si
n[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m
))/(a^3*f*(5 + 2*m))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2798

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
```

```
f*Cos[e + f*x])), Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2
)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rubi steps

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\Big(\int}{af}$$

$$= \frac{\left(2\sqrt{2}\,\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{4\sqrt{2}\,F_1\left(\frac{5}{2}+m;-\frac{3}{2},4;\frac{7}{2}+m;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)}{a^3f(5+2m)}$$

Mathematica [**F**]
time = 0.51, size = 0, normalized size = 0.00

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m,x]
[Out] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m, x]
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\cot^4(fx+e)\right)(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x)
[Out] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin{(e+fx)}+1))^m \cot^4{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{cot}\left(e+f\,x\right)}^4\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)^4*(a + a*sin(e + f*x))^m, x)
```

3.140 $\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{(2a+3b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-3b)\log(1+\sin(c+dx))}{4d}+\frac{3b\sin(c+dx)}{2d}+\frac{(a+b\sin(c+dx))\tan^2(c+$$

```
[Out] 1/4*(2*a+3*b)*ln(1-sin(d*x+c))/d+1/4*(2*a-3*b)*ln(1+sin(d*x+c))/d+3/2*b*sin
(d*x+c)/d+1/2*(a+b*sin(d*x+c))*tan(d*x+c)^2/d
```

Rubi [A]

time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2800, 833, 788, 647, 31}

$$\frac{(2a+3b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-3b)\log(\sin(c+dx)+1)}{4d}+\frac{\tan^2(c+dx)(a+b\sin(c+dx))}{2d}+\frac{3b\sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] ((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(4*d) + ((2*a - 3*b)*Log[1 + Sin[c + d*
x]])/(4*d) + (3*b*Sin[c + d*x])/(2*d) + ((a + b*Sin[c + d*x])*Tan[c + d*x]^
2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
```

```
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{(a+b\sin(c+dx))\tan^2(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{x(2ab^2+3b^2x)}{b^2-x^2}\,dx, x, b\sin(\right.}{2b^2d}$$
$$= \frac{3b\sin(c+dx)}{2d} + \frac{(a+b\sin(c+dx))\tan^2(c+dx)}{2d} + \frac{\text{Subst}\left(\int \frac{-3b^4-}{b^2-}\right.}{}$$
$$= \frac{3b\sin(c+dx)}{2d} + \frac{(a+b\sin(c+dx))\tan^2(c+dx)}{2d} - \frac{(2a-3b)\text{Subst}}{}$$
$$= \frac{(2a+3b)\log(1-\sin(c+dx))}{4d} + \frac{(2a-3b)\log(1+\sin(c+dx))}{4d} +$$

Mathematica [A]
time = 0.09, size = 77, normalized size = 0.88

$$-\frac{b\sin(c+dx)\tan^2(c+dx)}{d} - \frac{3b\left(\tanh^{-1}(\sin(c+dx)) - \sec(c+dx)\tan(c+dx)\right)}{2d} + \frac{a(2\log(\cos(c+dx)) + \tan^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((b*Sin[c + d*x]*Tan[c + d*x]^2)/d) - (3*b*(ArcTanh[Sin[c + d*x]] - Sec[c
+ d*x]*Tan[c + d*x]))/(2*d) + (a*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2
*d)
```

Maple [A]
time = 0.17, size = 81, normalized size = 0.92

method	result
derivativedivides	$\frac{a\left(\frac{\left(\tan^{2}(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+b\left(\frac{\sin^{5}(dx+c)}{2\cos(dx+c)^{2}}+\frac{\left(\sin^{3}(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a\left(\frac{\left(\tan^{2}(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+b\left(\frac{\sin^{5}(dx+c)}{2\cos(dx+c)^{2}}+\frac{\left(\sin^{3}(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$-iax-\frac{ib\,e^{i(dx+c)}}{2d}+\frac{ib\,e^{-i(dx+c)}}{2d}-\frac{2iac}{d}-\frac{i\left(b\,e^{3i(dx+c)}-b\,e^{i(dx+c)}+2ia\,e^{2i(dx+c)}\right)}{d\left(1+e^{2i(dx+c)}\right)^{2}}+\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{3\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+b*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1
/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 73, normalized size = 0.83

$$\frac{(2a-3b)\log(\sin(dx+c)+1)+(2a+3b)\log(\sin(dx+c)-1)+4b\sin(dx+c)-\frac{2(b\sin(dx+c)+a)}{\sin(dx+c)^{2}-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((2*a - 3*b)*log(sin(d*x + c) + 1) + (2*a + 3*b)*log(sin(d*x + c) - 1)
+ 4*b*sin(d*x + c) - 2*(b*sin(d*x + c) + a)/(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.38, size = 90, normalized size = 1.02

$$\frac{(2a-3b)\cos(dx+c)^{2}\log(\sin(dx+c)+1)+(2a+3b)\cos(dx+c)^{2}\log(-\sin(dx+c)+1)+2\left(2b\cos(dx+c)^{2}+b\right)\sin(dx+c)+2a}{4d\cos(dx+c)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (2*a + 3*b)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*(2*b*cos(d*x + c)^2 + b)*sin(d*x + c) +
2*a)/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))\tan^{3}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 26228 vs. $2(80) = 160$.
time = 230.66, size = 26228, normalized size = 298.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d
*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 3*b*log(2*(tan(1/2*d*x)^4*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + ta
n(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*t
an(c)^2 + 2*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*
tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 2*a*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/
2*c)^6*tan(c)^2 - 6*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c) + 6*b*log(2*(tan(1/2*d*
x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*
c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^6*tan(1/2
*c)^6*tan(c) - 4*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*
x)*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c) - 3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2
*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)
^2 + 3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
```

```
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d
*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 2*a*log(4*(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 2
4*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2
*tan(1/2*d*x)^5*tan(1/2*c)^5*tan(c)^2 + 24*b*log(2*(tan(1/2*d*x)^4*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^5*tan(1/2*c)^5*tan(c
)^2 - 16*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan
(1/2*d*x)^5*tan(1/2*c)^5*tan(c)^2 - 12*b*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*
c)^5*tan(c)^2 - 3*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*t
an(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
+ 1))*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 3*b*log(2*(tan(1/2*
d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/
2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
- 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^4*tan
(1/2*c)^6*tan(c)^2 - 2*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
 tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*
tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 - 12*b*tan(d*x)^2*tan(1/2*d
*x)^5*tan(1/2*c)^6*tan(c)^2 + 2*a*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 -
2*a*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 16*a*tan(d*x)^2*tan(1
/2*d*x)^5*tan(1/2*c)^5*tan(c)^2 - 2*a*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^
6*tan(c)^2 + 2*a*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 3*b*log(2*(tan(1/2*
d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/
2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*t...
```

Mupad [B]
time = 6.73, size = 176, normalized size = 2.00

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(a+\frac{3\,b}{2}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(a-\frac{3\,b}{2}\right)}{d}-\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{3\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) - 1)*(a + (3*b)/2))/d + (log(tan(c/2 + (d*x)/2) + 1
)*(a - (3*b)/2))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (3*b*tan(c/2 + (
d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2
 + (d*x)/2)^3 + 3*b*tan(c/2 + (d*x)/2)^5)/(d*(tan(c/2 + (d*x)/2)^2 + tan(c/
2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

3.141 $\int (a+b\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=55

$$-\frac{(a+b)\log(1-\sin(c+dx))}{2d}-\frac{(a-b)\log(1+\sin(c+dx))}{2d}-\frac{b\sin(c+dx)}{d}$$

```
[Out] -1/2*(a+b)*ln(1-sin(d*x+c))/d-1/2*(a-b)*ln(1+sin(d*x+c))/d-b*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2800, 788, 647, 31}

$$-\frac{(a+b)\log(1-\sin(c+dx))}{2d}-\frac{(a-b)\log(\sin(c+dx)+1)}{2d}-\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -1/2*((a + b)*Log[1 - Sin[c + d*x]])/d - ((a - b)*Log[1 + Sin[c + d*x]])/(2
*d) - (b*Sin[c + d*x])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\sin(c+dx)}{d} - \frac{\text{Subst}\left(\int \frac{-b^2-ax}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\sin(c+dx)}{d} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2d} + \frac{(a+b}{}$$
$$= -\frac{(a+b)\log(1-\sin(c+dx))}{2d} - \frac{(a-b)\log(1+\sin(c+dx))}{2d} - \frac{b\sin}{}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.69

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\log(\cos(c+dx))}{d} - \frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x],x]
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d - (b*Sin[c + d*x])/d
```

Maple [A]
time = 0.12, size = 41, normalized size = 0.75

method	result
derivativedivides	$\frac{-a\ln(\cos(dx+c))+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
default	$\frac{-a\ln(\cos(dx+c))+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
risch	$iax + \frac{ib\,e^{i(dx+c)}}{2d} - \frac{ib\,e^{-i(dx+c)}}{2d} + \frac{2iac}{d} - \frac{a\ln\left(e^{i(dx+c)}-i\right)}{d} - \frac{\ln\left(e^{i(dx+c)}-i\right)b}{d} - \frac{a\ln\left(e^{i(dx+c)}+i\right)}{d} + \frac{\ln(e^{i(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(-a*ln(cos(d*x+c))+b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 43, normalized size = 0.78

$$-\frac{(a-b)\log(\sin(dx+c)+1)+(a+b)\log(\sin(dx+c)-1)+2\,b\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*log(sin(d*x + c) + 1) + (a + b)*log(sin(d*x + c) - 1) + 2*b*s
in(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 45, normalized size = 0.82

$$-\frac{(a-b)\log(\sin(dx+c)+1)+(a+b)\log(-\sin(dx+c)+1)+2b\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*((a - b)*log(sin(d*x + c) + 1) + (a + b)*log(-sin(d*x + c) + 1) + 2*b*
sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1456 vs. $2(51) = 102$.
time = 7.52, size = 1456, normalized size = 26.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2
```

```
*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b*log(2*(tan(1/2*d*x)^4*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2
*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - b*log(2*(tan(1/2*d*x)
^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)
^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + a*log(4*(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)^2*tan
(1/2*c) + b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
n(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/
2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(1/2*c)^2 + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*
x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(1/2
*c)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c)^2 + b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d
*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*
c) + 1)/(tan(1/2*c)^2 + 1)) - b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(t
an(1/2*c)^2 + 1)) + a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 4
*b*tan(1/2*d*x) + 4*b*tan(1/2*c))/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/
2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

Mupad [B]
time = 6.64, size = 74, normalized size = 1.35

$$\frac{a\,\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(a-b\right)}{d}-\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(a+b\right)}{d}-\frac{b\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (log(tan(c/2 + (d*x)/2) + 1)*(a - b))
/d - (log(tan(c/2 + (d*x)/2) - 1)*(a + b))/d - (b*sin(c + d*x))/d
```

3.142 $\int \cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2800, 45}

$$\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a+x}{x}\,dx,x,b\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{a}{x}\right)\,dx,x,b\sin(c+dx)\right)}{d}\\
&= \frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 1.79

$$\frac{a(\log(\cos(c+dx))+\log(\tan(c+dx)))}{d}+\frac{b\cos(dx)\sin(c)}{d}+\frac{b\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
[Out] (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d + (b*Cos[d*x]*Sin[c])/d + (b*
Cos[c]*Sin[d*x])/d
```

Maple [A]
time = 0.10, size = 23, normalized size = 0.96

method	result	size
derivativedivides	$\frac{b\sin(dx+c)+a\ln(\sin(dx+c))}{d}$	23
default	$\frac{b\sin(dx+c)+a\ln(\sin(dx+c))}{d}$	23
risch	$-iax-\frac{2iac}{d}+\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{b\sin(dx+c)}{d}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*sin(d*x+c)+a*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.36, size = 22, normalized size = 0.92

$$\frac{a\log\left(\sin\left(dx+c\right)\right)+b\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] (a*log(sin(d*x + c)) + b*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 24, normalized size = 1.00

$$\frac{a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+b\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*log(1/2*sin(d*x + c)) + b*sin(d*x + c))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))\cot(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x)
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x), x)
```

Giac [**A**]
time = 9.45, size = 23, normalized size = 0.96

$$\frac{a\log(|\sin(dx + c)|) + b\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] (a*log(abs(sin(d*x + c))) + b*sin(d*x + c))/d
```

Mupad [**B**]
time = 6.56, size = 47, normalized size = 1.96

$$\frac{a\ln\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d} - \frac{a\ln\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)}{d} + \frac{b\sin(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x)),x)
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (b*si
n(c + d*x))/d
```

3.143 $\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{b\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}$$

```
[Out] -b*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-a*ln(sin(d*x+c))/d-b*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2800, 780}

$$-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (b*Sin[c + d*x])/d
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)}{x^3}\,dx, x, b\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-1+\frac{ab^2}{x^3}+\frac{b^2}{x^2}-\frac{a}{x}\right)\,dx, x, b\sin(c+dx)\right)}{d}\\
&= -\frac{b\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 60, normalized size = 1.11

$$-\frac{b\csc(c+dx)}{d}-\frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d}-\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Ta
n[c + d*x]]))/(2*d) - (b*Sin[c + d*x])/d
```

Maple [A]
time = 0.22, size = 67, normalized size = 1.24

method	result	siz
derivativedivides	$\frac{a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)}{d}$	67
default	$\frac{a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)}{d}$	67
risch	$iax+\frac{ib\,e^{i(dx+c)}}{2d}-\frac{ib\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}-\frac{2i\left(ia\,e^{2i(dx+c)}+b\,e^{3i(dx+c)}-b\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+b*(-1/sin(d*x+c)*cos(d*x+c)^4-(2+
cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 45, normalized size = 0.83

$$-\frac{2\,a\log\left(\sin\left(dx+c\right)\right)+2\,b\sin\left(dx+c\right)+\frac{2\,b\sin(dx+c)+a}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*log(sin(d*x + c)) + 2*b*sin(d*x + c) + (2*b*sin(d*x + c) + a)/sin
(d*x + c)^2)/d
```

Fricas [A]
time = 0.37, size = 69, normalized size = 1.28

$$-\frac{2\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+2\left(b\cos\left(dx+c\right)^2-2\,b\right)\sin\left(dx+c\right)-a}{2\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(b*cos(d*x + c)^2
- 2*b)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))\cot^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**3, x)
```

Giac [A]
time = 23.21, size = 60, normalized size = 1.11

$$-\frac{2\,a\log\left(|\sin\left(dx+c\right)|\right) + 2\,b\sin\left(dx+c\right) - \frac{3\,a\sin\left(dx+c\right)^2 - 2\,b\sin\left(dx+c\right) - a}{\sin\left(dx+c\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(sin(d*x + c))) + 2*b*sin(d*x + c) - (3*a*sin(d*x + c)^2 -
 2*b*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

Mupad [B]
time = 6.63, size = 146, normalized size = 2.70

$$\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{10\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\,\left(4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (b*tan(c/2 + (d*x)/2))/(2*d) - (a/2 +
 2*b*tan(c/2 + (d*x)/2) + (a*tan(c/2 + (d*x)/2)^2)/2 + 10*b*tan(c/2 + (d*x)
/2)^3)/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) - (a*tan(c/2 +
 (d*x)/2)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2)))/d
```

3.144 $\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{2b\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] 2*b*csc(d*x+c)/d+a*csc(d*x+c)^2/d-1/3*b*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d
+a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d
```

Rubi [A]

time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2800, 780}

$$-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (b*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{ab^4}{x^5}+\frac{b^4}{x^4}-\frac{2ab^2}{x^3}-\frac{2b^2}{x^2}+\frac{a}{x}\right)dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{2b\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 87, normalized size = 1.07

$$\frac{2b\csc(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{a(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx)))}{4d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (b*Sin[c
 + d*x])/d
```

Maple [A]
time = 0.22, size = 101, normalized size = 1.25

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin\right.}{d}$
default	$\frac{a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin\right.}{d}$
risch	$-iax-\frac{ib\,e^{i(dx+c)}}{2d}+\frac{ib\,e^{-i(dx+c)}}{2d}-\frac{2iac}{d}+\frac{4i\left(3ia\,e^{6i(dx+c)}+3b\,e^{7i(dx+c)}-3ia\,e^{4i(dx+c)}-7b\,e^{5i(dx+c)}+3ia\,e^{2i(dx\right.}{3d\left(e^{2i(dx+c)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+b*(-1/3/sin(d*x+
c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.43, size = 69, normalized size = 0.85

$$\frac{12\,a\log\left(\sin\left(dx+c\right)\right)+12\,b\sin\left(dx+c\right)+\frac{24\,b\sin(dx+c)^3+12\,a\sin(dx+c)^2-4\,b\sin(dx+c)-3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*b*sin(d*x + c) + (24*b*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*b*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.38, size = 110, normalized size = 1.36

$$-\frac{12\,a\cos\left(dx+c\right)^2-12\,\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\,\left(3\,b\cos\left(dx+c\right)^4-12\,b\cos\left(dx+c\right)^2+8\,b\right)\sin\left(dx+c\right)-9\,a}{12\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*b*cos(d*x + c)^4 - 12*b*cos(d*x + c)^2 + 8*b)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))\cot^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**5, x)
```

Giac [A]
time = 15.24, size = 82, normalized size = 1.01

$$\frac{12\,a\log\left(|\sin\left(dx+c\right)|\right)+12\,b\sin\left(dx+c\right)-\frac{25\,a\sin(dx+c)^4-24\,b\sin(dx+c)^3-12\,a\sin(dx+c)^2+4\,b\sin(dx+c)+3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*b*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*b*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*b*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

Mupad [B]
time = 6.67, size = 207, normalized size = 2.56

$$\frac{7\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{46\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{40\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a}{4}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b*sin(c + d*x)),x)
```

```
[Out] (7*b*tan(c/2 + (d*x)/2))/(8*d) + ((11*a*tan(c/2 + (d*x)/2)^2)/4 - (2*b*tan(
c/2 + (d*x)/2))/3 - a/4 + 3*a*tan(c/2 + (d*x)/2)^4 + (40*b*tan(c/2 + (d*x)/
2)^3)/3 + 46*b*tan(c/2 + (d*x)/2)^5)/(d*(16*tan(c/2 + (d*x)/2)^4 + 16*tan(c
/2 + (d*x)/2)^6)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (3*a*tan(c/2 + (d
*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^4)/(64*d) - (b*tan(c/2 + (d*x)/2)^
3)/(24*d) + (a*log(tan(c/2 + (d*x)/2)))/d
```

3.145 $\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=72

$$ax-\frac{b\cos(c+dx)}{d}-\frac{2b\sec(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] a*x-b*cos(d*x+c)/d-2*b*sec(d*x+c)/d+1/3*b*sec(d*x+c)^3/d-a*tan(d*x+c)/d+1/3
*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2801, 3554, 8, 2670, 276}

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+ax-\frac{b\cos(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{2b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (b*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx = \int \left(a\tan^4(c+dx)+b\sin(c+dx)\tan^4(c+dx)\right)\,dx$$
$$= a\int \tan^4(c+dx)\,dx + b\int \sin(c+dx)\tan^4(c+dx)\,dx$$
$$= \frac{a\tan^3(c+dx)}{3d} - a\int \tan^2(c+dx)\,dx - \frac{b\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, \cos\right.}{d}$$
$$= -\frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} + a\int 1\,dx - \frac{b\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\right.\right.}{}$$
$$= ax - \frac{b\cos(c+dx)}{d} - \frac{2b\sec(c+dx)}{d} + \frac{b\sec^3(c+dx)}{3d} - \frac{a\tan(c+d}{d}$$

Mathematica [A]
time = 0.03, size = 81, normalized size = 1.12

$$\frac{a\tan^{-1}(\tan(c+dx))}{d} - \frac{b\cos(c+dx)}{d} - \frac{2b\sec(c+dx)}{d} + \frac{b\sec^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (b
*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.16, size = 98, normalized size = 1.36

method	result
derivativedivides	$\frac{a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+b\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}$
default	$\frac{a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+b\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}$

risch	$ax - \frac{b\,\mathrm{e}^{i(dx+c)}}{2d} - \frac{b\,\mathrm{e}^{-i(dx+c)}}{2d} - \frac{4\left(3ia\,\mathrm{e}^{4i(dx+c)}+3b\,\mathrm{e}^{5i(dx+c)}+3ia\,\mathrm{e}^{2i(dx+c)}+4b\,\mathrm{e}^{3i(dx+c)}+2ia+3b\,\mathrm{e}^{i(dx+c)}\right)}{3d\left(1+\mathrm{e}^{2i(dx+c)}\right)^3}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+b*(1/3*sin(d*x+c)^6/cos(d*x+c)^3
-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.58, size = 65, normalized size = 0.90

$$\frac{\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a-b\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos\left(dx+c\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - b*((6*cos(d*x + c)
^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [A]
time = 0.34, size = 73, normalized size = 1.01

$$\frac{3\,adx\cos\left(dx+c\right)^3-3\,b\cos\left(dx+c\right)^4-6\,b\cos\left(dx+c\right)^2-\left(4\,a\cos\left(dx+c\right)^2-a\right)\sin\left(dx+c\right)+b}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*d*x*cos(d*x + c)^3 - 3*b*cos(d*x + c)^4 - 6*b*cos(d*x + c)^2 - (4*
a*cos(d*x + c)^2 - a)*sin(d*x + c) + b)/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\tan^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**4, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 10.18, size = 110, normalized size = 1.53

$$a\,x+\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\frac{14\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{14\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-\frac{32\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{16\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x)),x)
```

```
[Out] a*x + ((16*b)/3 + 2*a*tan(c/2 + (d*x)/2) - (14*a*tan(c/2 + (d*x)/2)^3)/3 -
(14*a*tan(c/2 + (d*x)/2)^5)/3 + 2*a*tan(c/2 + (d*x)/2)^7 - (32*b*tan(c/2 +
(d*x)/2)^2)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.146 $\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=38

$$-ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] -a*x+b*cos(d*x+c)/d+b*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2801, 3554, 8, 2670, 14}

$$\frac{a\tan(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx &= \int \left(a\tan^2(c+dx)+b\sin(c+dx)\tan^2(c+dx)\right)\,dx \\
&= a\int \tan^2(c+dx)\,dx+b\int \sin(c+dx)\tan^2(c+dx)\,dx \\
&= \frac{a\tan(c+dx)}{d}-a\int 1\,dx-\frac{b\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d} \\
&= -ax+\frac{a\tan(c+dx)}{d}-\frac{b\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,x,\cos(c+dx)\right)}{d} \\
&= -ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 1.24

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

Maple [A]
time = 0.16, size = 59, normalized size = 1.55

method	result	size
derivativedivides	$\frac{a(\tan(dx+c)-dx-c)+b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$	59
default	$\frac{a(\tan(dx+c)-dx-c)+b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$	59
risch	$-ax+\frac{b\,e^{i(dx+c)}}{2d}+\frac{b\,e^{-i(dx+c)}}{2d}+\frac{2ia+2b\,e^{i(dx+c)}}{d\left(1+e^{2i(dx+c)}\right)}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(tan(d*x+c)-d*x-c)+b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d
*x+c)))
```

Maxima [A]
time = 0.54, size = 39, normalized size = 1.03

$$-\frac{(dx+c-\tan(dx+c))a-b\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.35, size = 47, normalized size = 1.24

$$-\frac{adx\cos(dx+c)-b\cos(dx+c)^2-a\sin(dx+c)-b}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(a*d*x*cos(d*x + c) - b*cos(d*x + c)^2 - a*sin(d*x + c) - b)/(d*cos(d*x +
c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. 2(38) = 76.
time = 10.58, size = 1008, normalized size = 26.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -(a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - a*d*x*tan(1/2*d*x)^4*
tan(1/2*c)^4 - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*b*ta
n(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + a*tan(d*x)*tan(1/2*d*x)^4*tan(1
/2*c)^4 + a*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*a*d*x*tan(1/2*d*x)^3*tan
(1/2*c)^3 + 2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*d*x*tan(d*x)*tan(1/2*d*x)^4
*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*d*x*tan(d
*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 8*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*
c)^3*tan(c) - a*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x
)^3*tan(1/2*c)^3 - 4*a*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + a*d*x*tan(1/2*d
*x)^4 + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c)
^3 - 8*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + a*d*x*tan(1/2*c)^4 - 2*b*tan(d*x)*ta
n(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*b
*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 24*b*tan(d*x)*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(c) - 8*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 2*b*ta
n(d*x)*tan(1/2*c)^4*tan(c) - a*tan(d*x)*tan(1/2*d*x)^4 - 4*a*tan(d*x)*tan(1
/2*d*x)^3*tan(1/2*c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3 - a*tan(d*x)*
tan(1/2*c)^4 - a*tan(1/2*d*x)^4*tan(c) - 4*a*tan(1/2*d*x)^3*tan(1/2*c)*tan(
c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - a*tan(1/2*c)^4*tan(c) + 2*b*tan
(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*b*tan(1/2*d*x)^3*tan(1/2*
c) + 24*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*b*tan(1/2*d*x)*tan(1/2*c)^3 + 2*b
*tan(1/2*c)^4 + a*d*x*tan(d*x)*tan(c) + 8*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c
)*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c) - 4*a*tan(1/2*d*x)*tan(1/2*
c)*tan(c) - a*d*x - 8*b*tan(1/2*d*x)*tan(1/2*c) - 2*b*tan(d*x)*tan(c) + a*t
an(d*x) + a*tan(c) + 2*b)/(d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) -
d*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*ta
n(c) + 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3 - d*tan(d*x)*tan(1/2*d*x)^4*tan(c) -
 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*
tan(1/2*c)^3*tan(c) - d*tan(d*x)*tan(1/2*c)^4*tan(c) + d*tan(1/2*d*x)^4 + 4
*d*tan(1/2*d*x)^3*tan(1/2*c) + 4*d*tan(1/2*d*x)*tan(1/2*c)^3 + d*tan(1/2*c)
^4 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) + 4*d*tan(1/2*d*x)*tan(1/2
*c) + d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 6.60, size = 55, normalized size = 1.45

$$-a\,x-\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] - a*x - (4*b + 2*a*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c
/2 + (d*x)/2)^4 - 1))
```

3.147 $\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$-ax-\frac{b\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}$$

```
[Out] -a*x-b*arctanh(cos(d*x+c))/d+b*cos(d*x+c)/d-a*cot(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2801, 2672, 327, 212, 3554, 8}

$$-\frac{a\cot(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx &= \int \left(b\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)\,dx\\
&= a\int \cot^2(c+dx)\,dx+b\int \cos(c+dx)\cot(c+dx)\,dx\\
&= -\frac{a\cot(c+dx)}{d}-a\int 1\,dx-\frac{b\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d}\\
&= -ax+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c-\right.}{d}\\
&= -ax-\frac{b\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 75, normalized size = 1.83

$$
\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}-\frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d*x)/2]])/d
```

Maple [A]
time = 0.10, size = 49, normalized size = 1.20

method	result	size
derivativedivides	$\frac{a(-\cot(dx+c)-dx-c)+b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))}{d}$	49
default	$\frac{a(-\cot(dx+c)-dx-c)+b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))}{d}$	49
risch	$-ax+\frac{b\,e^{i(dx+c)}}{2d}+\frac{b\,e^{-i(dx+c)}}{2d}-\frac{2ia}{d(e^{2i(dx+c)}-1)}-\frac{b\ln(e^{i(dx+c)}+1)}{d}+\frac{b\ln(e^{i(dx+c)}-1)}{d}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-cot(d*x+c)-d*x-c)+b*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.65, size = 54, normalized size = 1.32

$$-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a-b(2\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - b*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(41) = 82$.
time = 0.39, size = 84, normalized size = 2.05

$$-\frac{b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+2\,a\cos(dx+c)+2\,(adx-b\cos(dx+c))\sin(dx+c)}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - b*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))\cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(41) = 82.

time = 9.08, size = 108, normalized size = 2.63

$$-\frac{6\,(dx+c)a-6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*b*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

Mupad [B]

time = 6.60, size = 158, normalized size = 3.85

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,b\,a}-\frac{4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,b\,a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) + (b*log(tan(c/2 + (d*x)/2)))/d - (a - 4*b*tan
(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)/(d*(2*tan(c/2 + (d*x)/2) + 2*tan(
c/2 + (d*x)/2)^3)) + (2*a*atan((4*a^2)/(4*a*b + 4*a^2*tan(c/2 + (d*x)/2)) -
 (4*a*b*tan(c/2 + (d*x)/2))/(4*a*b + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

3.148 $\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$ax+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3b\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}$$

```
[Out] a*x+3/2*b*arctanh(cos(d*x+c))/d-3/2*b*cos(d*x+c)/d+a*cot(d*x+c)/d-1/2*b*cos
(d*x+c)*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2801, 2672, 294, 327, 212, 3554, 8}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax-\frac{3b\cos(c+dx)}{2d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*b*ArcTanh[Cos[c + d*x]])/(2*d) - (3*b*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (b*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx &= \int \left(b\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)\,dx\\ &= a\int \cot^4(c+dx)\,dx+b\int \cos(c+dx)\cot^3(c+dx)\,dx\\ &= -\frac{a\cot^3(c+dx)}{3d}-a\int \cot^2(c+dx)\,dx-\frac{b\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\text{co}\right.}{d}\\ &= \frac{a\cot(c+dx)}{d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+a\int\\ &= ax-\frac{3b\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}-\\ &= ax+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3b\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{b}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.03, size = 125, normalized size = 1.52

$$-\frac{b\cos(c+dx)}{d}-\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d}+\frac{3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/d) - (b*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*b*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*b*Log[Sin[(c + d*x)/2]])/(2*d) + (b*Sec[(c + d*x)/2]^2
)/(8*d)
```

Maple [A]
time = 0.17, size = 86, normalized size = 1.05

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
risch	$ax-\frac{b\,e^{i(dx+c)}}{2d}-\frac{b\,e^{-i(dx+c)}}{2d}+\frac{12ia\,e^{4i(dx+c)}+3b\,e^{5i(dx+c)}-12ia\,e^{2i(dx+c)}+8ia-3b\,e^{i(dx+c)}}{3d\left(e^{2i(dx+c)}-1\right)^3}-\frac{3b\ln\left(e^{i(dx+c)}-1\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+b*(-1/2/sin(d*x+c)^2*cos(d*x+c)
^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.68, size = 92, normalized size = 1.12

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+3\,b\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*b*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(74) = 148.
time = 0.38, size = 160, normalized size = 1.95

$$\frac{16\,a\cos\left(dx+c\right)^3+9\left(b\cos\left(dx+c\right)^2-b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\left(b\cos\left(dx+c\right)^2-b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-12\,a\cos\left(dx+c\right)+6\left(2\,adx\cos\left(dx+c\right)^2-2\,b\cos\left(dx+c\right)^3-2\,adx+3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(b*cos(d*x + c)^2 - b)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(b*cos(d*x + c)^2 - b)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*b*cos(d*x
+ c)^3 - 2*a*d*x + 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))\cot^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**4, x)
```

Giac [A]
time = 9.99, size = 141, normalized size = 1.72

$$\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 3\,b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 24\,(dx + c)a - 36\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - 15\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \frac{48\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + \frac{66\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*b*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*b*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*b/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 6.29, size = 225, normalized size = 2.74

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d} - \frac{-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 17\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - \frac{14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3} + b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + \frac{a}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)} - \frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d} + \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d} - \frac{3\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d} - \frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+6\,b\,a} - \frac{6\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+6\,b\,a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a/3 + b*tan(c/2 + (d*x)/2) - (14*a*tan(c
/2 + (d*x)/2)^2)/3 - 5*a*tan(c/2 + (d*x)/2)^4 + 17*b*tan(c/2 + (d*x)/2)^3)/
(d*(8*tan(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) - (5*a*tan(c/2 + (d*x
)/2))/(8*d) + (b*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*b*log(tan(c/2 + (d*x)/2))
)/(2*d) - (2*a*atan((4*a^2)/(6*a*b + 4*a^2*tan(c/2 + (d*x)/2)) - (6*a*b*tan
(c/2 + (d*x)/2))/(6*a*b + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

3.149 $\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$-ax-\frac{15b\tanh^{-1}(\cos(c+dx))}{8d}+\frac{15b\cos(c+dx)}{8d}-\frac{a\cot(c+dx)}{d}+\frac{5b\cos(c+dx)\cot^2(c+dx)}{8d}+\frac{a\cot^3(c+dx}{3d}$$

```
[Out] -a*x-15/8*b*arctanh(cos(d*x+c))/d+15/8*b*cos(d*x+c)/d-a*cot(d*x+c)/d+5/8*b*
cos(d*x+c)*cot(d*x+c)^2/d+1/3*a*cot(d*x+c)^3/d-1/4*b*cos(d*x+c)*cot(d*x+c)^
4/d-1/5*a*cot(d*x+c)^5/d
```

Rubi [A]

time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2801, 2672, 294, 327, 212, 3554, 8}

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-ax+\frac{15b\cos(c+dx)}{8d}-\frac{b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5b\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{15b\tanh^{-1}(\cos(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (15*b*ArcTanh[Cos[c + d*x]])/(8*d) + (15*b*Cos[c + d*x])/(8*d) - (
a*Cot[c + d*x])/d + (5*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*
x]^3)/(3*d) - (b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx &= \int \left(b\cos(c+dx)\cot^5(c+dx)+a\cot^6(c+dx)\right)\,dx \\
&= a\int \cot^6(c+dx)\,dx+b\int \cos(c+dx)\cot^5(c+dx)\,dx \\
&= -\frac{a\cot^5(c+dx)}{5d}-a\int \cot^4(c+dx)\,dx-\frac{b\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx,x,\text{co}\right.}{d} \\
&= \frac{a\cot^3(c+dx)}{3d}-\frac{b\cos(c+dx)\cot^4(c+dx)}{4d}-\frac{a\cot^5(c+dx)}{5d}+a\int \\
&= -\frac{a\cot(c+dx)}{d}+\frac{5b\cos(c+dx)\cot^2(c+dx)}{8d}+\frac{a\cot^3(c+dx)}{3d}-\frac{b}{} \\
&= -ax+\frac{15b\cos(c+dx)}{8d}-\frac{a\cot(c+dx)}{d}+\frac{5b\cos(c+dx)\cot^2(c+d}{8d} \\
&= -ax-\frac{15b\tanh^{-1}(\cos(c+dx))}{8d}+\frac{15b\cos(c+dx)}{8d}-\frac{a\cot(c+dx)}{d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 164, normalized size = 1.34

$$\frac{b\cos(c+dx)}{d}+\frac{9b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d}-\frac{15b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{15b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{9b\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Cos[c + d*x])/d + (9*b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d*x)/2]^
4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x
]^2])/(5*d) - (15*b*Log[Cos[(c + d*x)/2]])/(8*d) + (15*b*Log[Sin[(c + d*x)/
2]])/(8*d) - (9*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(64*d
)
```

Maple [A]
time = 0.16, size = 129, normalized size = 1.06

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+b\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c)\right)}{8}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+b\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c)\right)}{8}\right.}{d}$
risch	$-ax+\frac{b\,e^{i(dx+c)}}{2d}+\frac{b\,e^{-i(dx+c)}}{2d}-\frac{360ia\,e^{8i(dx+c)}+135\,e^{9i(dx+c)}b-720ia\,e^{6i(dx+c)}-150b\,e^{7i(dx+c)}+1120ia\,e^{4i(dx+}}{60d\left(e^{2i(dx+c)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+b*(-1/4/sin(d*
x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos(d*x+c)^5+5/8*cos(
d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.67, size = 125, normalized size = 1.02

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+15\,b\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos\left(dx+c\right)+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a + 15*b*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4
```

```
 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*
log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(110) = 220$.
time = 0.40, size = 222, normalized size = 1.82

$$-\frac{368\,a\cos\left(dx+c\right)^{5}-560\,a\cos\left(dx+c\right)^{3}+225\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-225\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+240\,a\cos\left(dx+c\right)+30\left(8\,adx\cos\left(dx+c\right)^{4}-8\,b\cos\left(dx+c\right)^{5}-16\,adx\cos\left(dx+c\right)^{2}+25\,b\cos\left(dx+c\right)^{3}+8\,adx-15\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*a*cos(d*x + c)^5 - 560*a*cos(d*x + c)^3 + 225*(b*cos(d*x + c)^4
 - 2*b*cos(d*x + c)^2 + b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(
b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 240*a*cos(d*x + c) + 30*(8*a*d*x*cos(d*x + c)^4 - 8*b*cos(d*x +
 c)^5 - 16*a*d*x*cos(d*x + c)^2 + 25*b*cos(d*x + c)^3 + 8*a*d*x - 15*b*cos(
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(c+dx\right)\right)\cot^{6}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**6, x)
```

Giac [A]
time = 4.37, size = 199, normalized size = 1.63

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-240\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-960\left(dx+c\right)a+1800\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+660\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{1920\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}-\frac{4110\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+660\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-240\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*b*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 - 240*b*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 1800*
b*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*b/(tan
(1/2*d*x + 1/2*c)^2 + 1) - (4110*b*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*d
*x + 1/2*c)^4 - 240*b*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
+ 15*b*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 6.30, size = 288, normalized size = 2.36

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{22\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-72\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{59\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{15\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{32\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a}{5}}{d\,\left(32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}-\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{15\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}+\frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+\frac{15\,b\,a}{2}}-\frac{15\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+\frac{15\,b\,a}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b*sin(c + d*x)),x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(16*d) - (a/5 + (b*tan(c/2 + (d*x)/2))/2 - (32*a*
tan(c/2 + (d*x)/2)^2)/15 + (59*a*tan(c/2 + (d*x)/2)^4)/3 + 22*a*tan(c/2 + (
d*x)/2)^6 - (15*b*tan(c/2 + (d*x)/2)^3)/2 - 72*b*tan(c/2 + (d*x)/2)^5)/(d*(
32*tan(c/2 + (d*x)/2)^5 + 32*tan(c/2 + (d*x)/2)^7)) - (7*a*tan(c/2 + (d*x)/
2)^3)/(96*d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) - (b*tan(c/2 + (d*x)/2)^2)/
(4*d) + (b*tan(c/2 + (d*x)/2)^4)/(64*d) + (15*b*log(tan(c/2 + (d*x)/2)))/(8
*d) + (2*a*atan((4*a^2)/((15*a*b)/2 + 4*a^2*tan(c/2 + (d*x)/2)) - (15*a*b*t
an(c/2 + (d*x)/2))/(2*((15*a*b)/2 + 4*a^2*tan(c/2 + (d*x)/2)))))/d
```

3.150 $\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{(a+b)(a+2b)\log(1-\sin(c+dx))}{2d}+\frac{(a-2b)(a-b)\log(1+\sin(c+dx))}{2d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

```
[Out] 1/2*(a+b)*(a+2*b)*ln(1-sin(d*x+c))/d+1/2*(a-2*b)*(a-b)*ln(1+sin(d*x+c))/d+2
*a*b*sin(d*x+c)/d+1/2*b^2*sin(d*x+c)^2/d+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c))^
2/d
```

Rubi [A]
time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2800, 1659, 1643, 647, 31}

$$\frac{2ab\sin(c+dx)}{d}+\frac{(a+b)(a+2b)\log(1-\sin(c+dx))}{2d}+\frac{(a-2b)(a-b)\log(\sin(c+dx)+1)}{2d}+\frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d}+\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] ((a + b)*(a + 2*b)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - 2*b)*(a - b)*Log[1
+ Sin[c + d*x]])/(2*d) + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d
) + (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
```

```
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^2}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d} + \frac{\text{Subst}\left(\int \frac{(a+x)(-2b^4-2ab^2x-2b^2x^2)}{b^2-x^2}\,d\right.}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d} + \frac{\text{Subst}\left(\int \left(4ab^2+2b^2x-\frac{2(3ab^4+}{b}\right.\right.}{2b}$$

$$= \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^2(c+dx)(a+b\sin(c+dx))}{2d}$$

$$= \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^2(c+dx)(a+b\sin(c+dx))}{2d}$$

$$= \frac{(a+b)(a+2b)\log(1-\sin(c+dx))}{2d} + \frac{(a-2b)(a-b)\log(1+\sin(c}{2d}$$

Mathematica [A]
time = 0.29, size = 108, normalized size = 0.97

$$\frac{2(a+b)(a+2b)\log(1-\sin(c+dx))+2(a-2b)(a-b)\log(1+\sin(c+dx))-\frac{(a+b)^2}{-1+\sin(c+dx)}+8ab\sin(c+dx)+2b^2\sin^2(c+dx)+\frac{(a-b)^2}{1+\sin(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

[Out] (2*(a + b)*(a + 2*b)*Log[1 - Sin[c + d*x]] + 2*(a - 2*b)*(a - b)*Log[1 + Si
n[c + d*x]] - (a + b)^2/(-1 + Sin[c + d*x]) + 8*a*b*Sin[c + d*x] + 2*b^2*Si
n[c + d*x]^2 + (a - b)^2/(1 + Sin[c + d*x]))/(4*d)

Maple [A]
time = 0.19, size = 135, normalized size = 1.22

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+2ab\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+b^2\left(\frac{\mathrm{s}}{2}\right.}{d}$
default	$\frac{a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+2ab\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+b^2\left(\frac{\mathrm{s}}{2}\right.}{d}$
risch	$-\frac{4ib^2c}{d}-\frac{2ia^2c}{d}-\frac{b^2e^{2i(dx+c)}}{8d}-2ib^2x-\frac{2i\left(ia^2e^{2i(dx+c)}+ib^2e^{2i(dx+c)}+ab\,e^{3i(dx+c)}-a\,e^{i(dx+c)}b\right)}{d\left(1+e^{2i(dx+c)}\right)^2}-\frac{b^2e^{-2i}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+2*a*b*(1/2*sin(d*x+c)^5/cos(d*x+
c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c)))+b^2*(1/
2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c)))
)

Maxima [A]
time = 0.31, size = 105, normalized size = 0.95

$$\frac{b^2\sin(dx+c)^2+4\,ab\sin(dx+c)+(a^2-3\,ab+2\,b^2)\log(\sin(dx+c)+1)+(a^2+3\,ab+2\,b^2)\log(\sin(dx+c)-1)-\frac{2\,ab\sin(dx+c)+a^2+b^2}{\sin(dx+c)^2-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")

[Out] 1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + (a^2 - 3*a*b + 2*b^2)*log(si
n(d*x + c) + 1) + (a^2 + 3*a*b + 2*b^2)*log(sin(d*x + c) - 1) - (2*a*b*sin(
d*x + c) + a^2 + b^2)/(sin(d*x + c)^2 - 1))/d

Fricas [A]
time = 0.38, size = 140, normalized size = 1.26

$$-\frac{2\,b^2\cos(dx+c)^4-b^2\cos(dx+c)^2-2\,(a^2-3\,ab+2\,b^2)\cos(dx+c)^2\log(\sin(dx+c)+1)-2\,(a^2+3\,ab+2\,b^2)\cos(dx+c)^2\log(-\sin(dx+c)+1)-2\,a^2-2\,b^2-4\,(2\,ab\cos(dx+c)^2+ab)\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")

[Out] -1/4*(2*b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*(a^2 - 3*a*b + 2*b^2)*c
os(d*x + c)^2*log(sin(d*x + c) + 1) - 2*(a^2 + 3*a*b + 2*b^2)*cos(d*x + c)^

```
2*log(-sin(d*x + c) + 1) - 2*a^2 - 2*b^2 - 4*(2*a*b*cos(d*x + c)^2 + a*b)*s
in(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin{(c + dx)})^2 \tan^3{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.72, size = 232, normalized size = 2.09

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2+4b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(2a^2+4b^2)+4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+6ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+6ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)(a^2+2b^2)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)(a+b)(a+2b)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)(a-b)(a-2b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(2*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^6*(2*a^2 + 4*b^2
) + 4*a^2*tan(c/2 + (d*x)/2)^4 + 2*a*b*tan(c/2 + (d*x)/2)^3 + 2*a*b*tan(c/2
 + (d*x)/2)^5 + 6*a*b*tan(c/2 + (d*x)/2)^7 + 6*a*b*tan(c/2 + (d*x)/2))/(d*(
tan(c/2 + (d*x)/2)^8 - 2*tan(c/2 + (d*x)/2)^4 + 1)) - (log(tan(c/2 + (d*x)/
2)^2 + 1)*(a^2 + 2*b^2))/d + (log(tan(c/2 + (d*x)/2) - 1)*(a + b)*(a + 2*b)
)/d + (log(tan(c/2 + (d*x)/2) + 1)*(a - b)*(a - 2*b))/d
```

3.151 $\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=78

$$-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{(a-b)^2\log(1+\sin(c+dx))}{2d}-\frac{2ab\sin(c+dx)}{d}-\frac{b^2\sin^2(c+dx)}{2d}$$

```
[Out] -1/2*(a+b)^2*ln(1-sin(d*x+c))/d-1/2*(a-b)^2*ln(1+sin(d*x+c))/d-2*a*b*sin(d*
x+c)/d-1/2*b^2*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2800, 815, 647, 31}

$$-\frac{2ab\sin(c+dx)}{d}-\frac{(a-b)^2\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -1/2*((a + b)^2*Log[1 - Sin[c + d*x]])/d - ((a - b)^2*Log[1 + Sin[c + d*x]]
)/(2*d) - (2*a*b*Sin[c + d*x])/d - (b^2*Sin[c + d*x]^2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
```

```
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^2}{b^2-x^2}\,dx,x,b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a-x+\frac{2ab^2+(a^2+b^2)x}{b^2-x^2}\right)\,dx,x,b\sin(c+dx)\right)}{d}$$
$$= -\frac{2ab\sin(c+dx)}{d}-\frac{b^2\sin^2(c+dx)}{2d}+\frac{\text{Subst}\left(\int \frac{2ab^2+(a^2+b^2)x}{b^2-x^2}\,dx,x,b\,\text{s}\right.}{d}$$
$$= -\frac{2ab\sin(c+dx)}{d}-\frac{b^2\sin^2(c+dx)}{2d}+\frac{(a-b)^2\text{Subst}\left(\int \frac{1}{-b-x}\,dx,x,b\,\text{s}\right.}{2d}$$
$$= -\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{(a-b)^2\log(1+\sin(c+dx))}{2d}-\frac{2a}{}$$

Mathematica [A]
time = 0.09, size = 64, normalized size = 0.82

$$-\frac{(a+b)^2\log(1-\sin(c+dx))+(a-b)^2\log(1+\sin(c+dx))+4ab\sin(c+dx)+b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -1/2*((a + b)^2*Log[1 - Sin[c + d*x]] + (a - b)^2*Log[1 + Sin[c + d*x]] + 4
*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2)/d
```

Maple [A]
time = 0.19, size = 69, normalized size = 0.88

method	result
derivativedivides	$\frac{-a^2\ln(\cos(dx+c))+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{-a^2\ln(\cos(dx+c))+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$
risch	$ia^2x+ib^2x+\frac{iab\,\mathrm{e}^{i(dx+c)}}{d}-\frac{iab\,\mathrm{e}^{-i(dx+c)}}{d}+\frac{2ia^2c}{d}+\frac{2ib^2c}{d}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}+\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)ab}{d}-\mathrm{l}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*ln(cos(d*x+c))+2*a*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*
(-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.90

$$-\frac{b^2 \sin (dx+c)^2 + 4\,ab \sin (dx+c) + (a^2 - 2\,ab + b^2) \log (\sin (dx+c) + 1) + (a^2 + 2\,ab + b^2) \log (\sin (dx+c) - 1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + (a^2 - 2*a*b + b^2)*log(sin
(d*x + c) + 1) + (a^2 + 2*a*b + b^2)*log(sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.37, size = 74, normalized size = 0.95

$$\frac{b^2 \cos (dx+c)^2 - 4\,ab \sin (dx+c) - (a^2 - 2\,ab + b^2) \log (\sin (dx+c) + 1) - (a^2 + 2\,ab + b^2) \log (-\sin (dx+c) + 1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos(d*x + c)^2 - 4*a*b*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(sin(
d*x + c) + 1) - (a^2 + 2*a*b + b^2)*log(-sin(d*x + c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sin (c + dx))^2 \tan (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 7855 vs. $2(72) = 144$.
time = 36.44, size = 7855, normalized size = 100.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/4*(4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
n(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2
 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
+ 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(c)^2 + 2*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan
(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(
d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c
)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*t
an(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2
- b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a*b*log(2*(tan(1/
2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(
1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*t
an(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
+ 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*log(4*(tan(d*x)^4*tan(
c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*
tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*
log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + 2
*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d
*x)^2*tan(c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*ta
n(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 16*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2
*c)*tan(c)^2 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*
tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1
```

```
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a^2*
log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2*t
an(c)^2 + 2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^
2*tan(1/2*c)^2*tan(c)^2 - 16*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c
)^2 + 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c
) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*
d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2
*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a^2*l
og(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan
(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)...
```

Mupad [B]
time = 6.37, size = 150, normalized size = 1.92

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(a^2+b^2\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left(a+b\right)}^2}{d}-\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,{\left(a-b\right)}^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 + b^2))/d - (log(tan(c/2 + (d*x)/2) - 1
)*(a + b)^2)/d - (2*b^2*tan(c/2 + (d*x)/2)^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 +
 4*a*b*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^
4 + 1)) - (log(tan(c/2 + (d*x)/2) + 1)*(a - b)^2)/d
```

3.152 $\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=46

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

```
[Out] a^2*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d+1/2*b^2*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2800, 45}

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(
2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^2}{x}\,dx,x,b\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2}{x}+x\right)\,dx,x,b\sin(c+dx)\right)}{d}\\ &= \frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.00

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(
2*d)
```

Maple [A]
time = 0.09, size = 40, normalized size = 0.87

method	result	size
derivativedivides	$\frac{\frac{\left(\sin^2(dx+c)\right)b^2}{2}+2ab\sin(dx+c)+a^2\ln(\sin(dx+c))}{d}$	40
default	$\frac{\frac{\left(\sin^2(dx+c)\right)b^2}{2}+2ab\sin(dx+c)+a^2\ln(\sin(dx+c))}{d}$	40
risch	$-ia^2x-\frac{b^2e^{2i(dx+c)}}{8d}-\frac{b^2e^{-2i(dx+c)}}{8d}-\frac{2ia^2c}{d}+\frac{a^2\ln(e^{2i(dx+c)}-1)}{d}+\frac{2ab\sin(dx+c)}{d}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*sin(d*x+c)^2*b^2+2*a*b*sin(d*x+c)+a^2*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.32, size = 40, normalized size = 0.87

$$\frac{b^2\sin\left(dx+c\right)^2+2\,a^2\log\left(\sin\left(dx+c\right)\right)+4\,ab\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*sin(d*x + c)^2 + 2*a^2*log(sin(d*x + c)) + 4*a*b*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 42, normalized size = 0.91

$$-\frac{b^2\cos\left(dx+c\right)^2-2\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\,ab\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^2 - 2*a^2*log(1/2*sin(d*x + c)) - 4*a*b*sin(d*x + c)
)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin{(c + dx)})^2 \cot{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x), x)
```

Giac [A]
time = 13.32, size = 41, normalized size = 0.89

$$\frac{b^2 \sin{(dx + c)}^2 + 2\,a^2 \log{(|\sin{(dx + c)}|)} + 4\,ab\sin{(dx + c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*sin(d*x + c)^2 + 2*a^2*log(abs(sin(d*x + c))) + 4*a*b*sin(d*x + c)
)/d
```

Mupad [B]
time = 6.44, size = 117, normalized size = 2.54

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{a^2\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (2*b^2*tan(c/2 + (d*x)/2)^2 + 4*a*b*tan(c
/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + ta
n(c/2 + (d*x)/2)^4 + 1)) - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.153 $\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=84

$$-\frac{2ab\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{(a^2-b^2)\log(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{b^2\sin^2(c+dx)}{2d}$$

```
[Out] -2*a*b*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d-(a^2-b^2)*ln(sin(d*x+c))/d-2*a*b
*sin(d*x+c)/d-1/2*b^2*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 908}

$$-\frac{(a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((a^2 - b^2)*Log[Sin
[c + d*x]])/d - (2*a*b*Sin[c + d*x])/d - (b^2*Sin[c + d*x]^2)/(2*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)}{x^3}\,dx,x,b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^2b^2}{x^3}+\frac{2ab^2}{x^2}+\frac{-a^2+b^2}{x}-x\right)dx,x,b\sin(c+dx)\right)}{d}$$
$$= -\frac{2ab\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{(a^2-b^2)\log(\sin(c+dx))}{d}-$$

Mathematica [A]
time = 0.16, size = 70, normalized size = 0.83

$$-\frac{4ab\csc(c+dx)+a^2\csc^2(c+dx)+2(a^2-b^2)\log(\sin(c+dx))+4ab\sin(c+dx)+b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(4*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - b^2)*Log[Sin[c + d
*x]] + 4*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2)/d
```

Maple [A]
time = 0.24, size = 93, normalized size = 1.11

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-\left(2+\cos^2(dx+c)\right)\sin(dx+c)\right)+b^2\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c)\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-\left(2+\cos^2(dx+c)\right)\sin(dx+c)\right)+b^2\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c)\right.}{d}$
risch	$ia^2x-ib^2x+\frac{b^2e^{2i(dx+c)}}{8d}+\frac{iab\,e^{i(dx+c)}}{d}-\frac{iab\,e^{-i(dx+c)}}{d}+\frac{b^2e^{-2i(dx+c)}}{8d}+\frac{2ia^2c}{d}-\frac{2ib^2c}{d}-\frac{2ia(ia\,e^{2i(dx}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+2*a*b*(-1/sin(d*x+c)*cos(d*x+c)
^4-(2+cos(d*x+c)^2)*sin(d*x+c))+b^2*(1/2*cos(d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.56, size = 69, normalized size = 0.82

$$-\frac{b^2\sin(dx+c)^2+4\,ab\sin(dx+c)+2\,(a^2-b^2)\log(\sin(dx+c))+\frac{4\,ab\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + 2*(a^2 - b^2)*log(sin(d*x +
 c)) + (4*a*b*sin(d*x + c) + a^2)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.39, size = 115, normalized size = 1.37

$$\frac{2\,b^2\cos\left(dx+c\right)^4-3\,b^2\cos\left(dx+c\right)^2+2\,a^2+b^2-4\,\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2-a^2+b^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\,\left(ab\cos\left(dx+c\right)^2-2\,ab\right)\sin\left(dx+c\right)}{4\,\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*cos(d*x + c)^4 - 3*b^2*cos(d*x + c)^2 + 2*a^2 + b^2 - 4*((a^2 -
b^2)*cos(d*x + c)^2 - a^2 + b^2)*log(1/2*sin(d*x + c)) - 8*(a*b*cos(d*x + c
)^2 - 2*a*b)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\cot^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**3, x)
```

Giac [A]
time = 9.05, size = 99, normalized size = 1.18

$$-\frac{b^2\sin\left(dx+c\right)^2+4\,ab\sin\left(dx+c\right)+2\,\left(a^2-b^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{3\,a^2\sin\left(dx+c\right)^2-3\,b^2\sin\left(dx+c\right)^2-4\,ab\sin\left(dx+c\right)-a^2}{\sin\left(dx+c\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + 2*(a^2 - b^2)*log(abs(sin(d
*x + c))) - (3*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 - 4*a*b*sin(d*x +
c) - a^2)/sin(d*x + c)^2)/d
```

Mupad [B]
time = 6.39, size = 221, normalized size = 2.63

$$\frac{\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\,\left(a^2-b^2\right)}{d}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{a^2}{2}+8\,b^2\right)+a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\frac{a^2}{2}+24\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+20\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+4\,a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}-\frac{a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}-\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-b^2\right)}{d}-\frac{a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 - b^2))/d - (tan(c/2 + (d*x)/2)^4*(a^2/
2 + 8*b^2) + a^2*tan(c/2 + (d*x)/2)^2 + a^2/2 + 24*a*b*tan(c/2 + (d*x)/2)^3
 + 20*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 +
(d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6)) - (a^2*tan(c
/2 + (d*x)/2)^2)/(8*d) - (log(tan(c/2 + (d*x)/2))*(a^2 - b^2))/d - (a*b*tan
(c/2 + (d*x)/2))/d
```

3.154 $\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{4ab\csc(c+dx)}{d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}+$$

```
[Out] 4*a*b*csc(d*x+c)/d+1/2*(2*a^2-b^2)*csc(d*x+c)^2/d-2/3*a*b*csc(d*x+c)^3/d-1/
4*a^2*csc(d*x+c)^4/d+(a^2-2*b^2)*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d+1/2*b^
2*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 962}

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}+\frac{4ab\csc(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*a*b*Csc[c + d*x])/d + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[
c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) + ((a^2 - 2*b^2)*Log[Sin[c +
 d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2b^4}{x^5}+\frac{2ab^4}{x^4}+\frac{-2a^2b^2+b^4}{x^3}-\frac{4ab^2}{x^2}+\frac{a^2-2b^2}{x}+x\right)dx, x, b\right.}{d}$$
$$= \frac{4ab\csc(c+dx)}{d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{a^2 c}{}$$

Mathematica [A]
time = 0.48, size = 107, normalized size = 0.85

$$\frac{48ab\csc(c+dx)+6(2a^2-b^2)\csc^2(c+dx)-8ab\csc^3(c+dx)-3a^2\csc^4(c+dx)+6(2(a^2-2b^2)\log(\sin(c+dx))+4ab\sin(c+dx)+b^2\sin^2(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (48*a*b*Csc[c + d*x] + 6*(2*a^2 - b^2)*Csc[c + d*x]^2 - 8*a*b*Csc[c + d*x]^
3 - 3*a^2*Csc[c + d*x]^4 + 6*(2*(a^2 - 2*b^2)*Log[Sin[c + d*x]] + 4*a*b*Sin
[c + d*x] + b^2*Sin[c + d*x]^2))/(12*d)
```

Maple [A]
time = 0.24, size = 157, normalized size = 1.25

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
risch	$-ia^2x+2ib^2x-\frac{b^2 e^{2i(dx+c)}}{8d}-\frac{iab\,e^{i(dx+c)}}{d}+\frac{iab\,e^{-i(dx+c)}}{d}-\frac{b^2e^{-2i(dx+c)}}{8d}-\frac{2ia^2c}{d}+\frac{4ib^2c}{d}+\frac{2i(6ia^2e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+2*a*b*(-1/3/si
n(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c))+b^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)^4
-cos(d*x+c)^2-2*ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 105, normalized size = 0.83

$$\frac{6\,b^2\sin(dx+c)^2+24\,ab\sin(dx+c)+12\,(a^2-2\,b^2)\log(\sin(dx+c))+\frac{48\,ab\sin(dx+c)^3-8\,ab\sin(dx+c)+6\,(2\,a^2-b^2)\sin(dx+c)^2-3\,a^2}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(sin
(d*x + c)) + (48*a*b*sin(d*x + c)^3 - 8*a*b*sin(d*x + c) + 6*(2*a^2 - b^2)*
sin(d*x + c)^2 - 3*a^2)/sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.39, size = 177, normalized size = 1.40

$$-\frac{6\,b^2\cos\left(dx+c\right)^6-15\,b^2\cos\left(dx+c\right)^4+6\,\left(2\,a^2+b^2\right)\cos\left(dx+c\right)^2-9\,a^2+3\,b^2-12\,\left(\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^4-2\,\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^2+a^2-2\,b^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\,\left(3\,ab\cos\left(dx+c\right)^4-12\,ab\cos\left(dx+c\right)^2+8\,ab\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*cos(d*x + c)^6 - 15*b^2*cos(d*x + c)^4 + 6*(2*a^2 + b^2)*cos(d
*x + c)^2 - 9*a^2 + 3*b^2 - 12*((a^2 - 2*b^2)*cos(d*x + c)^4 - 2*(a^2 - 2*b
^2)*cos(d*x + c)^2 + a^2 - 2*b^2)*log(1/2*sin(d*x + c)) - 8*(3*a*b*cos(d*x
+ c)^4 - 12*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2
*d*cos(d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\cot^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**5, x)
```

Giac [A]
time = 8.73, size = 138, normalized size = 1.10

$$\frac{6\,b^2\sin\left(dx+c\right)^2+24\,ab\sin\left(dx+c\right)+12\,\left(a^2-2\,b^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{25\,a^2\sin(dx+c)^4-50\,b^2\sin(dx+c)^4-48\,ab\sin(dx+c)^3-12\,a^2\sin(dx+c)^2+6\,b^2\sin(dx+c)^2+8\,ab\sin(dx+c)+3\,a^2}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(abs
(sin(d*x + c))) - (25*a^2*sin(d*x + c)^4 - 50*b^2*sin(d*x + c)^4 - 48*a*b*s
in(d*x + c)^3 - 12*a^2*sin(d*x + c)^2 + 6*b^2*sin(d*x + c)^2 + 8*a*b*sin(d*
x + c) + 3*a^2)/sin(d*x + c)^4)/d
```

Mupad [B]
time = 6.44, size = 310, normalized size = 2.46

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{5a^2}{2}-2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{23a^2}{4}-4b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(3a^2+30b^2\right)-\frac{a^2}{4}+\frac{76ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{356ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3}+92ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-\frac{4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}}{d\left(16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\left(a^2-2b^2\right)}{d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2-2b^2\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3a^2}{16}-\frac{b^2}{8}\right)}{d}-\frac{ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12d}+\frac{7ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*((5*a^2)/2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*((23*a^2)/
4 - 4*b^2) + tan(c/2 + (d*x)/2)^6*(3*a^2 + 30*b^2) - a^2/4 + (76*a*b*tan(c/
2 + (d*x)/2)^3)/3 + (356*a*b*tan(c/2 + (d*x)/2)^5)/3 + 92*a*b*tan(c/2 + (d*
x)/2)^7 - (4*a*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 + 32*ta
n(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8)) - (log(tan(c/2 + (d*x)/2)^2
+ 1)*(a^2 - 2*b^2))/d - (a^2*tan(c/2 + (d*x)/2)^4)/(64*d) + (log(tan(c/2 +
(d*x)/2))*(a^2 - 2*b^2))/d + (tan(c/2 + (d*x)/2)^2*((3*a^2)/16 - b^2/8))/d
- (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) + (7*a*b*tan(c/2 + (d*x)/2))/(4*d)
```

3.155 $\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=149

$$a^2x+\frac{5b^2x}{2}-\frac{2ab\cos(c+dx)}{d}-\frac{4ab\sec(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}-\frac{5b^2\tan(c+dx)}{2d}+\frac{a^2\tan^3}{3\ldots}$$

```
[Out] a^2*x+5/2*b^2*x-2*a*b*cos(d*x+c)/d-4*a*b*sec(d*x+c)/d+2/3*a*b*sec(d*x+c)^3/
d-a^2*tan(d*x+c)/d-5/2*b^2*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d+5/6*b^2*tan(
d*x+c)^3/d-1/2*b^2*sin(d*x+c)^2*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {2801, 3554, 8, 2670, 276, 2671, 294, 308, 209}

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x-\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{4ab\sec(c+dx)}{d}+\frac{5b^2\tan^3(c+dx)}{6d}-\frac{5b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan^3(c+dx)}{2d}+\frac{5b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] a^2*x + (5*b^2*x)/2 - (2*a*b*Cos[c + d*x])/d - (4*a*b*Sec[c + d*x])/d + (2*
a*b*Sec[c + d*x]^3)/(3*d) - (a^2*Tan[c + d*x])/d - (5*b^2*Tan[c + d*x])/(2*
d) + (a^2*Tan[c + d*x]^3)/(3*d) + (5*b^2*Tan[c + d*x]^3)/(6*d) - (b^2*Sin[c
 + d*x]^2*Tan[c + d*x]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx = \int \left(a^2\tan^4(c+dx)+2ab\sin(c+dx)\tan^4(c+dx)+b^2\sin^2(c+dx\right.$$

$$= a^2\int \tan^4(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan^4(c+dx)\,dx+b^2\int$$

$$= \frac{a^2\tan^3(c+dx)}{3d}-a^2\int \tan^2(c+dx)\,dx-\frac{(2ab)\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx\right.}{d}$$

$$= -\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}-\frac{b^2\sin^2(c+dx)\tan^3(c+dx)}{2d}$$

$$= a^2x-\frac{2ab\cos(c+dx)}{d}-\frac{4ab\sec(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{a^2}{}$$

$$= a^2x-\frac{2ab\cos(c+dx)}{d}-\frac{4ab\sec(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{a^2}{}$$

$$= a^2x+\frac{5b^2x}{2}-\frac{2ab\cos(c+dx)}{d}-\frac{4ab\sec(c+dx)}{d}+\frac{2ab\sec^3(c+d}{3d}$$

Mathematica [A]
time = 0.47, size = 176, normalized size = 1.18

$$-\frac{\sec^3(c+dx)\left(200ab-36(2a^2+5b^2)(c+dx)\cos(c+dx)+288ab\cos(2(c+dx))-24a^2c\cos(3(c+dx))-60b^2c\cos(3(c+dx))-24a^2dx\cos(3(c+dx))-60b^2dx\cos(3(c+dx))+24ab\cos(4(c+dx))+30b^2\sin(c+dx)+32a^2\sin(3(c+dx))+65b^2\sin(3(c+dx))+3b^2\sin(5(c+dx))\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -1/96*(Sec[c + d*x]^3*(200*a*b - 36*(2*a^2 + 5*b^2)*(c + d*x)*Cos[c + d*x]
+ 288*a*b*Cos[2*(c + d*x)] - 24*a^2*c*Cos[3*(c + d*x)] - 60*b^2*c*Cos[3*(c
+ d*x)] - 24*a^2*d*x*Cos[3*(c + d*x)] - 60*b^2*d*x*Cos[3*(c + d*x)] + 24*a*
b*Cos[4*(c + d*x)] + 30*b^2*Sin[c + d*x] + 32*a^2*Sin[3*(c + d*x)] + 65*b^2
*Sin[3*(c + d*x)] + 3*b^2*Sin[5*(c + d*x)]))/d
```

Maple [A]
time = 0.22, size = 185, normalized size = 1.24

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+2ab\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}$

default	$\frac{a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+2ab\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+}{d}$
risch	$a^2x+\frac{5b^2x}{2}+\frac{ib^2e^{2i(dx+c)}}{8d}-\frac{ab\,e^{i(dx+c)}}{d}-\frac{ab\,e^{-i(dx+c)}}{d}-\frac{ib^2e^{-2i(dx+c)}}{8d}-\frac{2(6ia^2e^{4i(dx+c)}+9ib^2e^{4i(dx+c)}+1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+2*a*b*(1/3*sin(d*x+c)^6/cos(d*
x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+
c))+b^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin
(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A]
time = 0.57, size = 119, normalized size = 0.80

$$\frac{2\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^2+\left(2\,\tan\left(dx+c\right)^3+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^2+1}-12\,\tan\left(dx+c\right)\right)b^2-4\,ab\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2 + (2*tan(d*x + c
)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x + c)
)*b^2 - 4*a*b*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [A]
time = 0.35, size = 118, normalized size = 0.79

$$\frac{3\,(2\,a^2+5\,b^2)dx\cos\left(dx+c\right)^3-12\,ab\cos\left(dx+c\right)^4-24\,ab\cos\left(dx+c\right)^2+4\,ab-\left(3\,b^2\cos\left(dx+c\right)^4+2\,(4\,a^2+7\,b^2)\cos\left(dx+c\right)^2-2\,a^2-2\,b^2\right)\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*a^2 + 5*b^2)*d*x*cos(d*x + c)^3 - 12*a*b*cos(d*x + c)^4 - 24*a*b*
cos(d*x + c)^2 + 4*a*b - (3*b^2*cos(d*x + c)^4 + 2*(4*a^2 + 7*b^2)*cos(d*x
+ c)^2 - 2*a^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\tan^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**4,x)
[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**4, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 10.04, size = 235, normalized size = 1.58

$$\frac{x\,\left(2\,a^2+5\,b^2\right)}{2}-\frac{\left(-2\,a^2-5\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,a^2}{3}+\frac{20\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{28\,a^2}{3}+\frac{22\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{64\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\left(\frac{8\,a^2}{3}+\frac{20\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{32\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\left(-2\,a^2-5\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{32\,a\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x))^2,x)
[Out] (x*(2*a^2 + 5*b^2))/2 - (tan(c/2 + (d*x)/2)^3*((8*a^2)/3 + (20*b^2)/3) - ta
n(c/2 + (d*x)/2)^9*(2*a^2 + 5*b^2) - (32*a*b)/3 + tan(c/2 + (d*x)/2)^7*((8*
a^2)/3 + (20*b^2)/3) + tan(c/2 + (d*x)/2)^5*((28*a^2)/3 + (22*b^2)/3) - tan
(c/2 + (d*x)/2)*(2*a^2 + 5*b^2) + (32*a*b*tan(c/2 + (d*x)/2)^2)/3 + (64*a*b
*tan(c/2 + (d*x)/2)^4)/3)/(d*(tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^4
 - 2*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 -
1))
```

3.156 $\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=94

$$-a^2x-\frac{3b^2x}{2}+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}$$

```
[Out] -a^2*x-3/2*b^2*x+2*a*b*cos(d*x+c)/d+2*a*b*sec(d*x+c)/d+a^2*tan(d*x+c)/d+3/2
*b^2*tan(d*x+c)/d-1/2*b^2*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {2801, 3554, 8, 2670, 14, 2671, 294, 327, 209}

$$\frac{a^2\tan(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (3*b^2*x)/2 + (2*a*b*Cos[c + d*x])/d + (2*a*b*Sec[c + d*x])/d +
(a^2*Tan[c + d*x])/d + (3*b^2*Tan[c + d*x])/(2*d) - (b^2*Sin[c + d*x]^2*Tan
[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = \int \left(a^2\tan^2(c+dx)+2ab\sin(c+dx)\tan^2(c+dx)+b^2\sin^2(c+dx)\right.$$

$$= a^2\int \tan^2(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan^2(c+dx)\,dx+b^2\int$$

$$= \frac{a^2\tan(c+dx)}{d}-a^2\int 1\,dx-\frac{(2ab)\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d}$$

$$= -a^2x+\frac{a^2\tan(c+dx)}{d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{(2ab)\mathrm{Subst}}{}$$

$$= -a^2x+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{3b^2\mathrm{t}}{}$$

$$= -a^2x-\frac{3b^2x}{2}+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}$$

Mathematica [A]
time = 0.33, size = 77, normalized size = 0.82

$$\frac{-4(2a^2+3b^2)(c+dx)+b\sec(c+dx)(24a+8a\cos(2(c+dx))+b\sin(3(c+dx)))+(8a^2+9b^2)\tan(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-4*(2*a^2 + 3*b^2)*(c + d*x) + b*Sec[c + d*x]*(24*a + 8*a*Cos[2*(c + d*x)]
 + b*Sin[3*(c + d*x)]) + (8*a^2 + 9*b^2)*Tan[c + d*x])/(8*d)
```

Maple [A]
time = 0.23, size = 116, normalized size = 1.23

method	result
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(d\right.}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(d\right.}{d}$
risch	$-a^2x-\frac{3b^2x}{2}-\frac{ib^2e^{2i(dx+c)}}{8d}+\frac{ab\,e^{i(dx+c)}}{d}+\frac{ab\,e^{-i(dx+c)}}{d}+\frac{ib^2e^{-2i(dx+c)}}{8d}+\frac{2ia^2+2ib^2+4a\,e^{i(dx+c)}b}{d(1+e^{2i(dx+c)})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)
*cos(d*x+c))+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos
(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]
time = 0.72, size = 83, normalized size = 0.88

$$-\frac{2\,(dx+c-\tan\,(dx+c))a^2+\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\,(dx+c)\right)b^2-4\,ab\left(\frac{1}{\cos(dx+c)}+\cos\,(dx+c)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*a^2 + (3*d*x + 3*c - tan(d*x + c)/(tan(d*x
 + c)^2 + 1) - 2*tan(d*x + c))*b^2 - 4*a*b*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

Fricas [A]
time = 0.40, size = 81, normalized size = 0.86

$$-\frac{(2\,a^2+3\,b^2)dx\cos\,(dx+c)-4\,ab\cos\,(dx+c)^2-4\,ab-\left(b^2\cos\,(dx+c)^2+2\,a^2+2\,b^2\right)\sin\,(dx+c)}{2\,d\cos\,(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 + 3*b^2)*d*x*cos(d*x + c) - 4*a*b*cos(d*x + c)^2 - 4*a*b - (b^
2*cos(d*x + c)^2 + 2*a^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin\,(c+dx))^2\tan^2\,(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 7670 vs. $2(88) = 176$.
time = 29.43, size = 7670, normalized size = 81.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 3*b^2*d*x
*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*a^2*d*x*tan(d*x)^3*tan
```

(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 3*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1
/2*c)^4*tan(c) - 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2
- 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 8*a^2*d*x*tan
(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/
2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 2*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*
c)^4*tan(c)^3 + 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 - 8
*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*a^2*tan(d*x)^3*tan
(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 3*b^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2
*c)^4*tan(c)^2 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 3*
b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 - 2*a^2*d*x*tan(d*x)^2*
tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c
)^4 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 12*b^2*d*x*
tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 2*a^2*d*x*tan(d*x)*tan(1/2*
d*x)^4*tan(1/2*c)^4*tan(c) + 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4
*tan(c) - 8*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 8*a^2*d*x*t
an(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 12*b^2*d*x*tan(d*x)^2*tan(
1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 2*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*ta
n(c)^2 - 3*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 8*a*b*tan(d*x)^2*
tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*
tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*d*x*tan(d*x
)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)
^3*tan(1/2*c)*tan(c)^3 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan
(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 8*a^2*d*
x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(1
/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 32*a*b*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*
c)^3*tan(c)^3 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 3*b^2*d*x*tan(
d*x)^3*tan(1/2*c)^4*tan(c)^3 - 8*a*b*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*t
an(c)^3 + 2*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4 + 2*b^2*tan(d*x)^3*t
an(1/2*d*x)^4*tan(1/2*c)^4 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*t
an(c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 12*b^2*tan(
d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*tan(d*x)*tan(1/2*d*x)^4
*tan(1/2*c)^4*tan(c)^2 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c
)^3 - 12*b^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 2*a^2*tan(1/
2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3
 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 12*b^2*d*x*tan(d*x)^2
*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*b^
2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 8*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2
*c)^4 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 3*b^2*d*x*tan(d*x)^3*t
an(1/2*d*x)^4*tan(c) - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c
) - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 8*a^2*d*x*tan(
d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x
)*tan(1/2*c)^3*tan(c) - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(
c) - 12*b^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 32*a*b*tan(d*
x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4
*tan(c) - 3*b^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c) - 8*a*b*tan(d*x)*tan(1/2

```
*d*x)^4*tan(1/2*c)^4*tan(c) + 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2
+ 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 8*a^2*d*x*tan(d*x)^2*tan(1
/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/
2*c)*tan(c)^2 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 1
2*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 8*a^2*d*x*tan(1/2
*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 12*b^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(
c)^2 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*d*x*t
an(d*x)^2*tan(1/2*c)^4*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*c)^4*tan(c)^
2 + 8*a*b*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 2*a^2*d*x*tan(d*x)*tan(1/2
*d*x)^4*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 8*a*b*tan(d
*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c
)*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 8*a^2
*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(
1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 32*a*b*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c
)*tan(c)^3 - 96*a*b*tan(d*x)^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^3 - 8*a^2
*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(
1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 32*a*b*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^
3*tan(c)^3 + 32*a*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 2*a^2*d
*x*tan(d*x)*tan(1/2*c)^4*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(1/2*c)^4*tan(c)^
3 - 8*a*b*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 8*...
```

Mupad [B]
time = 9.29, size = 147, normalized size = 1.56

$$\frac{(2a^2+3b^2)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+(4a^2+2b^2)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+8ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+(2a^2+3b^2)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+8ab}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{x\,(2a^2+3b^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (8*a*b + tan(c/2 + (d*x)/2)^3*(4*a^2 + 2*b^2) + tan(c/2 + (d*x)/2)^5*(2*a^2
 + 3*b^2) + tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2) + 8*a*b*tan(c/2 + (d*x)/2)^2
)/(d*(tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 +
1)) - (x*(2*a^2 + 3*b^2))/2
```

3.157 $\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=78

$$-a^2x+\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] -a^2*x+1/2*b^2*x-2*a*b*arctanh(cos(d*x+c))/d+2*a*b*cos(d*x+c)/d-a^2*cot(d*x
+c)/d+1/2*b^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2801, 2715, 8, 2672, 327, 212, 3554}

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (b^2*x)/2 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c + d*x
])/d - (a^2*Cot[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)+2ab\cos(c+dx)\cot(c+dx)+a^2\cot^2(c+dx)\right.$$

$$= a^2\int \cot^2(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot(c+dx)\,dx+b^2\int$$

$$= -\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}-a^2\int 1\,dx+\frac{1}{2}b^2\int$$

$$= -a^2x+\frac{b^2x}{2}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\,\mathrm{s}}{2d}$$

$$= -a^2x+\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\,\mathrm{co}}{}$$

Mathematica [A]
time = 0.28, size = 116, normalized size = 1.49

$$\frac{-4a^2c+2b^2c-4a^2dx+2b^2dx+8ab\cos(c+dx)-2a^2\cot\left(\frac{1}{2}(c+dx)\right)-8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+8ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+b^2\sin(2(c+dx))+2a^2\tan\left(\frac{1}{2}(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a^2*c + 2*b^2*c - 4*a^2*d*x + 2*b^2*d*x + 8*a*b*Cos[c + d*x] - 2*a^2*Co
t[(c + d*x)/2] - 8*a*b*Log[Cos[(c + d*x)/2]] + 8*a*b*Log[Sin[(c + d*x)/2]]
+ b^2*Sin[2*(c + d*x)] + 2*a^2*Tan[(c + d*x)/2])/(4*d)
```

Maple [A]
time = 0.15, size = 79, normalized size = 1.01

method	result
derivativedivides	$\frac{a^2(-\cot(dx+c)-dx-c)+2ab(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2(-\cot(dx+c)-dx-c)+2ab(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$-a^2x+\frac{b^2x}{2}-\frac{ib^2e^{2i(dx+c)}}{8d}+\frac{ab\,e^{i(dx+c)}}{d}+\frac{ab\,e^{-i(dx+c)}}{d}+\frac{ib^2e^{-2i(dx+c)}}{8d}-\frac{2ia^2}{d\left(e^{2i(dx+c)}-1\right)}+\frac{2ab\ln\left(e^{i(dx+\right.}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-cot(d*x+c)-d*x-c)+2*a*b*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+b
^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.50, size = 79, normalized size = 1.01

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-(2\,dx+2\,c+\sin(2\,dx+2\,c))b^2-4\,ab(2\,\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*a^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*b
^2 - 4*a*b*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1))
)/d
```

Fricas [A]
time = 0.38, size = 118, normalized size = 1.51

$$-\frac{b^2\cos(dx+c)^3+2\,ab\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,ab\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+(2\,a^2-b^2)\cos(dx+c)+((2\,a^2-b^2)dx-4\,ab\cos(dx+c))\sin(dx+c)}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^3 + 2*a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c) + ((2*a^2 - b^2)*d*x - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2\cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**2, x)
```

Giac [A]
time = 14.78, size = 148, normalized size = 1.90

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-(2\,a^2-b^2)(dx+c)-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + a^2*tan(1/2*d*x + 1/2*c) - (2*a
^2 - b^2)*(d*x + c) - (4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*
c) - 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 7.27, size = 277, normalized size = 3.55

$$\frac{b^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}\right)-2\,a^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}\right)+2\,a\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\cos\left(c+d\,x\right)-\frac{b^2\,\cos(c+d\,x)}{8}+\frac{b^2\,\cos(3\,c+3\,d\,x)}{8}-a\,b\,\sin\left(2\,c+2\,d\,x\right)}{d\,\sin\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (b^2*atan((b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 4*a*b*sin(c/
2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - b^2*sin(c/2 + (d*x)/2) + 4*a*b*co
s(c/2 + (d*x)/2))) - 2*a^2*atan((b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (
d*x)/2) + 4*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - b^2*sin(c/2
 + (d*x)/2) + 4*a*b*cos(c/2 + (d*x)/2))) + 2*a*b*log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/d - (a^2*cos(c + d*x) - (b^2*cos(c + d*x))/8 + (b^2*cos(3
*c + 3*d*x))/8 - a*b*sin(2*c + 2*d*x))/(d*sin(c + d*x))
```

3.158 $\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=133

$$a^2x-\frac{3b^2x}{2}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d}-\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+dx)\cot}{2d}$$

```
[Out] a^2*x-3/2*b^2*x+3*a*b*arctanh(cos(d*x+c))/d-3*a*b*cos(d*x+c)/d+a^2*cot(d*x+
c)/d-3/2*b^2*cot(d*x+c)/d+1/2*b^2*cos(d*x+c)^2*cot(d*x+c)/d-a*b*cos(d*x+c)*
cot(d*x+c)^2/d-1/3*a^2*cot(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2801, 2671, 294, 327, 209, 2672, 212, 3554, 8}

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{3ab\cos(c+dx)}{d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (3*b^2*x)/2 + (3*a*b*ArcTanh[Cos[c + d*x]])/d - (3*a*b*Cos[c + d*x]
)/d + (a^2*Cot[c + d*x])/d - (3*b^2*Cot[c + d*x])/(2*d) + (b^2*Cos[c + d*x]
^2*Cot[c + d*x])/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^2)/d - (a^2*Cot[c +
 d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx &= \int \left(b^2\cos^2(c+dx)\cot^2(c+dx)+2ab\cos(c+dx)\cot^3(c+dx)+a^2\right. \\
&= a^2\int \cot^4(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot^3(c+dx)\,dx+b^2\int \\
&= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx-\frac{(2ab)\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx\right.}{d} \\
&= \frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{ab\cos(c+dx)\cot^2(c}{d} \\
&= a^2x-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d}-\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2}{} \\
&= a^2x-\frac{3b^2x}{2}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(}{d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 293 vs. 2(133) = 266.
time = 6.18, size = 293, normalized size = 2.20

$$\frac{(2a^2-3b^2)(c+dx)}{2d}-\frac{2ab\cos(c+dx)}{d}+\frac{\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6d}-\frac{ab\csc^2\left(\frac{1}{2}(c+dx)\right)}{4d}-\frac{a^2\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d}+\frac{3ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{3ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{ab\sec^2\left(\frac{1}{2}(c+dx)\right)}{4d}+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^2\sin\left(\frac{1}{2}(c+dx)\right)+3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d}-\frac{b^2\sin(2(c+dx))}{4d}+\frac{a^2\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - 3*b^2)*(c + d*x))/(2*d) - (2*a*b*Cos[c + d*x])/d + ((4*a^2*Cos[(c
 + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (a*b*Csc[(c
+ d*x)/2]^2)/(4*d) - (a^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (3*
a*b*Log[Cos[(c + d*x)/2]])/d - (3*a*b*Log[Sin[(c + d*x)/2]])/d + (a*b*Sec[(
c + d*x)/2]^2)/(4*d) + (Sec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 3*b^2*S
in[(c + d*x)/2]))/(6*d) - (b^2*Sin[2*(c + d*x)])/(4*d) + (a^2*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/(24*d)
```

Maple [A]
time = 0.24, size = 145, normalized size = 1.09

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b}{d}$

risch	$a^2x-\frac{3b^2x}{2}+\frac{ib^2\mathrm{e}^{2i(dx+c)}}{8d}-\frac{ab\,\mathrm{e}^{i(dx+c)}}{d}-\frac{ab\,\mathrm{e}^{-i(dx+c)}}{d}-\frac{ib^2\mathrm{e}^{-2i(dx+c)}}{8d}+\frac{4ia^2\mathrm{e}^{4i(dx+c)}-2ib^2\mathrm{e}^{4i(dx+c)}+2c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a*b*(-1/2/sin(d*x+c)^2*cos(
d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+b^2
*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)-3/2*d
*x-3/2*c))
```

Maxima [A]
time = 0.56, size = 138, normalized size = 1.04

$$\frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b^2+3\,ab\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c)-1)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*(3*d*x
 + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*b^2 + 3*a*
b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c
) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.38, size = 218, normalized size = 1.64

$$\frac{3\,b^2\cos(dx+c)^5+4\,(2\,a^2-3\,b^2)\cos(dx+c)^3+9\,(ab\cos(dx+c)^2-ab)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-9\,(ab\cos(dx+c)^2-ab)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3\,(2\,a^2-3\,b^2)\cos(dx+c)+3\,((2\,a^2-3\,b^2)dx\cos(dx+c)^2-4\,ab\cos(dx+c)^3-(2\,a^2-3\,b^2)dx+6\,ab\cos(dx+c))\sin(dx+c)}{6\,(d\cos(dx+c)^2-d)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*cos(d*x + c)^5 + 4*(2*a^2 - 3*b^2)*cos(d*x + c)^3 + 9*(a*b*cos(d
*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a*b*cos(d*x
+ c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(2*a^2 - 3*b^2)
*cos(d*x + c) + 3*((2*a^2 - 3*b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)^
3 - (2*a^2 - 3*b^2)*d*x + 6*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^2 - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2\cot^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**4, x)
```

Giac [A]
time = 18.62, size = 241, normalized size = 1.81

$$\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-72ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)-15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12\left(2a^2-3b^2\right)(dx+c)+\frac{24\left(b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4ab\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}+\frac{132ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 72*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c))) - 15*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2
*d*x + 1/2*c) + 12*(2*a^2 - 3*b^2)*(d*x + c) + 24*(b^2*tan(1/2*d*x + 1/2*c)
^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan(1/2*d*x + 1/2*c) - 4*a*b)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a*b*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/
2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b*tan(1/2*d*x + 1/2*
c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 9.00, size = 584, normalized size = 4.39

$$-\frac{\frac{5b^2\cos(c+dx)}{16}+\frac{a^2\cos(3c+3dx)}{3}-\frac{11b^2\cos(3c+3dx)}{32}+\frac{b^2\cos(5c+5dx)}{32}+\frac{a^2\,\mathrm{atan}\left(\frac{-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)ab+3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab-3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}\right)\sin(3c+3dx)}{2}-\frac{3b^2\,\mathrm{atan}\left(\frac{-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)ab+3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab-3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}\right)\sin(3c+3dx)}{4}+\frac{3ab\sin(c+dx)}{2}-\frac{3a^2\,\mathrm{atan}\left(\frac{-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)ab+3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab-3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}\right)\sin(c+dx)}{2}+\frac{9b^2\,\mathrm{atan}\left(\frac{-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)ab+3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab-3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}\right)\sin(c+dx)}{4}+ab\sin\left(2c+2dx\right)-\frac{ab\sin(3c+3dx)}{2}-\frac{ab\sin(4c+4dx)}{4}+\frac{9ab\sin(c+dx)\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{4}-\frac{3ab\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin(3c+3dx)}{4}}{d\sin\left(c+dx\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^2,x)
```

```
[Out] -((5*b^2*cos(c + d*x))/16 + (a^2*cos(3*c + 3*d*x))/3 - (11*b^2*cos(3*c + 3*
d*x))/32 + (b^2*cos(5*c + 5*d*x))/32 + (a^2*atan((3*b^2*cos(c/2 + (d*x)/2)
- 2*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*
x)/2) - 3*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d
*x))/2 - (3*b^2*atan((3*b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) +
 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x
)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/4 + (3*a*b*sin(c + d*x)
)/2 - (3*a^2*atan((3*b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 6*
a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x)/2
) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(c + d*x))/2 + (9*b^2*atan((3*b^2*cos(c/2
 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*s
in(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*s
in(c + d*x))/4 + a*b*sin(2*c + 2*d*x) - (a*b*sin(3*c + 3*d*x))/2 - (a*b*sin
(4*c + 4*d*x))/4 + (9*a*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2)))/4 - (3*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d
*x))/4)/(d*sin(c + d*x)^3)
```

3.159 $\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=202

$$-a^2x+\frac{5b^2x}{2}-\frac{15ab\tanh^{-1}(\cos(c+dx))}{4d}+\frac{15ab\cos(c+dx)}{4d}-\frac{a^2\cot(c+dx)}{d}+\frac{5b^2\cot(c+dx)}{2d}+\frac{5ab\cos(c+d}{}$$

```
[Out] -a^2*x+5/2*b^2*x-15/4*a*b*arctanh(cos(d*x+c))/d+15/4*a*b*cos(d*x+c)/d-a^2*c
ot(d*x+c)/d+5/2*b^2*cot(d*x+c)/d+5/4*a*b*cos(d*x+c)*cot(d*x+c)^2/d+1/3*a^2*
cot(d*x+c)^3/d-5/6*b^2*cot(d*x+c)^3/d+1/2*b^2*cos(d*x+c)^2*cot(d*x+c)^3/d-1
/2*a*b*cos(d*x+c)*cot(d*x+c)^4/d-1/5*a^2*cot(d*x+c)^5/d
```

Rubi [A]
time = 0.13, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2801, 2671, 294, 308, 209, 2672, 327, 212, 3554, 8}

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}+\frac{5ab\cos(c+dx)\cot^2(c+dx)}{4d}-\frac{15ab\tanh^{-1}(\cos(c+dx))}{4d}-\frac{5b^2\cot^3(c+dx)}{6d}+\frac{5b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+dx)\cot^3(c+dx)}{2d}+\frac{5b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (5*b^2*x)/2 - (15*a*b*ArcTanh[Cos[c + d*x]])/(4*d) + (15*a*b*Cos
[c + d*x])/(4*d) - (a^2*Cot[c + d*x])/d + (5*b^2*Cot[c + d*x])/(2*d) + (5*a
*b*Cos[c + d*x]*Cot[c + d*x]^2)/(4*d) + (a^2*Cot[c + d*x]^3)/(3*d) - (5*b^2
*Cot[c + d*x]^3)/(6*d) + (b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*b*C
os[c + d*x]*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)\cot^4(c+dx) + 2ab\cos(c+dx)\cot^5(c+dx) + a\right.$$

$$= a^2\int \cot^6(c+dx)\,dx + (2ab)\int \cos(c+dx)\cot^5(c+dx)\,dx + b^2\int$$

$$= -\frac{a^2\cot^5(c+dx)}{5d} - a^2\int \cot^4(c+dx)\,dx - \frac{(2ab)\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,d\right.}{d}$$

$$= \frac{a^2\cot^3(c+dx)}{3d} + \frac{b^2\cos^2(c+dx)\cot^3(c+dx)}{2d} - \frac{ab\cos(c+dx)\,\text{co}}{2d}$$

$$= -\frac{a^2\cot(c+dx)}{d} + \frac{5ab\cos(c+dx)\cot^2(c+dx)}{4d} + \frac{a^2\cot^3(c+dx)}{3d}$$

$$= -a^2x + \frac{15ab\cos(c+dx)}{4d} - \frac{a^2\cot(c+dx)}{d} + \frac{5b^2\cot(c+dx)}{2d} + \frac{5a}{}$$

$$= -a^2x + \frac{5b^2x}{2} - \frac{15ab\tanh^{-1}(\cos(c+dx))}{4d} + \frac{15ab\cos(c+dx)}{4d} - \frac{a}{}$$

Mathematica [A]
time = 0.75, size = 351, normalized size = 1.74

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-480*a^2*c + 1200*b^2*c - 480*a^2*d*x + 1200*b^2*d*x + 960*a*b*Cos[c + d*x
] + (-368*a^2 + 560*b^2)*Cot[(c + d*x)/2] + 270*a*b*Csc[(c + d*x)/2]^2 - 15
*a*b*Csc[(c + d*x)/2]^4 - 1800*a*b*Log[Cos[(c + d*x)/2]] + 1800*a*b*Log[Sin
[(c + d*x)/2]] - 270*a*b*Sec[(c + d*x)/2]^2 + 15*a*b*Sec[(c + d*x)/2]^4 - 3
28*a^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 160*b^2*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + 96*a^2*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + (41*a^2*Csc[(c + d*x
)/2]^4*Sin[c + d*x])/2 - 10*b^2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - (3*a^2*Cs
c[(c + d*x)/2]^6*Sin[c + d*x])/2 + 120*b^2*Sin[2*(c + d*x)] + 368*a^2*Tan[(
c + d*x)/2] - 560*b^2*Tan[(c + d*x)/2])/(480*d)
```

Maple [A]
time = 0.25, size = 216, normalized size = 1.07

method	result

derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+\right.}{8}}{}$
default	$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+\right.}{8}}{}$
risch	$-a^2x+\frac{5b^2x}{2}-\frac{ib^2\mathrm{e}^{2i(dx+c)}}{8d}+\frac{ab\,\mathrm{e}^{i(dx+c)}}{d}+\frac{ab\,\mathrm{e}^{-i(dx+c)}}{d}+\frac{ib^2\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{180ia^2\mathrm{e}^{8i(dx+c)}-180ib^2\mathrm{e}^{8i(dx+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+2*a*b*(-1/4/
sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos(d*x+c)^5+5/
8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))+b^2*(-1/3/si
n(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(cos(d*x+c)^5+5/4*c
os(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A]
time = 0.50, size = 183, normalized size = 0.91

$$-\frac{8\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-20\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)b^2+15\,ab\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos(dx+c)+15\,\log(\cos(dx+c)+1)-15\,\log(\cos(dx+c)-1)\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/120*(8*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d
*x + c)^5)*a^2 - 20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2
 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*b^2 + 15*a*b*(2*(9*cos(d*x + c)^3
- 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c)
 + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.39, size = 306, normalized size = 1.51

$$-\frac{60\,b^2\cos(dx+c)^7+92\,(2\,a^2-5\,b^2)\cos(dx+c)^5-140\,(2\,a^2-5\,b^2)\cos(dx+c)^3+225\,(ab\cos(dx+c)^4-2\,ab\cos(dx+c)^2+ab)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-225\,(ab\cos(dx+c)^4-2\,ab\cos(dx+c)^2+ab)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+60\,(2\,a^2-5\,b^2)\cos(dx+c)+30\,(2\,(2\,a^2-5\,b^2)dx\cos(dx+c)^4-8\,ab\cos(dx+c)^5-4\,(2\,a^2-5\,b^2)dx\cos(dx+c)^2+25\,ab\cos(dx+c)^3+2\,(2\,a^2-5\,b^2)dx-15\,ab\cos(dx+c))\sin(dx+c)}{120\,(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/120*(60*b^2*cos(d*x + c)^7 + 92*(2*a^2 - 5*b^2)*cos(d*x + c)^5 - 140*(2*
a^2 - 5*b^2)*cos(d*x + c)^3 + 225*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^
```

```
2 + a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(a*b*cos(d*x + c)^4
 - 2*a*b*cos(d*x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
60*(2*a^2 - 5*b^2)*cos(d*x + c) + 30*(2*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^4
- 8*a*b*cos(d*x + c)^5 - 4*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^2 + 25*a*b*cos(
d*x + c)^3 + 2*(2*a^2 - 5*b^2)*d*x - 15*a*b*cos(d*x + c))*sin(d*x + c))/((d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^2 \cot^6(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**6, x)
```

Giac [**A**]
time = 11.45, size = 337, normalized size = 1.67

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1800\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-540\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-240\,(2\,a^2-5\,b^2)(dx+c)-\frac{480\,\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}-\frac{4110\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-540\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^
2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*a*b*tan(1/2*
d*x + 1/2*c)^2 + 1800*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 330*a^2*tan(1/2*
d*x + 1/2*c) - 540*b^2*tan(1/2*d*x + 1/2*c) - 240*(2*a^2 - 5*b^2)*(d*x + c)
 - 480*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (4110*a*b*tan(1
/2*d*x + 1/2*c)^5 + 330*a^2*tan(1/2*d*x + 1/2*c)^4 - 540*b^2*tan(1/2*d*x +
1/2*c)^4 - 240*a*b*tan(1/2*d*x + 1/2*c)^3 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 +
 20*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a*b*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan(1
/2*d*x + 1/2*c)^5)/d
```

Mupad [**B**]
time = 11.28, size = 888, normalized size = 4.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((95*b^2*cos(c + d*x))/384 - (5*a^2*cos(c + d*x))/24 + (5*a^2*cos(3*c + 3*d
*x))/48 - (23*a^2*cos(5*c + 5*d*x))/240 - (163*b^2*cos(3*c + 3*d*x))/384 +
```

(71*b^2*cos(5*c + 5*d*x))/384 - (b^2*cos(7*c + 7*d*x))/128 + (5*a^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/8 - (a^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(5*c + 5*d*x))/8 - (25*b^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/16 + (5*b^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(5*c + 5*d*x))/16 + (5*a*b*sin(c + d*x))/4 - (5*a^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(c + d*x))/4 + (25*b^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(c + d*x))/8 + (5*a*b*sin(2*c + 2*d*x))/8 - (5*a*b*sin(3*c + 3*d*x))/8 - (17*a*b*sin(4*c + 4*d*x))/32 + (a*b*sin(5*c + 5*d*x))/8 + (a*b*sin(6*c + 6*d*x))/16 + (75*a*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/32 - (75*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/64 + (15*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(5*c + 5*d*x))/64)/(d*sin(c + d*x)^5)

3.160 $\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=150

$$\frac{(a+b)^2(2a+5b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-5b)(a-b)^2\log(1+\sin(c+dx))}{4d}+\frac{b(6a^2+5b^2)\sin(c+dx)}{2d}+$$

```
[Out] 1/4*(a+b)^2*(2*a+5*b)*ln(1-sin(d*x+c))/d+1/4*(2*a-5*b)*(a-b)^2*ln(1+sin(d*x
+c))/d+1/2*b*(6*a^2+5*b^2)*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3*si
n(d*x+c)^3/d+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3/d
```

Rubi [A]

time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2800, 1659, 1643, 647, 31}

$$\frac{b(6a^2+5b^2)\sin(c+dx)}{2d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{(a+b)^2(2a+5b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-5b)(a-b)^2\log(\sin(c+dx)+1)}{4d}+\frac{\sec^2(c+dx)(a+b\sin(c+dx))^3}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] ((a + b)^2*(2*a + 5*b)*Log[1 - Sin[c + d*x]])/(4*d) + ((2*a - 5*b)*(a - b)^
2*Log[1 + Sin[c + d*x]])/(4*d) + (b*(6*a^2 + 5*b^2)*Sin[c + d*x])/(2*d) + (
3*a*b^2*Sin[c + d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d) + (Sec[c + d*x]^
2*(a + b*Sin[c + d*x])^3)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
```

```
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^3}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^3}{2d} + \frac{\text{Subst}\left(\int \frac{(a+x)^2(-3b^4-2ab^2x-2b^2x^2)}{b^2-x^2}\right.}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^3}{2d} + \frac{\text{Subst}\left(\int \left(6a^2b^2+5b^4+6ab^2x+\right.\right.}{}$$

$$= \frac{b(6a^2+5b^2)\sin(c+dx)}{2d} + \frac{3ab^2\sin^2(c+dx)}{2d} + \frac{b^3\sin^3(c+dx)}{3d} + \frac{\text{se}}{}$$

$$= \frac{b(6a^2+5b^2)\sin(c+dx)}{2d} + \frac{3ab^2\sin^2(c+dx)}{2d} + \frac{b^3\sin^3(c+dx)}{3d} + \frac{\text{se}}{}$$

$$= \frac{(a+b)^2(2a+5b)\log(1-\sin(c+dx))}{4d} + \frac{(2a-5b)(a-b)^2\log(1+}{4d}$$

Mathematica [A]
time = 0.17, size = 141, normalized size = 0.94

$$\frac{3(a+b)^2(2a+5b)\log(1-\sin(c+dx))+3(2a-5b)(a-b)^2\log(1+\sin(c+dx))-\frac{3(a+b)^3}{-1+\sin(c+dx)}+12b(3a^2+2b^2)\sin(c+dx)+18ab^2\sin^2(c+dx)+4b^3\sin^3(c+dx)+\frac{3(a-b)^3}{1+\sin(c+dx)}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (3*(a + b)^2*(2*a + 5*b)*Log[1 - Sin[c + d*x]] + 3*(2*a - 5*b)*(a - b)^2*Lo
g[1 + Sin[c + d*x]] - (3*(a + b)^3)/(-1 + Sin[c + d*x]) + 12*b*(3*a^2 + 2*b
```

```
^2)*Sin[c + d*x] + 18*a*b^2*Sin[c + d*x]^2 + 4*b^3*Sin[c + d*x]^3 + (3*(a -
 b)^3)/(1 + Sin[c + d*x]))/(12*d)
```

Maple [A]
time = 0.19, size = 206, normalized size = 1.37

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a^2b\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2}{}$
default	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a^2b\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2}{}$
risch	$\frac{ib^3\mathrm{e}^{3i(dx+c)}}{24d}-\frac{3ib\,\mathrm{e}^{i(dx+c)}a^2}{2d}-\frac{i\left(2ia^3\mathrm{e}^{2i(dx+c)}+6ia\,b^2\mathrm{e}^{2i(dx+c)}+3a^2b\,\mathrm{e}^{3i(dx+c)}+b^3\mathrm{e}^{3i(dx+c)}-3a^2b\,\mathrm{e}^{i(dx+c)}-b^3\mathrm{e}^{i(}\right.}{d\left(1+\mathrm{e}^{2i(dx+c)}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+3*a^2*b*(1/2*sin(d*x+c)^5/cos(d*
x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a*b
^2*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*
x+c)))+b^3*(1/2*sin(d*x+c)^7/cos(d*x+c)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+c)^3
+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 162, normalized size = 1.08

$$\frac{4\,b^3\sin\left(dx+c\right)^3+18\,ab^2\sin\left(dx+c\right)^2+3\left(2\,a^3-9\,a^2b+12\,ab^2-5\,b^3\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(2\,a^3+9\,a^2b+12\,ab^2+5\,b^3\right)\log\left(\sin\left(dx+c\right)-1\right)+12\left(3\,a^2b+2\,b^3\right)\sin\left(dx+c\right)-\frac{6\left(a^3+3\,ab^2+\left(3\,a^2b+b^3\right)\sin(dx+c)\right)}{\sin(dx+c)^2-1}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 3*(2*a^3 - 9*a^2*b +
 12*a*b^2 - 5*b^3)*log(sin(d*x + c) + 1) + 3*(2*a^3 + 9*a^2*b + 12*a*b^2 +
5*b^3)*log(sin(d*x + c) - 1) + 12*(3*a^2*b + 2*b^3)*sin(d*x + c) - 6*(a^3 +
 3*a*b^2 + (3*a^2*b + b^3)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.37, size = 194, normalized size = 1.29

$$-\frac{18\,ab^2\cos\left(dx+c\right)^4-9\,ab^2\cos\left(dx+c\right)^2-3\left(2\,a^3-9\,a^2b+12\,ab^2-5\,b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,a^3+9\,a^2b+12\,ab^2+5\,b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)-6\,a^3-18\,ab^2+2\left(2\,b^3\cos\left(dx+c\right)^4-9\,a^2b-3\,b^3-2\left(9\,a^2b+7\,b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(18*a*b^2*cos(d*x + c)^4 - 9*a*b^2*cos(d*x + c)^2 - 3*(2*a^3 - 9*a^2*
b + 12*a*b^2 - 5*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(2*a^3 + 9*a
^2*b + 12*a*b^2 + 5*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 6*a^3 - 18
*a*b^2 + 2*(2*b^3*cos(d*x + c)^4 - 9*a^2*b - 3*b^3 - 2*(9*a^2*b + 7*b^3)*co
s(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*tan(c + d*x)**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.98, size = 366, normalized size = 2.44

$$\frac{\left(9\,a^2\,b+5\,b^3\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+\left(2\,a^3+12\,a\,b^2\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+\left(12\,a^2\,b+\frac{20\,b^3}{3}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+\left(6\,a^3+12\,a\,b^2\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+\left(6\,a^2\,b-\frac{22\,b^3}{3}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(6\,a^3+12\,a\,b^2\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\left(12\,a^2\,b+\frac{20\,b^3}{3}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(2\,a^3+12\,a\,b^2\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\left(9\,a^2\,b+5\,b^3\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\,\left(a^3+6\,a\,b^2\right)}{d}+\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,{\left(a-b\right)}^2\,\left(a-\frac{5\,b}{2}\right)}{d}+\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left(a+b\right)}^2\,\left(a+\frac{5\,b}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(9*a^2*b + 5*b^3) + tan(c/2 + (d*x)/2)^2*(12*a*b^2 + 2*
a^3) + tan(c/2 + (d*x)/2)^4*(12*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^8*(12*a
*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^6*(12*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2
)^9*(9*a^2*b + 5*b^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*b - (22*b^3)/3) + tan(c
/2 + (d*x)/2)^3*(12*a^2*b + (20*b^3)/3) + tan(c/2 + (d*x)/2)^7*(12*a^2*b +
(20*b^3)/3))/(d*(tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2
+ (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) - (log(ta
n(c/2 + (d*x)/2)^2 + 1)*(6*a*b^2 + a^3))/d + (log(tan(c/2 + (d*x)/2) + 1)*(
a - b)^2*(a - (5*b)/2))/d + (log(tan(c/2 + (d*x)/2) - 1)*(a + b)^2*(a + (5*
b)/2))/d
```

3.161 $\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=105

$$-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{(a-b)^3\log(1+\sin(c+dx))}{2d}-\frac{b(3a^2+b^2)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-$$

```
[Out] -1/2*(a+b)^3*ln(1-sin(d*x+c))/d-1/2*(a-b)^3*ln(1+sin(d*x+c))/d-b*(3*a^2+b^2
)*sin(d*x+c)/d-3/2*a*b^2*sin(d*x+c)^2/d-1/3*b^3*sin(d*x+c)^3/d
```

Rubi [A]

time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2800, 815, 647, 31}

$$-\frac{b(3a^2+b^2)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{(a-b)^3\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -1/2*((a + b)^3*Log[1 - Sin[c + d*x]])/d - ((a - b)^3*Log[1 + Sin[c + d*x]]
)/(2*d) - (b*(3*a^2 + b^2)*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d*x]^2)/(2*d)
 - (b^3*Sin[c + d*x]^3)/(3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
```

```
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^3}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^2-b^2-3ax-x^2+\frac{3a^2b^2+b^4+a(a^2+3b^2)x}{b^2-x^2}\right)\,dx, x, b\sin(c$$
$$= -\frac{b(3a^2+b^2)\sin(c+dx)}{d} - \frac{3ab^2\sin^2(c+dx)}{2d} - \frac{b^3\sin^3(c+dx)}{3d} + \text{Su}$$
$$= -\frac{b(3a^2+b^2)\sin(c+dx)}{d} - \frac{3ab^2\sin^2(c+dx)}{2d} - \frac{b^3\sin^3(c+dx)}{3d} + (a$$
$$= -\frac{(a+b)^3\log(1-\sin(c+dx))}{2d} - \frac{(a-b)^3\log(1+\sin(c+dx))}{2d} - b(3$$

Mathematica [A]
time = 0.13, size = 90, normalized size = 0.86

$$-\frac{3((a+b)^3\log(1-\sin(c+dx))+(a-b)^3\log(1+\sin(c+dx)))+6b(3a^2+b^2)\sin(c+dx)+9ab^2\sin^2(c+dx)+2b^3\sin^3(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
[Out] -1/6*(3*((a + b)^3*Log[1 - Sin[c + d*x]] + (a - b)^3*Log[1 + Sin[c + d*x]])
 + 6*b*(3*a^2 + b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 + 2*b^3*Sin[c +
d*x]^3)/d
```

Maple [A]
time = 0.19, size = 110, normalized size = 1.05

method	result
derivativedivides	$\frac{-a^3\ln(\cos(dx+c))+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3ab^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^3\left(-\frac{\left(\sin\right.}{}\right.}{d}$
default	$\frac{-a^3\ln(\cos(dx+c))+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3ab^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^3\left(-\frac{\left(\sin\right.}{}\right.}{d}$
risch	$\frac{3ib\,e^{i(dx+c)}a^2}{2d}+\frac{6ia\,b^2c}{d}+\frac{2ia^3c}{d}+\frac{5ib^3e^{i(dx+c)}}{8d}+3ia\,b^2x-\frac{5ib^3e^{-i(dx+c)}}{8d}-\frac{3ib\,e^{-i(dx+c)}a^2}{2d}+ia^3x-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*ln(cos(d*x+c))+3*a^2*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*
a*b^2*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+b^3*(-1/3*sin(d*x+c)^3-sin(d*x+c)+
ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 113, normalized size = 1.08

$$-\frac{2 b^3 \sin (dx+c)^3 + 9 a b^2 \sin (dx+c)^2 + 3 (a^3 - 3 a^2 b + 3 a b^2 - b^3) \log (\sin (dx+c)+1) + 3 (a^3 + 3 a^2 b + 3 a b^2 + b^3) \log (\sin (dx+c)-1) + 6 (3 a^2 b + b^3) \sin (dx+c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 3*(a^3 - 3*a^2*b + 3*
a*b^2 - b^3)*log(sin(d*x + c) + 1) + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(
sin(d*x + c) - 1) + 6*(3*a^2*b + b^3)*sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 116, normalized size = 1.10

$$\frac{9 a b^2 \cos (dx+c)^2 - 3 (a^3 - 3 a^2 b + 3 a b^2 - b^3) \log (\sin (dx+c)+1) - 3 (a^3 + 3 a^2 b + 3 a b^2 + b^3) \log (-\sin (dx+c)+1) + 2 (b^3 \cos (dx+c)^2 - 9 a^2 b - 4 b^3) \sin (dx+c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(9*a*b^2*cos(d*x + c)^2 - 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*log(sin(d*x
 + c) + 1) - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(-sin(d*x + c) + 1) + 2*(
b^3*cos(d*x + c)^2 - 9*a^2*b - 4*b^3)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sin (c + dx))^3 \tan (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*tan(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 55225 vs. $2(97) = 194$.
time = 31.93, size = 55225, normalized size = 525.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
[Out] -1/12*(18*a^2*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1
/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)
)*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 6*b^3*log(2*(tan(1/2*d*
x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*
c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 +
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1
/2*c)^6*tan(c)^2 - 18*a^2*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1
/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 6*b^3*log(2
*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x
)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*
d*x)^6*tan(1/2*c)^6*tan(c)^2 + 6*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x
)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan
(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 18*a*b^2*log(
4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*
x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan
(1/2*c)^6*tan(c)^2 - 9*a*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^
2 + 18*a^2*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*
c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*t
an(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1
/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*
x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(
1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 - 18
*a^2*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2
*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x
)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
- 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x
)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
 + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6*a^3*lo
g(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(
```

```
d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*t
an(1/2*c)^6 + 18*a*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*ta
n(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 + 54*a^2*b*log(2*(tan(1/2*d*x)^4*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan
(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*
d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*ta
n(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*ta
n(c)^2 + 18*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1
/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1)
)*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 54*a^2*b*log(2*(tan(1/2
*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*ta
n(1/2*c)^4*tan(c)^2 - 18*b^3*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 18*a^3*log
(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*...
```

Mupad [B]
time = 6.75, size = 226, normalized size = 2.15

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(a^3+3\,a\,b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^2\,b+2\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(6\,a^2\,b+2\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(12\,a^2\,b+\frac{20\,b^3}{3}\right)+6\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(a-b\right)^3}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(a+b\right)^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(3*a*b^2 + a^3))/d - (tan(c/2 + (d*x)/2)*(6*
a^2*b + 2*b^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*b + 2*b^3) + tan(c/2 + (d*x)/2
)^3*(12*a^2*b + (20*b^3)/3) + 6*a*b^2*tan(c/2 + (d*x)/2)^2 + 6*a*b^2*tan(c/
2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c
/2 + (d*x)/2)^6 + 1)) - (log(tan(c/2 + (d*x)/2) + 1)*(a - b)^3)/d - (log(ta
n(c/2 + (d*x)/2) - 1)*(a + b)^3)/d
```

3.162 $\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=67

$$\frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

```
[Out] a^3*ln(sin(d*x+c))/d+3*a^2*b*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3*
sin(d*x+c)^3/d
```

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2800, 45}

$$\frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^2*b*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*x
]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^3}{x}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(3a^2+\frac{a^3}{x}+3ax+x^2\right)\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(}{3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 1.00

$$\frac{a^3 \log(\sin(c+dx))}{d}+\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^2*b*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*x
]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.12, size = 56, normalized size = 0.84

method	result
derivativedivides	$\frac{\frac{b^3\left(\sin^3(dx+c)\right)}{3}+\frac{3a\,b^2\left(\sin^2(dx+c)\right)}{2}+3a^2b\sin(dx+c)+a^3\ln(\sin(dx+c))}{d}$
default	$\frac{\frac{b^3\left(\sin^3(dx+c)\right)}{3}+\frac{3a\,b^2\left(\sin^2(dx+c)\right)}{2}+3a^2b\sin(dx+c)+a^3\ln(\sin(dx+c))}{d}$
risch	$-ia^3x-\frac{3a\,b^2\mathrm{e}^{2i(dx+c)}}{8d}-\frac{3a\,b^2\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{2ia^3c}{d}+\frac{a^3\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}+\frac{3a^2b\sin(dx+c)}{d}+\frac{b^3\sin(dx+c)}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b^3*sin(d*x+c)^3+3/2*a*b^2*sin(d*x+c)^2+3*a^2*b*sin(d*x+c)+a^3*ln(
sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.85

$$\frac{2\,b^3\sin\left(dx+c\right)^3+9\,ab^2\sin\left(dx+c\right)^2+6\,a^3\log\left(\sin\left(dx+c\right)\right)+18\,a^2b\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*a^3*log(sin(d*x + c)
) + 18*a^2*b*sin(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 66, normalized size = 0.99

$$-\frac{9\,ab^2\cos\left(dx+c\right)^2-6\,a^3\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+2\,\left(b^3\cos\left(dx+c\right)^2-9\,a^2b-b^3\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(9*a*b^2*cos(d*x + c)^2 - 6*a^3*log(1/2*sin(d*x + c)) + 2*(b^3*cos(d*x
 + c)^2 - 9*a^2*b - b^3)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^3 \cot(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x), x)
```

Giac [A]
time = 6.38, size = 58, normalized size = 0.87

$$\frac{2\,b^3\sin(dx+c)^3 + 9\,ab^2\sin(dx+c)^2 + 6\,a^3\log(|\sin(dx+c)|) + 18\,a^2b\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*a^3*log(abs(sin(d*x
+ c))) + 18*a^2*b*sin(d*x + c))/d
```

Mupad [B]
time = 6.68, size = 118, normalized size = 1.76

$$\frac{b^3\sin(c+d\,x)}{3\,d} - \frac{a^3\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d} + \frac{a^3\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{3\,a\,b^2\cos(c+d\,x)^2}{2\,d} - \frac{b^3\cos(c+d\,x)^2\sin(c+d\,x)}{3\,d} + \frac{3\,a^2\,b\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (b^3*sin(c + d*x))/(3*d) - (a^3*log(1/cos(c/2 + (d*x)/2)^2))/d + (a^3*log(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (3*a*b^2*cos(c + d*x)^2)/(2*d) -
 (b^3*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (3*a^2*b*sin(c + d*x))/d
```

3.163 $\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=116

$$-\frac{3a^2b\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{a(a^2-3b^2)\log(\sin(c+dx))}{d}-\frac{b(3a^2-b^2)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}$$

[Out] -3*a^2*b*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d-a*(a^2-3*b^2)*ln(sin(d*x+c))/d -b*(3*a^2-b^2)*sin(d*x+c)/d-3/2*a*b^2*sin(d*x+c)^2/d-1/3*b^3*sin(d*x+c)^3/d

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 908}

$$-\frac{a^3\csc^2(c+dx)}{2d}-\frac{b(3a^2-b^2)\sin(c+dx)}{d}-\frac{a(a^2-3b^2)\log(\sin(c+dx))}{d}-\frac{3a^2b\csc(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]

[Out] (-3*a^2*b*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (a*(a^2 - 3*b^2)*Log[Sin[c + d*x]])/d - (b*(3*a^2 - b^2)*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d*x]^2)/(2*d) - (b^3*Sin[c + d*x]^3)/(3*d)

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3(b^2-x^2)}{x^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^2\left(1-\frac{b^2}{3a^2}\right)+\frac{a^3b^2}{x^3}+\frac{3a^2b^2}{x^2}+\frac{-a^3+3ab^2}{x}-3ax-x^2\right)\,dx\right.}{d}$$
$$= -\frac{3a^2b\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{a(a^2-3b^2)\log(\sin(c+dx))}{d}$$

Mathematica [A]
time = 0.20, size = 97, normalized size = 0.84

$$-\frac{18a^2b\csc(c+dx)+3a^3\csc^2(c+dx)+6a(a^2-3b^2)\log(\sin(c+dx))-6b(-3a^2+b^2)\sin(c+dx)+9ab^2\sin^2(c+dx)+2b^3\sin^3(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*(18*a^2*b*Csc[c + d*x] + 3*a^3*Csc[c + d*x]^2 + 6*a*(a^2 - 3*b^2)*Log[
Sin[c + d*x]] - 6*b*(-3*a^2 + b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 +
2*b^3*Sin[c + d*x]^3)/d
```

Maple [A]
time = 0.26, size = 118, normalized size = 1.02

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3a^2b\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+3a\,b^2\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx\right.}{d}$
default	$\frac{a^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3a^2b\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+3a\,b^2\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx\right.}{d}$
risch	$\frac{3ib\,e^{i(dx+c)}a^2}{2d}-\frac{2ia^2\left(ia\,e^{2i(dx+c)}+3b\,e^{3i(dx+c)}-3b\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{3ib^3e^{i(dx+c)}}{8d}+\frac{3a\,b^2e^{2i(dx+c)}}{8d}+\frac{3ib^3e^{-i(dx+c)}}{8d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+3*a^2*b*(-1/sin(d*x+c)*cos(d*x+
c)^4-(2+cos(d*x+c)^2)*sin(d*x+c))+3*a*b^2*(1/2*cos(d*x+c)^2+ln(sin(d*x+c)))
+1/3*b^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.34, size = 98, normalized size = 0.84

$$-\frac{2\,b^3\sin(dx+c)^3+9\,ab^2\sin(dx+c)^2+6\,(a^3-3\,ab^2)\log(\sin(dx+c))+6\,(3\,a^2b-b^3)\sin(dx+c)+\frac{3\,(6\,a^2b\sin(dx+c)+a^3)}{\sin(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*(a^3 - 3*a*b^2)*log
(sin(d*x + c)) + 6*(3*a^2*b - b^3)*sin(d*x + c) + 3*(6*a^2*b*sin(d*x + c) +
 a^3)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.39, size = 153, normalized size = 1.32

$$\frac{18\,ab^2\cos\left(dx+c\right)^4-27\,ab^2\cos\left(dx+c\right)^2+6\,a^3+9\,ab^2+12\,\left(a^3-3\,ab^2-\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\,\left(b^3\cos\left(dx+c\right)^4+18\,a^2b-2\,b^3-\left(9\,a^2b-b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a*b^2*cos(d*x + c)^4 - 27*a*b^2*cos(d*x + c)^2 + 6*a^3 + 9*a*b^2 +
 12*(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2)*log(1/2*sin(d*x + c))
+ 4*(b^3*cos(d*x + c)^4 + 18*a^2*b - 2*b^3 - (9*a^2*b - b^3)*cos(d*x + c)^2
)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(c+dx\right)\right)^3\cot^3\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**3, x)
```

Giac [A]
time = 4.23, size = 131, normalized size = 1.13

$$-\frac{2\,b^3\sin\left(dx+c\right)^3+9\,ab^2\sin\left(dx+c\right)^2+18\,a^2b\sin\left(dx+c\right)-6\,b^3\sin\left(dx+c\right)+6\,\left(a^3-3\,ab^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{3\,\left(3\,a^3\sin(dx+c)^2-9\,ab^2\sin(dx+c)^2-6\,a^2b\sin(dx+c)-a^3\right)}{\sin(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 18*a^2*b*sin(d*x + c)
 - 6*b^3*sin(d*x + c) + 6*(a^3 - 3*a*b^2)*log(abs(sin(d*x + c))) - 3*(3*a^3
*sin(d*x + c)^2 - 9*a*b^2*sin(d*x + c)^2 - 6*a^2*b*sin(d*x + c) - a^3)/sin(
d*x + c)^2)/d
```

Mupad [B]
time = 6.95, size = 312, normalized size = 2.69

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,a\,b^2-a^3\right)}{d}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\,\left(3\,a\,b^2-a^3\right)}{d}-\frac{\frac{3\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{3\,a^3}{2}+24\,a\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{a^3}{2}+24\,a\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(30\,a^2\,b-8\,b^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(42\,a^2\,b-8\,b^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(66\,a^2\,b-\frac{16\,b^3}{3}\right)+\frac{a^3}{2}+6\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+12\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+12\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}-\frac{a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}-\frac{3\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*(3*a*b^2 - a^3))/d - (log(tan(c/2 + (d*x)/2)^2 + 1
)*(3*a*b^2 - a^3))/d - ((3*a^3*tan(c/2 + (d*x)/2)^2)/2 + tan(c/2 + (d*x)/2)
^4*(24*a*b^2 + (3*a^3)/2) + tan(c/2 + (d*x)/2)^6*(24*a*b^2 + a^3/2) + tan(c
/2 + (d*x)/2)^7*(30*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^3*(42*a^2*b - 8*b^3
) + tan(c/2 + (d*x)/2)^5*(66*a^2*b - (16*b^3)/3) + a^3/2 + 6*a^2*b*tan(c/2
+ (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c
/2 + (d*x)/2)^6 + 4*tan(c/2 + (d*x)/2)^8)) - (a^3*tan(c/2 + (d*x)/2)^2)/(8*
d) - (3*a^2*b*tan(c/2 + (d*x)/2))/(2*d)
```

3.164 $\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=165

$$\frac{b(6a^2-b^2)\csc(c+dx)}{d}+\frac{a(2a^2-3b^2)\csc^2(c+dx)}{2d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{a^3\csc^4(c+dx)}{4d}+\frac{a(a^2-6b^2)\log(\sin(c+dx))}{d}$$

```
[Out] b*(6*a^2-b^2)*csc(d*x+c)/d+1/2*a*(2*a^2-3*b^2)*csc(d*x+c)^2/d-a^2*b*csc(d*x
+c)^3/d-1/4*a^3*csc(d*x+c)^4/d+a*(a^2-6*b^2)*ln(sin(d*x+c))/d+b*(3*a^2-2*b^
2)*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 962}

$$-\frac{a^3\csc^4(c+dx)}{4d}+\frac{b(3a^2-2b^2)\sin(c+dx)}{d}+\frac{a(2a^2-3b^2)\csc^2(c+dx)}{2d}+\frac{b(6a^2-b^2)\csc(c+dx)}{d}+\frac{a(a^2-6b^2)\log(\sin(c+dx))}{d}-\frac{a^2b\csc^3(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(6*a^2 - b^2)*Csc[c + d*x])/d + (a*(2*a^2 - 3*b^2)*Csc[c + d*x]^2)/(2*d)
 - (a^2*b*Csc[c + d*x]^3)/d - (a^3*Csc[c + d*x]^4)/(4*d) + (a*(a^2 - 6*b^2)
*Log[Sin[c + d*x]])/d + (b*(3*a^2 - 2*b^2)*Sin[c + d*x])/d + (3*a*b^2*Sin[c
 + d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3(b^2-x^2)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)+\frac{a^3b^4}{x^5}+\frac{3a^2b^4}{x^4}+\frac{-2a^3b^2+3ab^4}{x^3}+\frac{-6a^2b^2+b^4}{x^2}+\cdots\right.\right.}{d}$$
$$= \frac{b(6a^2-b^2)\csc(c+dx)}{d}+\frac{a(2a^2-3b^2)\csc^2(c+dx)}{2d}-\frac{a^2b\csc^3(c+\cdots}{d}$$

Mathematica [A]
time = 0.70, size = 144, normalized size = 0.87

$$\frac{-12b(-6a^2+b^2)\csc(c+dx)+6a(2a^2-3b^2)\csc^2(c+dx)-12a^2b\csc^3(c+dx)-3a^3\csc^4(c+dx)+2(6a(a^2-6b^2)\log(\sin(c+dx))+6b(3a^2-2b^2)\sin(c+dx)+9ab^2\sin^2(c+dx)+2b^3\sin^3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-12*b*(-6*a^2 + b^2)*Csc[c + d*x] + 6*a*(2*a^2 - 3*b^2)*Csc[c + d*x]^2 - 1
2*a^2*b*Csc[c + d*x]^3 - 3*a^3*Csc[c + d*x]^4 + 2*(6*a*(a^2 - 6*b^2)*Log[Si
n[c + d*x]] + 6*b*(3*a^2 - 2*b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 + 2
*b^3*Sin[c + d*x]^3))/(12*d)
```

Maple [A]
time = 0.25, size = 212, normalized size = 1.28

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3a^2b\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\cdots\right.\right.}{\cdots}$
default	$\frac{a^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3a^2b\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\cdots\right.\right.}{\cdots}$
risch	$-\frac{7ib^3e^{-i(dx+c)}}{8d}+\frac{ib^3e^{3i(dx+c)}}{24d}-\frac{3ib\,e^{i(dx+c)}a^2}{2d}-\frac{3a\,b^2e^{2i(dx+c)}}{8d}-\frac{ib^3e^{-3i(dx+c)}}{24d}+\frac{7ib^3e^{i(dx+c)}}{8d}+6ia\,b^2x\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+3*a^2*b*(-1/3/
sin(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c))+3*a*b^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*
x+c)^4-cos(d*x+c)^2-2*ln(sin(d*x+c)))+b^3*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.45, size = 142, normalized size = 0.86

$$\frac{4\,b^3\sin\left(dx+c\right)^3+18\,ab^2\sin\left(dx+c\right)^2+12\left(a^3-6\,ab^2\right)\log\left(\sin\left(dx+c\right)\right)+12\left(3\,a^2b-2\,b^3\right)\sin\left(dx+c\right)-\frac{3\left(4\,a^2b\sin(dx+c)-4\left(6\,a^2b-b^3\right)\sin(dx+c)^3+a^3-2\left(2\,a^3-3\,ab^2\right)\sin(dx+c)^2\right)}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 12*(a^3 - 6*a*b^2)*l
og(sin(d*x + c)) + 12*(3*a^2*b - 2*b^3)*sin(d*x + c) - 3*(4*a^2*b*sin(d*x +
 c) - 4*(6*a^2*b - b^3)*sin(d*x + c)^3 + a^3 - 2*(2*a^3 - 3*a*b^2)*sin(d*x
+ c)^2)/sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.37, size = 225, normalized size = 1.36

$$-\frac{18\,ab^2\cos\left(dx+c\right)^6-45\,ab^2\cos\left(dx+c\right)^4-9\,a^3+9\,ab^2+6\left(2\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^2-12\left(\left(a^3-6\,ab^2\right)\cos\left(dx+c\right)^4+a^3-6\,ab^2-2\left(a^3-6\,ab^2\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+4\left(b^3\cos\left(dx+c\right)^6-3\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^4-24\,a^2b+8\,b^3+12\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(18*a*b^2*cos(d*x + c)^6 - 45*a*b^2*cos(d*x + c)^4 - 9*a^3 + 9*a*b^2
+ 6*(2*a^3 + 3*a*b^2)*cos(d*x + c)^2 - 12*((a^3 - 6*a*b^2)*cos(d*x + c)^4 +
 a^3 - 6*a*b^2 - 2*(a^3 - 6*a*b^2)*cos(d*x + c)^2)*log(1/2*sin(d*x + c)) +
4*(b^3*cos(d*x + c)^6 - 3*(3*a^2*b - b^3)*cos(d*x + c)^4 - 24*a^2*b + 8*b^3
 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d
*cos(d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(c+dx\right)\right)^3\cot^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**5, x)
```

Giac [A]
time = 3.30, size = 185, normalized size = 1.12

$$\frac{4\,b^3\sin\left(dx+c\right)^3+18\,ab^2\sin\left(dx+c\right)^2+36\,a^2b\sin\left(dx+c\right)-24\,b^3\sin\left(dx+c\right)+12\left(a^3-6\,ab^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{25\,a^3\sin(dx+c)^4-150\,ab^2\sin(dx+c)^4-72\,a^2b\sin(dx+c)^3+12\,b^3\sin(dx+c)^3-12\,a^3\sin(dx+c)^2+18\,ab^2\sin(dx+c)^2+12\,a^2b\sin(dx+c)+3\,a^3}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 36*a^2*b*sin(d*x + c
) - 24*b^3*sin(d*x + c) + 12*(a^3 - 6*a*b^2)*log(abs(sin(d*x + c))) - (25*a
^3*sin(d*x + c)^4 - 150*a*b^2*sin(d*x + c)^4 - 72*a^2*b*sin(d*x + c)^3 + 12
*b^3*sin(d*x + c)^3 - 12*a^3*sin(d*x + c)^2 + 18*a*b^2*sin(d*x + c)^2 + 12*
a^2*b*sin(d*x + c) + 3*a^3)/sin(d*x + c)^4)/d
```

Mupad [B]
time = 6.97, size = 424, normalized size = 2.57

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(6\,a\,b^2-a^3\right)}{d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{3\,a\,b^2}{8}-\frac{3\,a^3}{16}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(6\,a\,b^2-a^3\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(3\,a^3+90\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(18\,a\,b^2-\frac{33\,a^3}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(6\,a\,b^2-\frac{9\,a^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{35\,a^3}{4}+78\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(36\,a^2\,b-8\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\left(138\,a^2\,b-72\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(216\,a^2\,b-88\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(316\,a^2\,b-\frac{328\,b^3}{3}\right)-\frac{a^3}{4}-2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{21\,a^2\,b}{8}-\frac{b^3}{2}\right)}{d}-\frac{a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(6*a*b^2 - a^3))/d - (a^3*tan(c/2 + (d*x)/2)
^4)/(64*d) - (tan(c/2 + (d*x)/2)^2*((3*a*b^2)/8 - (3*a^3)/16))/d - (log(tan
(c/2 + (d*x)/2))*(6*a*b^2 - a^3))/d + (tan(c/2 + (d*x)/2)^8*(90*a*b^2 + 3*a
^3) - tan(c/2 + (d*x)/2)^4*(18*a*b^2 - (33*a^3)/4) - tan(c/2 + (d*x)/2)^2*(
6*a*b^2 - (9*a^3)/4) + tan(c/2 + (d*x)/2)^6*(78*a*b^2 + (35*a^3)/4) + tan(c
/2 + (d*x)/2)^3*(36*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^9*(138*a^2*b - 72*b
^3) + tan(c/2 + (d*x)/2)^5*(216*a^2*b - 88*b^3) + tan(c/2 + (d*x)/2)^7*(316
*a^2*b - (328*b^3)/3) - a^3/4 - 2*a^2*b*tan(c/2 + (d*x)/2))/(d*(16*tan(c/2
+ (d*x)/2)^4 + 48*tan(c/2 + (d*x)/2)^6 + 48*tan(c/2 + (d*x)/2)^8 + 16*tan(c
/2 + (d*x)/2)^10)) + (tan(c/2 + (d*x)/2)*((21*a^2*b)/8 - b^3/2))/d - (a^2*b
*tan(c/2 + (d*x)/2)^3)/(8*d)
```

3.165 $\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=220

$$a^3x+\frac{15}{2}ab^2x-\frac{3a^2b\cos(c+dx)}{d}-\frac{3b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}-\frac{6a^2b\sec(c+dx)}{d}-\frac{3b^3\sec(c+dx)}{d}+\frac{a^2b}{}$$

```
[Out] a^3*x+15/2*a*b^2*x-3*a^2*b*cos(d*x+c)/d-3*b^3*cos(d*x+c)/d+1/3*b^3*cos(d*x+
c)^3/d-6*a^2*b*sec(d*x+c)/d-3*b^3*sec(d*x+c)/d+a^2*b*sec(d*x+c)^3/d+1/3*b^3
*sec(d*x+c)^3/d-a^3*tan(d*x+c)/d-15/2*a*b^2*tan(d*x+c)/d+1/3*a^3*tan(d*x+c)
^3/d+5/2*a*b^2*tan(d*x+c)^3/d-3/2*a*b^2*sin(d*x+c)^2*tan(d*x+c)^3/d
```

Rubi [A]

time = 0.17, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2801, 3554, 8, 2670, 276, 2671, 294, 308, 209}

$$\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x-\frac{3a^2b\cos(c+dx)}{d}+\frac{a^2b\sec^3(c+dx)}{d}-\frac{6a^2b\sec(c+dx)}{d}+\frac{5ab^2\tan^3(c+dx)}{2d}-\frac{15ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan^3(c+dx)}{2d}+\frac{15}{2}ab^2x+\frac{b^3\cos^3(c+dx)}{3d}-\frac{3b^3\cos(c+dx)}{d}+\frac{b^3\sec^3(c+dx)}{3d}-\frac{3b^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] a^3*x + (15*a*b^2*x)/2 - (3*a^2*b*Cos[c + d*x])/d - (3*b^3*Cos[c + d*x])/d
+ (b^3*Cos[c + d*x]^3)/(3*d) - (6*a^2*b*Sec[c + d*x])/d - (3*b^3*Sec[c + d*
x])/d + (a^2*b*Sec[c + d*x]^3)/d + (b^3*Sec[c + d*x]^3)/(3*d) - (a^3*Tan[c
+ d*x])/d - (15*a*b^2*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d) + (5
*a*b^2*Tan[c + d*x]^3)/(2*d) - (3*a*b^2*Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx = \int \left(a^3\tan^4(c+dx)+3a^2b\sin(c+dx)\tan^4(c+dx)+3ab^2\sin^2(c+\right.$$

$$= a^3\int \tan^4(c+dx)\,dx + \left(3a^2b\right)\int \sin(c+dx)\tan^4(c+dx)\,dx + (3$$

$$= \frac{a^3\tan^3(c+dx)}{3d} - a^3\int \tan^2(c+dx)\,dx - \frac{\left(3a^2b\right)\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\right.}{d}$$

$$= -\frac{a^3\tan(c+dx)}{d} + \frac{a^3\tan^3(c+dx)}{3d} - \frac{3ab^2\sin^2(c+dx)\tan^3(c+d}{2d}$$

$$= a^3x - \frac{3a^2b\cos(c+dx)}{d} - \frac{3b^3\cos(c+dx)}{d} + \frac{b^3\cos^3(c+dx)}{3d} - 6a^2$$

$$= a^3x - \frac{3a^2b\cos(c+dx)}{d} - \frac{3b^3\cos(c+dx)}{d} + \frac{b^3\cos^3(c+dx)}{3d} - 6a^2$$

$$= a^3x + \frac{15}{2}ab^2x - \frac{3a^2b\cos(c+dx)}{d} - \frac{3b^3\cos(c+dx)}{d} + \frac{b^3\cos^3(c+}{3d}$$

Mathematica [A]

time = 0.46, size = 226, normalized size = 1.03

$$\frac{\sec^3(c+dx)\left(-300a^2b-210b^3+36a\left(2a^2+15b^2\right)(c+dx)\cos(c+dx)-3\left(144a^2b+91b^3\right)\cos(2(c+dx))+24a^3c\cos(3(c+dx))+180ab^2c\cos(3(c+dx))+24a^3dx\cos(3(c+dx))+180ab^2dx\cos(3(c+dx))-36a^2b\cos(4(c+dx))-30b^3\cos(4(c+dx))+b^3\cos(6(c+dx))-90ab^2\sin(c+dx)-32a^3\sin(3(c+dx))-195ab^2\sin(3(c+dx))-9ab^2\sin(5(c+dx))\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (Sec[c + d*x]^3*(-300*a^2*b - 210*b^3 + 36*a*(2*a^2 + 15*b^2)*(c + d*x)*Cos
[c + d*x] - 3*(144*a^2*b + 91*b^3)*Cos[2*(c + d*x)] + 24*a^3*c*Cos[3*(c + d
*x)] + 180*a*b^2*c*Cos[3*(c + d*x)] + 24*a^3*d*x*Cos[3*(c + d*x)] + 180*a*b
^2*d*x*Cos[3*(c + d*x)] - 36*a^2*b*Cos[4*(c + d*x)] - 30*b^3*Cos[4*(c + d*x
)] + b^3*Cos[6*(c + d*x)] - 90*a*b^2*Sin[c + d*x] - 32*a^3*Sin[3*(c + d*x)]
 - 195*a*b^2*Sin[3*(c + d*x)] - 9*a*b^2*Sin[5*(c + d*x)]))/(96*d)
```

Maple [A]

time = 0.24, size = 268, normalized size = 1.22

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+3a^2b\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right.}{}$

default	$\frac{a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+3a^2b\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{}$
risch	$a^3x+\frac{15a\,b^2x}{2}+\frac{b^3\mathrm{e}^{3i(dx+c)}}{24d}+\frac{3ia\,b^2\mathrm{e}^{2i(dx+c)}}{8d}-\frac{3b\,\mathrm{e}^{i(dx+c)}a^2}{2d}-\frac{11b^3\mathrm{e}^{i(dx+c)}}{8d}-\frac{3b\,\mathrm{e}^{-i(dx+c)}a^2}{2d}-\frac{11b^3\mathrm{e}^{-i(}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+3*a^2*b*(1/3*sin(d*x+c)^6/cos(
d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*
x+c))+3*a*b^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/
3*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)
+b^3*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5+s
in(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.64, size = 167, normalized size = 0.76

$$\frac{2\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^3+3\left(2\,\tan\left(dx+c\right)^3+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^2+1}-12\,\tan\left(dx+c\right)\right)ab^2+2\left(\cos\left(dx+c\right)^3-\frac{9\,\cos(dx+c)^2-1}{\cos(dx+c)^3}-9\,\cos\left(dx+c\right)\right)b^3-6\,a^2b\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3 + 3*(2*tan(d*x +
 c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x +
c))*a*b^2 + 2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*c
os(d*x + c))*b^3 - 6*a^2*b*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d
*x + c)))/d
```

Fricas [A]
time = 0.34, size = 157, normalized size = 0.71

$$\frac{2\,b^3\cos\left(dx+c\right)^6+3\left(2\,a^3+15\,ab^2\right)dx\cos\left(dx+c\right)^3-18\left(a^2b+b^3\right)\cos\left(dx+c\right)^4+6\,a^2b+2\,b^3-18\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)^2-\left(9\,ab^2\cos\left(dx+c\right)^4-2\,a^3-6\,ab^2+2\left(4\,a^3+21\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*cos(d*x + c)^6 + 3*(2*a^3 + 15*a*b^2)*d*x*cos(d*x + c)^3 - 18*(a
^2*b + b^3)*cos(d*x + c)^4 + 6*a^2*b + 2*b^3 - 18*(2*a^2*b + b^3)*cos(d*x +
 c)^2 - (9*a*b^2*cos(d*x + c)^4 - 2*a^3 - 6*a*b^2 + 2*(4*a^3 + 21*a*b^2)*co
s(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*tan(c + d*x)**4, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.20, size = 297, normalized size = 1.35

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+15\,b^2\right)}{2\,a^3+15\,a\,b^2}\right)\,\left(2\,a^2+15\,b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{2\,a^3}{3}+5\,a\,b^2\right)-16\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^3+15\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(\frac{2\,a^3}{3}+5\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(2\,a^3+15\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(12\,a^3+42\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(12\,a^3+42\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(48\,a^2\,b+32\,b^3\right)-\frac{32\,b^3}{3}+32\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(2*a^2 + 15*b^2))/(15*a*b^2 + 2*a^3))*(2*a^2
+ 15*b^2))/d - (tan(c/2 + (d*x)/2)^3*(5*a*b^2 + (2*a^3)/3) - 16*a^2*b - tan
(c/2 + (d*x)/2)*(15*a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^9*(5*a*b^2 + (2*a^3
)/3) - tan(c/2 + (d*x)/2)^11*(15*a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^5*(42*
a*b^2 + 12*a^3) + tan(c/2 + (d*x)/2)^7*(42*a*b^2 + 12*a^3) + tan(c/2 + (d*x
)/2)^4*(48*a^2*b + 32*b^3) - (32*b^3)/3 + 32*a^2*b*tan(c/2 + (d*x)/2)^6)/(d
*(3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^12 -
 1))
```

3.166 $\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=146

$$-a^3x-\frac{9}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}+\frac{a^3\tan}{}$$

```
[Out] -a^3*x-9/2*a*b^2*x+3*a^2*b*cos(d*x+c)/d+2*b^3*cos(d*x+c)/d-1/3*b^3*cos(d*x+
c)^3/d+3*a^2*b*sec(d*x+c)/d+b^3*sec(d*x+c)/d+a^3*tan(d*x+c)/d+9/2*a*b^2*tan
(d*x+c)/d-3/2*a*b^2*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]

time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2801, 3554, 8, 2670, 14, 2671, 294, 327, 209, 276}

$$\frac{a^3\tan(c+dx)}{d}+a^3(-x)+\frac{3a^2b\cos(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{9}{2}ab^2x-\frac{b^3\cos^3(c+dx)}{3d}+\frac{2b^3\cos(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -(a^3*x) - (9*a*b^2*x)/2 + (3*a^2*b*Cos[c + d*x])/d + (2*b^3*Cos[c + d*x])/
d - (b^3*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Sec[c + d*x])/d + (b^3*Sec[c + d*
x])/d + (a^3*Tan[c + d*x])/d + (9*a*b^2*Tan[c + d*x])/(2*d) - (3*a*b^2*Sin[
c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \int \left(a^3\tan^2(c+dx)+3a^2b\sin(c+dx)\tan^2(c+dx)+3ab^2\sin^2(c+\right.$$

$$= a^3\int \tan^2(c+dx)\,dx+\left(3a^2b\right)\int \sin(c+dx)\tan^2(c+dx)\,dx+\left(3a\right.$$

$$= \frac{a^3\tan(c+dx)}{d}-a^3\int 1\,dx-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx\right.}{d}$$

$$= -a^3x+\frac{a^3\tan(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{\left(3a^2b\right)\text{Su}}{}$$

$$= -a^3x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}+\frac{3a^2}{}$$

$$= -a^3x-\frac{9}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+d}{3d}$$

Mathematica [A]

time = 0.52, size = 113, normalized size = 0.77

$$\frac{b\sec(c+dx)\left(108a^2+45b^2+4\left(9a^2+5b^2\right)\cos(2(c+dx))-b^2\cos(4(c+dx))+9ab\sin(3(c+dx))\right)+3a\left(-4\left(2a^2+9b^2\right)(c+dx)+\left(8a^2+27b^2\right)\tan(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (b*Sec[c + d*x]*(108*a^2 + 45*b^2 + 4*(9*a^2 + 5*b^2)*Cos[2*(c + d*x)] - b^
2*Cos[4*(c + d*x)] + 9*a*b*Sin[3*(c + d*x)]) + 3*a*(-4*(2*a^2 + 9*b^2)*(c +
 d*x) + (8*a^2 + 27*b^2)*Tan[c + d*x]))/(24*d)
```

Maple [A]

time = 0.27, size = 169, normalized size = 1.16

method	result
derivativedivides	$\frac{a^3(\tan(dx+c)-dx-c)+3a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a\,b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\text{c}\right.}{d}$
default	$\frac{a^3(\tan(dx+c)-dx-c)+3a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a\,b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\text{c}\right.}{d}$
risch	$-a^3x-\frac{9a\,b^2x}{2}-\frac{3ia\,b^2\text{e}^{2i(dx+c)}}{8d}+\frac{3b\,\text{e}^{i(dx+c)}a^2}{2d}+\frac{7b^3\text{e}^{i(dx+c)}}{8d}+\frac{3b\,\text{e}^{-i(dx+c)}a^2}{2d}+\frac{7b^3\text{e}^{-i(dx+c)}}{8d}+\frac{3ia\,b^2\text{e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(tan(d*x+c)-d*x-c)+3*a^2*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^
2)*cos(d*x+c))+3*a*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c
))*cos(d*x+c)-3/2*d*x-3/2*c)+b^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4
+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.59, size = 119, normalized size = 0.82

$$-\frac{6\,(dx+c-\tan(dx+c))a^3+9\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)ab^2+2\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\,\cos(dx+c)\right)b^3-18\,a^2b\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(d*x + c - tan(d*x + c))*a^3 + 9*(3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*a*b^2 + 2*(cos(d*x + c)^3 - 3/cos(d*x + c)
 - 6*cos(d*x + c))*b^3 - 18*a^2*b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.38, size = 116, normalized size = 0.79

$$-\frac{2\,b^3\cos(dx+c)^4+3\,(2\,a^3+9\,ab^2)dx\cos(dx+c)-18\,a^2b-6\,b^3-6\,(3\,a^2b+2\,b^3)\cos(dx+c)^2-3\,\left(3\,ab^2\cos(dx+c)^2+2\,a^3+6\,ab^2\right)\sin(dx+c)}{6\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*cos(d*x + c)^4 + 3*(2*a^3 + 9*a*b^2)*d*x*cos(d*x + c) - 18*a^2*
b - 6*b^3 - 6*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 - 3*(3*a*b^2*cos(d*x + c)^2
+ 2*a^3 + 6*a*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*tan(c + d*x)**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 9.15, size = 249, normalized size = 1.71

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^3+9\,a\,b^2\right)+12\,a^2\,b+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(2\,a^3+9\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(6\,a^3+15\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(6\,a^3+15\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(24\,a^2\,b+\frac{32\,b^3}{3}\right)+\frac{16\,b^3}{3}+12\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{a\,\mathrm{atan}\left(\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+9\,b^2\right)}{2\,a^3+9\,a\,b^2}\right)\,\left(2\,a^2+9\,b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(9*a*b^2 + 2*a^3) + 12*a^2*b + tan(c/2 + (d*x)/2)^7*(9*
a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^3*(15*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/
2)^5*(15*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^2*(24*a^2*b + (32*b^3)/3) + (1
6*b^3)/3 + 12*a^2*b*tan(c/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*ta
n(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1)) - (a*atan((a*tan(c/2 + (d*x
)/2)*(2*a^2 + 9*b^2))/(9*a*b^2 + 2*a^3))*(2*a^2 + 9*b^2))/d
```

3.167 $\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=102

$$-a^3x+\frac{3}{2}ab^2x-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos(c+$$

```
[Out] -a^3*x+3/2*a*b^2*x-3*a^2*b*arctanh(cos(d*x+c))/d+3*a^2*b*cos(d*x+c)/d-1/3*b
^3*cos(d*x+c)^3/d-a^3*cot(d*x+c)/d+3/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2801, 2715, 8, 2672, 327, 212, 3554, 2645, 30}

$$-\frac{a^3\cot(c+dx)}{d}+a^3(-x)+\frac{3a^2b\cos(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + (3*a^2*b*Cos
[c + d*x])/d - (b^3*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a*b^2
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx &= \int \left(3ab^2\cos^2(c+dx)+3a^2b\cos(c+dx)\cot(c+dx)+a^3\cot^2(c+\right. \\
&= a^3\int \cot^2(c+dx)\,dx+\left(3a^2b\right)\int \cos(c+dx)\cot(c+dx)\,dx+\left(3a\right. \\
&= -\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos(c+dx)\sin(c+dx)}{2d}-a^3\int 1\,dx+\frac{1}{2}(3 \\
&= -a^3x+\frac{3}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}-\frac{a^3\cot(c+}{d} \\
&= -a^3x+\frac{3}{2}ab^2x-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}-
\end{aligned}
$$

Mathematica [A]
time = 0.87, size = 143, normalized size = 1.40

$$
\frac{(36a^2b-3b^3)\cos(c+dx)-b^3\cos(3(c+dx))-6a^3\cot\left(\frac{1}{2}(c+dx)\right)+9ab^2\sin(2(c+dx))+6a\left(-2a^2c+3b^2c-2a^2dx+3b^2dx-6ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2\tan\left(\frac{1}{2}(c+dx)\right)\right)}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((36*a^2*b - 3*b^3)*Cos[c + d*x] - b^3*Cos[3*(c + d*x)] - 6*a^3*Cot[(c + d*
x)/2] + 9*a*b^2*Sin[2*(c + d*x)] + 6*a*(-2*a^2*c + 3*b^2*c - 2*a^2*d*x + 3*
b^2*d*x - 6*a*b*Log[Cos[(c + d*x)/2]] + 6*a*b*Log[Sin[(c + d*x)/2]] + a^2*T
an[(c + d*x)/2]))/(12*d)
```

Maple [A]
time = 0.15, size = 96, normalized size = 0.94

method	result
derivativedivides	$\frac{a^3(-\cot(dx+c)-dx-c)+3a^2b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a\,b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{b^3\left(\cos^3(d\right.}{3}}{d}$
default	$\frac{a^3(-\cot(dx+c)-dx-c)+3a^2b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a\,b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{b^3\left(\cos^3(d\right.}{3}}{d}$
risch	$-a^3x+\frac{3a\,b^2x}{2}-\frac{3ia\,b^2\mathrm{e}^{2i(dx+c)}}{8d}+\frac{3b\,\mathrm{e}^{i(dx+c)}a^2}{2d}-\frac{b^3\mathrm{e}^{i(dx+c)}}{8d}+\frac{3b\,\mathrm{e}^{-i(dx+c)}a^2}{2d}-\frac{b^3\mathrm{e}^{-i(dx+c)}}{8d}+\frac{3ia\,b^2\mathrm{e}^{-}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-cot(d*x+c)-d*x-c)+3*a^2*b*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))
+3*a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-1/3*b^3*cos(d*x+c)^3)
```

Maxima [A]
time = 0.62, size = 95, normalized size = 0.93

$$-\frac{4\,b^{3}\cos\left(dx+c\right)^{3}+12\,\left(dx+c+\frac{1}{\tan\left(dx+c\right)}\right)a^{3}-9\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)ab^{2}-18\,a^{2}b\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] -1/12*(4*b^3*cos(d*x + c)^3 + 12*(d*x + c + 1/tan(d*x + c))*a^3 - 9*(2*d*x
+ 2*c + sin(2*d*x + 2*c))*a*b^2 - 18*a^2*b*(2*cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.40, size = 143, normalized size = 1.40

$$-\frac{9\,ab^{2}\cos\left(dx+c\right)^{3}+9\,a^{2}b\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a^{2}b\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(2\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)+\left(2\,b^{3}\cos\left(dx+c\right)^{3}-18\,a^{2}b\cos\left(dx+c\right)+3\left(2\,a^{3}-3\,ab^{2}\right)dx\right)\sin\left(dx+c\right)}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] -1/6*(9*a*b^2*cos(d*x + c)^3 + 9*a^2*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 9*a^2*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(2*a^3 - 3*a*b
^2)*cos(d*x + c) + (2*b^3*cos(d*x + c)^3 - 18*a^2*b*cos(d*x + c) + 3*(2*a^3
 - 3*a*b^2)*d*x)*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^{3}\cot^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(96) = 192.
time = 12.66, size = 199, normalized size = 1.95

$$\frac{18\,a^{2}b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\left(2\,a^{3}-3\,ab^{2}\right)\left(dx+c\right)-\frac{3\left(6\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(9\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-18\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+6\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-36\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-18\,a^{2}b+2\,b^{3}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(18*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/2*d*x + 1/2*c) -
 3*(2*a^3 - 3*a*b^2)*(d*x + c) - 3*(6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan
(1/2*d*x + 1/2*c) - 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*
x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)
^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 2*b^3)/(tan(1/2*d*x + 1/2*c)
^2 + 1)^3)/d
```

Mupad [B]
time = 6.86, size = 289, normalized size = 2.83

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)\,\left(\frac{a\,b^2\,3\mathrm{i}}{2}-a^3\,1\mathrm{i}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,a^2\,b-\frac{4\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(a^3+6\,a\,b^2\right)-3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(6\,a\,b^2-3\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(12\,a^2\,b-4\,b^3\right)-a^3+24\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{3\,a^2\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)\,\left(2\,a^2-3\,b^2\right)\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2))/(2*d) - (log(tan(c/2 + (d*x)/2) - 1i)*((a*b^2*3i)/
2 - a^3*1i))/d + (tan(c/2 + (d*x)/2)*(12*a^2*b - (4*b^3)/3) - tan(c/2 + (d*
x)/2)^6*(6*a*b^2 + a^3) - 3*a^3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^2
*(6*a*b^2 - 3*a^3) + tan(c/2 + (d*x)/2)^5*(12*a^2*b - 4*b^3) - a^3 + 24*a^2
*b*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3
+ 6*tan(c/2 + (d*x)/2)^5 + 2*tan(c/2 + (d*x)/2)^7)) + (3*a^2*b*log(tan(c/2
+ (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2) + 1i)*(2*a^2 - 3*b^2)*1i)/(2*d)
```

3.168 $\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$a^3x-\frac{9}{2}ab^2x+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{9a^2b\cos(c+dx)}{2d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c}{3c}$$

```
[Out] a^3*x-9/2*a*b^2*x+9/2*a^2*b*arctanh(cos(d*x+c))/d-b^3*arctanh(cos(d*x+c))/d
-9/2*a^2*b*cos(d*x+c)/d+b^3*cos(d*x+c)/d+1/3*b^3*cos(d*x+c)^3/d+a^3*cot(d*x
+c)/d-9/2*a*b^2*cot(d*x+c)/d+3/2*a*b^2*cos(d*x+c)^2*cot(d*x+c)/d-3/2*a^2*b*
cos(d*x+c)*cot(d*x+c)^2/d-1/3*a^3*cot(d*x+c)^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.476$, Rules used = {2801, 2672, 308, 212, 2671, 294, 327, 209, 3554, 8}

$$-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+a^3x-\frac{9a^2b\cos(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{9ab^2\cot(c+dx)}{2d}+\frac{3ab^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{9}{2}ab^2x+\frac{b^3\cos^3(c+dx)}{3d}+\frac{b^3\cos(c+dx)}{d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] a^3*x - (9*a*b^2*x)/2 + (9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTan
h[Cos[c + d*x]])/d - (9*a^2*b*Cos[c + d*x])/(2*d) + (b^3*Cos[c + d*x])/d +
(b^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/
(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (3*a^2*b*Cos[c + d*x]
*Cot[c + d*x]^2)/(2*d) - (a^3*Cot[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\cos^3(c+dx)\cot(c+dx)+3ab^2\cos^2(c+dx)\cot^2(c+dx)+3a\right.$$
$$= a^3\int \cot^4(c+dx)\,dx+\left(3a^2b\right)\int \cos(c+dx)\cot^3(c+dx)\,dx+\left(3ab\right.$$
$$= -\frac{a^3\cot^3(c+dx)}{3d}-a^3\int \cot^2(c+dx)\,dx-\frac{\left(3a^2b\right)\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,d\right.}{d}$$
$$= \frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\,c}{2d}$$
$$= a^3x-\frac{9a^2b\cos(c+dx)}{2d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}+\frac{a^3\cot(}{}$$
$$= a^3x-\frac{9}{2}ab^2x+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}$$

Mathematica [A]
time = 6.18, size = 355, normalized size = 1.83

$$\frac{a(2a^2-9b^2)(c+dx)}{2d}+\frac{b(-12a^2+5b^2)\cos(c+dx)}{4d}+\frac{b^3\cos(3(c+dx))}{12d}+\frac{\left(4a^3\cos\left(\frac{1}{2}(c+dx)\right)-9ab^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6d}-\frac{3a^2b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^3\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d}+\frac{(9a^2b-2b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{(-9a^2b+2b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{3a^2b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^3\sin\left(\frac{1}{2}(c+dx)\right)+9ab^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d}-\frac{3ab^2\sin(2(c+dx))}{4d}+\frac{a^3\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 9*b^2)*(c + d*x))/(2*d) + (b*(-12*a^2 + 5*b^2)*Cos[c + d*x])/(4
*d) + (b^3*Cos[3*(c + d*x)])/(12*d) + ((4*a^3*Cos[(c + d*x)/2] - 9*a*b^2*Co
s[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (3*a^2*b*Csc[(c + d*x)/2]^2)/(8*d
) - (a^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + ((9*a^2*b - 2*b^3)*L
og[Cos[(c + d*x)/2]])/(2*d) + ((-9*a^2*b + 2*b^3)*Log[Sin[(c + d*x)/2]])/(2
*d) + (3*a^2*b*Sec[(c + d*x)/2]^2)/(8*d) + (Sec[(c + d*x)/2]*(-4*a^3*Sin[(c
 + d*x)/2] + 9*a*b^2*Sin[(c + d*x)/2]))/(6*d) - (3*a*b^2*Sin[2*(c + d*x)])/
(4*d) + (a^3*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

Maple [A]
time = 0.27, size = 186, normalized size = 0.96

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3a^2b\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+}{}$
default	$\frac{a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3a^2b\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+}{}$

risch	$a^3x-\frac{9a\,b^2x}{2}+\frac{3ia\,b^2\mathrm{e}^{2i(dx+c)}}{8d}-\frac{3b\,\mathrm{e}^{i(dx+c)}a^2}{2d}+\frac{5b^3\mathrm{e}^{i(dx+c)}}{8d}-\frac{3b\,\mathrm{e}^{-i(dx+c)}a^2}{2d}+\frac{5b^3\mathrm{e}^{-i(dx+c)}}{8d}-\frac{3ia\,b^2\mathrm{e}^{-}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+3*a^2*b*(-1/2/sin(d*x+c)^2*co
s(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+3
*a*b^2*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
-3/2*d*x-3/2*c)+b^3*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))
)
```

Maxima [A]
time = 0.51, size = 187, normalized size = 0.96

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3-18\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)ab^2+2\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)b^3+9\,a^2b\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 - 18*(3*d
*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a*b^2 +
2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(cos(
d*x + c) - 1))*b^3 + 9*a^2*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d
*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.38, size = 293, normalized size = 1.51

$$\frac{18\,ab^2\cos\left(dx+c\right)^5+8\left(2\,a^3-9\,ab^2\right)\cos\left(dx+c\right)^3-3\left(9\,a^2b-2\,b^3-\left(9\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(9\,a^2b-2\,b^3-\left(9\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-6\left(2\,a^3-9\,ab^2\right)\cos\left(dx+c\right)+2\left(2\,b^3\cos\left(dx+c\right)^5+3\left(2\,a^3-9\,ab^2\right)dx\cos\left(dx+c\right)^2-2\left(9\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^3-3\left(2\,a^3-9\,ab^2\right)dx+3\left(9\,a^2b-2\,b^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a*b^2*cos(d*x + c)^5 + 8*(2*a^3 - 9*a*b^2)*cos(d*x + c)^3 - 3*(9*a
^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(9*a^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log
(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(2*a^3 - 9*a*b^2)*cos(d*x + c) +
 2*(2*b^3*cos(d*x + c)^5 + 3*(2*a^3 - 9*a*b^2)*d*x*cos(d*x + c)^2 - 2*(9*a^
2*b - 2*b^3)*cos(d*x + c)^3 - 3*(2*a^3 - 9*a*b^2)*d*x + 3*(9*a^2*b - 2*b^3)
*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^3\cot^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(178) = 356.
time = 6.46, size = 421, normalized size = 2.17

$$\frac{3a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^3+27a^2b\tan(\frac{1}{2}dx+\frac{1}{2}c)^2-45a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)+108ab^2\tan(\frac{1}{2}dx+\frac{1}{2}c)+36(2a^3-9ab^2)(dx+c)-36(9a^2b-2b^3)\log(|\tan(\frac{1}{2}dx+\frac{1}{2}c)|)+\frac{198a^2b\tan(\frac{1}{2}dx+\frac{1}{2}c)^9-44b^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^9+45a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^8+108ab^2\tan(\frac{1}{2}dx+\frac{1}{2}c)^8+135a^2b\tan(\frac{1}{2}dx+\frac{1}{2}c)^7+156b^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^7+132a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^6-324ab^2\tan(\frac{1}{2}dx+\frac{1}{2}c)^6-351a^2b\tan(\frac{1}{2}dx+\frac{1}{2}c)^5+156b^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^5+126a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^4-540ab^2\tan(\frac{1}{2}dx+\frac{1}{2}c)^4-315a^2b\tan(\frac{1}{2}dx+\frac{1}{2}c)^3+148b^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^3+36a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^2-108ab^2\tan(\frac{1}{2}dx+\frac{1}{2}c)^2-27a^2b\tan(\frac{1}{2}dx+\frac{1}{2}c)-3a^3}{(\tan(\frac{1}{2}dx+\frac{1}{2}c)^3+\tan(\frac{1}{2}dx+\frac{1}{2}c))^3}}{72d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*a^3*tan(1/2*d*x + 1/2*c)^3 + 27*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 45*a
^3*tan(1/2*d*x + 1/2*c) + 108*a*b^2*tan(1/2*d*x + 1/2*c) + 36*(2*a^3 - 9*a*
b^2)*(d*x + c) - 36*(9*a^2*b - 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (198
*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 44*b^3*tan(1/2*d*x + 1/2*c)^9 + 45*a^3*tan(
1/2*d*x + 1/2*c)^8 + 108*a*b^2*tan(1/2*d*x + 1/2*c)^8 + 135*a^2*b*tan(1/2*d
*x + 1/2*c)^7 + 156*b^3*tan(1/2*d*x + 1/2*c)^7 + 132*a^3*tan(1/2*d*x + 1/2*
c)^6 - 324*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 351*a^2*b*tan(1/2*d*x + 1/2*c)^5
+ 156*b^3*tan(1/2*d*x + 1/2*c)^5 + 126*a^3*tan(1/2*d*x + 1/2*c)^4 - 540*a*b
^2*tan(1/2*d*x + 1/2*c)^4 - 315*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 148*b^3*tan(
1/2*d*x + 1/2*c)^3 + 36*a^3*tan(1/2*d*x + 1/2*c)^2 - 108*a*b^2*tan(1/2*d*x
+ 1/2*c)^2 - 27*a^2*b*tan(1/2*d*x + 1/2*c) - 3*a^3)/(tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))^3)/d
```

Mupad [B]
time = 6.81, size = 405, normalized size = 2.09

$$\frac{a^3\tan(\frac{c}{2}+\frac{dx}{2})^3}{24d}-\frac{\ln(\tan(\frac{c}{2}+\frac{dx}{2}))\left(\frac{9a^2b}{2}-b^3\right)}{d}-\frac{\ln(\tan(\frac{c}{2}+\frac{dx}{2})+1\mathrm{i})\left(\frac{ab^2\,9\mathrm{i}}{2}-a^3\,1\mathrm{i}\right)}{d}+\frac{\tan(\frac{c}{2}+\frac{dx}{2})\left(\frac{3ab^2}{2}-\frac{5a^3}{8}\right)}{d}-\frac{\tan(\frac{c}{2}+\frac{dx}{2})^2(12ab^2-4a^3)-\tan(\frac{c}{2}+\frac{dx}{2})^8(5a^3+12ab^2)+\tan(\frac{c}{2}+\frac{dx}{2})^4(60ab^2-14a^3)+\tan(\frac{c}{2}+\frac{dx}{2})^6\left(36ab^2-\frac{44a^3}{3}\right)+\tan(\frac{c}{2}+\frac{dx}{2})^7(51a^2b-32b^3)+\tan(\frac{c}{2}+\frac{dx}{2})^3\left(57a^2b-\frac{64b^3}{3}\right)+\tan(\frac{c}{2}+\frac{dx}{2})^5(105a^2b-32b^3)+\frac{a^3}{3}+3a^2b\tan(\frac{c}{2}+\frac{dx}{2})}{d\left(8\tan(\frac{c}{2}+\frac{dx}{2})^9+24\tan(\frac{c}{2}+\frac{dx}{2})^7+24\tan(\frac{c}{2}+\frac{dx}{2})^5+8\tan(\frac{c}{2}+\frac{dx}{2})^3\right)}+\frac{3a^2b\tan(\frac{c}{2}+\frac{dx}{2})^2}{8d}-\frac{a\ln(\tan(\frac{c}{2}+\frac{dx}{2})-\mathrm{i})(2a^2-9b^2)\,1\mathrm{i}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (log(tan(c/2 + (d*x)/2))*((9*a^2*b)/2 -
 b^3))/d - (log(tan(c/2 + (d*x)/2) + 1i)*((a*b^2*9i)/2 - a^3*1i))/d + (tan(
c/2 + (d*x)/2)*((3*a*b^2)/2 - (5*a^3)/8))/d - (tan(c/2 + (d*x)/2)^2*(12*a*b
^2 - 4*a^3) - tan(c/2 + (d*x)/2)^8*(12*a*b^2 + 5*a^3) + tan(c/2 + (d*x)/2)^
4*(60*a*b^2 - 14*a^3) + tan(c/2 + (d*x)/2)^6*(36*a*b^2 - (44*a^3)/3) + tan(
c/2 + (d*x)/2)^7*(51*a^2*b - 32*b^3) + tan(c/2 + (d*x)/2)^3*(57*a^2*b - (64
*b^3)/3) + tan(c/2 + (d*x)/2)^5*(105*a^2*b - 32*b^3) + a^3/3 + 3*a^2*b*tan(
c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 24*tan(c/2 + (d*x)/2)^5 + 24*t
an(c/2 + (d*x)/2)^7 + 8*tan(c/2 + (d*x)/2)^9)) + (3*a^2*b*tan(c/2 + (d*x)/2
)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2) - 1i)*(2*a^2 - 9*b^2)*1i)/(2*d)
```

3.169 $\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=291

$$-a^3x+\frac{15}{2}ab^2x-\frac{45a^2b\tanh^{-1}(\cos(c+dx))}{8d}+\frac{5b^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{45a^2b\cos(c+dx)}{8d}-\frac{5b^3\cos(c+dx)}{2d}-$$

```
[Out] -a^3*x+15/2*a*b^2*x-45/8*a^2*b*arctanh(cos(d*x+c))/d+5/2*b^3*arctanh(cos(d*
x+c))/d+45/8*a^2*b*cos(d*x+c)/d-5/2*b^3*cos(d*x+c)/d-5/6*b^3*cos(d*x+c)^3/d
-a^3*cot(d*x+c)/d+15/2*a*b^2*cot(d*x+c)/d+15/8*a^2*b*cos(d*x+c)*cot(d*x+c)^
2/d-1/2*b^3*cos(d*x+c)^3*cot(d*x+c)^2/d+1/3*a^3*cot(d*x+c)^3/d-5/2*a*b^2*co
t(d*x+c)^3/d+3/2*a*b^2*cos(d*x+c)^2*cot(d*x+c)^3/d-3/4*a^2*b*cos(d*x+c)*cot
(d*x+c)^4/d-1/5*a^3*cot(d*x+c)^5/d
```

Rubi [A]
time = 0.17, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2801, 2672, 294, 308, 212, 2671, 209, 327, 3554, 8}

$$-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-a^3x+\frac{45a^2b\cos(c+dx)}{8d}-\frac{3a^2b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{15a^2b\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{45a^2b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{5ab^2\cot^3(c+dx)}{2d}+\frac{15ab^2\cot(c+dx)}{2d}+\frac{3ab^2\cos^2(c+dx)\cot^3(c+dx)}{2d}+\frac{15}{2}ab^2x-\frac{5b^3\cos^3(c+dx)}{6d}-\frac{5b^3\cos(c+dx)}{2d}-\frac{b^3\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{5b^3\tanh^{-1}(\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (15*a*b^2*x)/2 - (45*a^2*b*ArcTanh[Cos[c + d*x]])/(8*d) + (5*b^3
*ArcTanh[Cos[c + d*x]])/(2*d) + (45*a^2*b*Cos[c + d*x])/(8*d) - (5*b^3*Cos[
c + d*x])/(2*d) - (5*b^3*Cos[c + d*x]^3)/(6*d) - (a^3*Cot[c + d*x])/d + (15
*a*b^2*Cot[c + d*x])/(2*d) + (15*a^2*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) -
 (b^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(2*d) + (a^3*Cot[c + d*x]^3)/(3*d) - (
5*a*b^2*Cot[c + d*x]^3)/(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*
d) - (3*a^2*b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a^3*Cot[c + d*x]^5)/(5*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx &= \int \left(b^3\cos^3(c+dx)\cot^3(c+dx)+3ab^2\cos^2(c+dx)\cot^4(c+dx)+\right. \\
&= a^3\int \cot^6(c+dx)\,dx+\left(3a^2b\right)\int \cos(c+dx)\cot^5(c+dx)\,dx+\left(3a\right. \\
&= -\frac{a^3\cot^5(c+dx)}{5d}-a^3\int \cot^4(c+dx)\,dx-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\right.}{d} \\
&= -\frac{b^3\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{a^3\cot^3(c+dx)}{3d}+\frac{3ab^2\cos^2(c+d}{2} \\
&= -\frac{a^3\cot(c+dx)}{d}+\frac{15a^2b\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{b^3\cos^3(c+dx}{2} \\
&= -a^3x+\frac{45a^2b\cos(c+dx)}{8d}-\frac{5b^3\cos(c+dx)}{2d}-\frac{5b^3\cos^3(c+dx)}{6d}- \\
&= -a^3x+\frac{15}{2}ab^2x-\frac{45a^2b\tanh^{-1}(\cos(c+dx))}{8d}+\frac{5b^3\tanh^{-1}(\cos(c-}{2d}
\end{aligned}$$

Mathematica [A]
time = 1.73, size = 346, normalized size = 1.19

$$\frac{-600a(2a^2-15b^2)(c+dx)\csc^4(c+dx)+1200b(-9a^2+4b^2)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+5\cot(c+dx)\csc^4(c+dx)\left(-80a^3+285ab^2+12b(60a^2-29b^2)\sin(c+dx)\right)+\csc^5(c+dx)\left(5(40a^3-489ab^2)\cos(3(c+dx))+(-184a^3+1065ab^2)\cos(5(c+dx))+5(-9ab^2\cos(7(c+dx))+60a(2a^2-15b^2)(c+dx)\sin(3(c+dx))-306a^2b\sin(4(c+dx))+122b^3\sin(4(c+dx))-24a^3c\sin(5(c+dx))+180ab^2c\sin(5(c+dx))-24a^3dx\sin(5(c+dx))+180ab^2dx\sin(5(c+dx))+36a^2b\sin(6(c+dx))-22b^3\sin(6(c+dx))-b^3\sin(8(c+dx)))\right)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-600*a*(2*a^2 - 15*b^2)*(c + d*x)*Csc[c + d*x]^4 + 1200*b*(-9*a^2 + 4*b^2)
*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 5*Cot[c + d*x]*Csc[c + d
*x]^4*(-80*a^3 + 285*a*b^2 + 12*b*(60*a^2 - 29*b^2)*Sin[c + d*x]) + Csc[c +
 d*x]^5*(5*(40*a^3 - 489*a*b^2)*Cos[3*(c + d*x)] + (-184*a^3 + 1065*a*b^2)*
Cos[5*(c + d*x)] + 5*(-9*a*b^2*Cos[7*(c + d*x)] + 60*a*(2*a^2 - 15*b^2)*(c
+ d*x)*Sin[3*(c + d*x)] - 306*a^2*b*Sin[4*(c + d*x)] + 122*b^3*Sin[4*(c + d
*x)] - 24*a^3*c*Sin[5*(c + d*x)] + 180*a*b^2*c*Sin[5*(c + d*x)] - 24*a^3*d*
x*Sin[5*(c + d*x)] + 180*a*b^2*d*x*Sin[5*(c + d*x)] + 36*a^2*b*Sin[6*(c + d
*x)] - 22*b^3*Sin[6*(c + d*x)] - b^3*Sin[8*(c + d*x)])))/(1920*d)
```

Maple [A]
time = 0.29, size = 289, normalized size = 0.99

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+3a^2b\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx\right.}{8}}{}$
default	$\frac{a^3\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+3a^2b\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx\right.}{8}}{}$
risch	$-a^3x+\frac{15a\,b^2x}{2}-\frac{b^3\mathrm{e}^{3i(dx+c)}}{24d}-\frac{3ia\,b^2\mathrm{e}^{2i(dx+c)}}{8d}+\frac{3b\,\mathrm{e}^{i(dx+c)}a^2}{2d}-\frac{9b^3\mathrm{e}^{i(dx+c)}}{8d}+\frac{3b\,\mathrm{e}^{-i(dx+c)}a^2}{2d}-\frac{9b^3\mathrm{e}^{-i(}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+3*a^2*b*(-1/
4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos(d*x+c)^5+
5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))+3*a*b^2*(-
1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(cos(d*x+c)^5
+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+b^3*(-1/2/sin(
d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*cos(d*x+c)-5/2*
ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.66, size = 252, normalized size = 0.87

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3-120\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)ab^2+20\left(4\,\cos(dx+c)^3-\frac{6\,\cos(dx+c)}{\cos(dx+c)^2-1}+24\,\cos(dx+c)-15\,\log(\cos(dx+c)+1)+15\,\log(\cos(dx+c)-1)\right)b^3+45\,a^2b\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos(dx+c)+15\,\log(\cos(dx+c)+1)-15\,\log(\cos(dx+c)-1)\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a^3 - 120*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)
^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a*b^2 + 20*(4*cos(d*x + c)^3 - 6
*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c)
+ 1) + 15*log(cos(d*x + c) - 1))*b^3 + 45*a^2*b*(2*(9*cos(d*x + c)^3 - 7*co
s(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*
log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.40, size = 412, normalized size = 1.42

$$-\frac{360\,ab^2\cos(dx+c)^7+184\,(2\,a^3-15\,ab^2)\cos(dx+c)^5-280\,(2\,a^3-15\,ab^2)\cos(dx+c)^3+75\left((9\,a^2b-4\,b^3)\cos(dx+c)^4+9\,a^2b-4\,b^3-2\,(9\,a^2b-4\,b^3)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-75\left((9\,a^2b-4\,b^3)\cos(dx+c)^4+9\,a^2b-4\,b^3-2\,(9\,a^2b-4\,b^3)\cos(dx+c)^2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+120\,(2\,a^3-15\,ab^2)\cos(dx+c)+10\left(8\,b^3\cos(dx+c)^7+12\,(2\,a^3-15\,ab^2)dx\cos(dx+c)^4-8\,(9\,a^2b-4\,b^3)\cos(dx+c)^5-24\,(2\,a^3-15\,ab^2)dx\cos(dx+c)^2+25\,(9\,a^2b-4\,b^3)\cos(dx+c)^3+12\,(2\,a^3-15\,ab^2)dx-15\,(9\,a^2b-4\,b^3)\cos(dx+c)\right)\sin(dx+c)}{240\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(360*a*b^2*cos(d*x + c)^7 + 184*(2*a^3 - 15*a*b^2)*cos(d*x + c)^5 -
280*(2*a^3 - 15*a*b^2)*cos(d*x + c)^3 + 75*((9*a^2*b - 4*b^3)*cos(d*x + c)^
4 + 9*a^2*b - 4*b^3 - 2*(9*a^2*b - 4*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 75*((9*a^2*b - 4*b^3)*cos(d*x + c)^4 + 9*a^2*b -
4*b^3 - 2*(9*a^2*b - 4*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*si
n(d*x + c) + 120*(2*a^3 - 15*a*b^2)*cos(d*x + c) + 10*(8*b^3*cos(d*x + c)^7
 + 12*(2*a^3 - 15*a*b^2)*d*x*cos(d*x + c)^4 - 8*(9*a^2*b - 4*b^3)*cos(d*x +
 c)^5 - 24*(2*a^3 - 15*a*b^2)*d*x*cos(d*x + c)^2 + 25*(9*a^2*b - 4*b^3)*cos
(d*x + c)^3 + 12*(2*a^3 - 15*a*b^2)*d*x - 15*(9*a^2*b - 4*b^3)*cos(d*x + c)
)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^3 \cot^6(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + dx))**3*cot(c + d*x)**6, x)
```

Giac [**A**]
time = 9.47, size = 471, normalized size = 1.62

$$\frac{6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+45a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-70a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+120ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-720a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+120b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+660a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3240ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-480(2a^3-15ab^2)(dx+c)+600(9a^2b-4b^3)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)-\frac{320\left(9ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-18a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+18b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-36a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-18a^2b+14b^3\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3}-\frac{12330a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-5480b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+660a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-3240ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-720a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+120b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-70a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+120ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+45a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6a^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 70*
a^3*tan(1/2*d*x + 1/2*c)^3 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 720*a^2*b*t
an(1/2*d*x + 1/2*c)^2 + 120*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^3*tan(1/2*d*
x + 1/2*c) - 3240*a*b^2*tan(1/2*d*x + 1/2*c) - 480*(2*a^3 - 15*a*b^2)*(d*x
+ c) + 600*(9*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) - 320*(9*a*b^2*
tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 18*b^3*tan(1/2*d
*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 24*b^3*tan(1/2*d*x + 1/2*
c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 14*b^3)/(tan(1/2*d*x + 1/2
*c)^2 + 1)^3 - (12330*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 5480*b^3*tan(1/2*d*x +
 1/2*c)^5 + 660*a^3*tan(1/2*d*x + 1/2*c)^4 - 3240*a*b^2*tan(1/2*d*x + 1/2*c
)^4 - 720*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan(1/2*d*x + 1/2*c)^3 - 7
0*a^3*tan(1/2*d*x + 1/2*c)^2 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 45*a^2*b*
tan(1/2*d*x + 1/2*c) + 6*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]

time = 7.06, size = 507, normalized size = 1.74

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(12\,a\,b^2-22\,a^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(4\,a\,b^2-\frac{26\,a^3}{15}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(96\,a\,b^2-\frac{78\,a^3}{5}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(320\,a\,b^2-\frac{191\,a^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(408\,a\,b^2-\frac{296\,a^3}{5}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{39\,a^2\,b}{2}-4\,b^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(216\,a^2\,b-196\,b^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{519\,a^2\,b}{2}-\frac{484\,b^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{909\,a^2\,b}{2}-268\,b^3\right)-\frac{a^3}{5}-\frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}}{d\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+96\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+96\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a\,b^2}{8}-\frac{7\,a^3}{96}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3\,a^2\,b}{4}-\frac{b^3}{8}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{45\,a^2\,b}{8}-\frac{5\,b^3}{2}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)\left(\frac{a\,b^2\,15\mathrm{i}}{2}-a^3\,1\mathrm{i}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{27\,a\,b^2}{8}-\frac{11\,a^3}{16}\right)}{d}+\frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,d}-\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\left(2\,a^2-15\,b^2\right)\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^5)/(160*d) + (tan(c/2 + (d*x)/2)^10*(12*a*b^2 - 22*
a^3) - tan(c/2 + (d*x)/2)^2*(4*a*b^2 - (26*a^3)/15) + tan(c/2 + (d*x)/2)^4*
(96*a*b^2 - (78*a^3)/5) + tan(c/2 + (d*x)/2)^8*(320*a*b^2 - (191*a^3)/3) +
tan(c/2 + (d*x)/2)^6*(408*a*b^2 - (296*a^3)/5) + tan(c/2 + (d*x)/2)^3*((39*
a^2*b)/2 - 4*b^3) + tan(c/2 + (d*x)/2)^9*(216*a^2*b - 196*b^3) + tan(c/2 +
(d*x)/2)^5*((519*a^2*b)/2 - (484*b^3)/3) + tan(c/2 + (d*x)/2)^7*((909*a^2*b
)/2 - 268*b^3) - a^3/5 - (3*a^2*b*tan(c/2 + (d*x)/2))/2)/(d*(32*tan(c/2 + (
d*x)/2)^5 + 96*tan(c/2 + (d*x)/2)^7 + 96*tan(c/2 + (d*x)/2)^9 + 32*tan(c/2
+ (d*x)/2)^11)) + (tan(c/2 + (d*x)/2)^3*((a*b^2)/8 - (7*a^3)/96))/d - (tan(
c/2 + (d*x)/2)^2*((3*a^2*b)/4 - b^3/8))/d + (log(tan(c/2 + (d*x)/2))*((45*a
^2*b)/8 - (5*b^3)/2))/d - (log(tan(c/2 + (d*x)/2) - 1i)*((a*b^2*15i)/2 - a^
3*1i))/d - (tan(c/2 + (d*x)/2)*((27*a*b^2)/8 - (11*a^3)/16))/d + (3*a^2*b*t
an(c/2 + (d*x)/2)^4)/(64*d) - (a*log(tan(c/2 + (d*x)/2) + 1i)*(2*a^2 - 15*b
^2)*1i)/(2*d)
```

3.170 $\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=204

$$-\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{16(a+b)^3d}-\frac{(8a^2-9ab+3b^2)\log(1+\sin(c+dx))}{16(a-b)^3d}+\frac{a^5\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}$$

```
[Out] -1/16*(8*a^2+9*a*b+3*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d-1/16*(8*a^2-9*a*b+3*b^
2)*ln(1+sin(d*x+c))/(a-b)^3/d+a^5*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+1/4*sec(
d*x+c)^4*(a-b*sin(d*x+c))/(a^2-b^2)/d-1/8*sec(d*x+c)^2*(4*a*(2*a^2-b^2)-b*(
9*a^2-5*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.28, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2800, 1661, 815}

$$-\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(8a^2-9ab+3b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d\,(a^2-b^2)}-\frac{\sec^2(c+dx)\,(4a(2a^2-b^2)-b(9a^2-5b^2)\sin(c+dx))}{8d\,(a^2-b^2)^2}+\frac{a^5\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/((a + b)^3*d) - ((8*a
^2 - 9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^5*Log[a +
b*Sin[c + d*x]])/((a^2 - b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/
(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(2*a^2 - b^2) - b*(9*a^2 - 5*b^2)*
Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)\,d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^6}{a^2-b^2} - \frac{b^4(4a^2-b^2)x}{a^2-b^2} - 4b^2x^3}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+d\right.}{4b^2d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)\,d} - \frac{\sec^2(c+dx)\,(4a(2a^2-b^2) - b(9a^2-5b^2)\sin(c-}{8(a^2-b^2)^2\,d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)\,d} - \frac{\sec^2(c+dx)\,(4a(2a^2-b^2) - b(9a^2-5b^2)\sin(c-}{8(a^2-b^2)^2\,d}$$

$$= -\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{(8a^2-9ab+3b^2)\log(1+\sin(c+dx))}{16(a-b)^3d}$$

Mathematica [A]
time = 0.93, size = 184, normalized size = 0.90

$$\frac{-\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{(a+b)^3} - \frac{(8a^2-9ab+3b^2)\log(1+\sin(c+dx))}{(a-b)^3} + \frac{16a^5\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{1}{(a+b)(-1+\sin(c+dx))^2} + \frac{7a+5b}{(a+b)^2(-1+\sin(c+dx))} + \frac{1}{(a-b)(1+\sin(c+dx))^2} + \frac{-7a+5b}{(a-b)^2(1+\sin(c+dx))}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) - ((8*a^2 - 9
*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 + (16*a^5*Log[a + b*Sin[c +
d*x]])/((a - b)^3*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (7*a + 5
*b)/((a + b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])^2) + (-
7*a + 5*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

Maple [A]
time = 0.43, size = 189, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{1}{2(8a-8b)(1+\sin(dx+c))^2}-\frac{7a-5b}{16(a-b)^2(1+\sin(dx+c))}+\frac{\left(-8a^2+9ab-3b^2\right)\ln(1+\sin(dx+c))}{16(a-b)^3}+\frac{a^5\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{2(8a+8b)(\sin(dx}}{d}$
default	$\frac{\frac{1}{2(8a-8b)(1+\sin(dx+c))^2}-\frac{7a-5b}{16(a-b)^2(1+\sin(dx+c))}+\frac{\left(-8a^2+9ab-3b^2\right)\ln(1+\sin(dx+c))}{16(a-b)^3}+\frac{a^5\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{2(8a+8b)(\sin(dx}}{d}$
risch	$\frac{3ib^2x}{8(a^3+3a^2b+3ab^2+b^3)}+\frac{i(16ia^3e^{6i(dx+c)}-8iab^2e^{6i(dx+c)}-9a^2be^{7i(dx+c)}+5b^3e^{7i(dx+c)}+16ia^3e^{4i(dx+c)}-a^2be^{5i(dx}}{4(a^4-2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(8*a-8*b)/(1+sin(d*x+c))^2-1/16*(7*a-5*b)/(a-b)^2/(1+sin(d*x+c))+1
/16/(a-b)^3*(-8*a^2+9*a*b-3*b^2)*ln(1+sin(d*x+c))+a^5/(a+b)^3/(a-b)^3*ln(a+
b*sin(d*x+c))+1/2/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16*(-7*a-5*b)/(a+b)^2/(sin(d
*x+c)-1)+1/16/(a+b)^3*(-8*a^2-9*a*b-3*b^2)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.31, size = 288, normalized size = 1.41

$$\frac{\frac{16\,a^5\log\left(b\sin\left(dx+c\right)+a\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(8\,a^2-9\,ab+3\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+9\,ab+3\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(\left(9\,a^2b-5\,b^3\right)\sin\left(dx+c\right)^3+6\,a^3-2\,ab^2-4\left(2\,a^3-ab^2\right)\sin\left(dx+c\right)^2-\left(7\,a^2b-3\,b^3\right)\sin\left(dx+c\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^4+a^4-2\,a^2b^2+b^4-2\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^5*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
(8*a^2 - 9*a*b + 3*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3) - (8*a^2 + 9*a*b + 3*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2
 + b^3) - 2*((9*a^2*b - 5*b^3)*sin(d*x + c)^3 + 6*a^3 - 2*a*b^2 - 4*(2*a^3
- a*b^2)*sin(d*x + c)^2 - (7*a^2*b - 3*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2
 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*
sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.49, size = 261, normalized size = 1.28

$$\frac{16\,a^5\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)-\left(8\,a^5+15\,a^4b-10\,a^2b^3+3\,b^5\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(8\,a^5-15\,a^4b+10\,a^2b^3-3\,b^5\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+4\,a^5-8\,a^3b^2+4\,ab^4-8\left(2\,a^5-3\,a^3b^2+ab^4\right)\cos\left(dx+c\right)^2-2\left(2\,a^4b-4\,a^2b^3+2\,b^5-\left(9\,a^4b-14\,a^2b^3+5\,b^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^5*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (8*a^5 + 15*a^4*b - 1
0*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^5 - 15*a^4*b
```

```
 + 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 - 8*a^
3*b^2 + 4*a*b^4 - 8*(2*a^5 - 3*a^3*b^2 + a*b^4)*cos(d*x + c)^2 - 2*(2*a^4*b
 - 4*a^2*b^3 + 2*b^5 - (9*a^4*b - 14*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d
*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5{(c + dx)}}{a + b\sin{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 15.80, size = 343, normalized size = 1.68

$$\frac{\frac{16\,a^5 b\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^6 b-3\,a^4 b^3+3\,a^2 b^5-b^7}-\frac{\left(8\,a^2-9\,ab+3\,b^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)}{a^3-3\,a^2 b+3\,ab^2-b^3}-\frac{\left(8\,a^2+9\,ab+3\,b^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)}{a^3+3\,a^2 b+3\,ab^2+b^3}+\frac{2\left(6\,a^5\sin\left(dx+c\right)^4-9\,a^4 b\sin\left(dx+c\right)^3+14\,a^2 b^3\sin\left(dx+c\right)^3-5\,b^5\sin\left(dx+c\right)^3-4\,a^5\sin\left(dx+c\right)^2-12\,a^3 b^2\sin\left(dx+c\right)^2+4\,ab^4\sin\left(dx+c\right)^2+7\,a^4 b\sin\left(dx+c\right)-10\,a^2 b^3\sin\left(dx+c\right)+3\,b^5\sin\left(dx+c\right)+8\,a^3 b^2-2\,ab^4\right)}{\left(a^6-3\,a^4 b^2+3\,a^2 b^4-b^6\right)\left(\sin\left(dx+c\right)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^5*b*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7) - (8*a^2 - 9*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3) - (8*a^2 + 9*a*b + 3*b^2)*log(abs(sin(d*x + c) - 1))/(a^3
+ 3*a^2*b + 3*a*b^2 + b^3) + 2*(6*a^5*sin(d*x + c)^4 - 9*a^4*b*sin(d*x + c)
^3 + 14*a^2*b^3*sin(d*x + c)^3 - 5*b^5*sin(d*x + c)^3 - 4*a^5*sin(d*x + c)^
2 - 12*a^3*b^2*sin(d*x + c)^2 + 4*a*b^4*sin(d*x + c)^2 + 7*a^4*b*sin(d*x +
c) - 10*a^2*b^3*sin(d*x + c) + 3*b^5*sin(d*x + c) + 8*a^3*b^2 - 2*a*b^4)/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 7.43, size = 498, normalized size = 2.44

$$\frac{a^5\,\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{1}{a+b}-\frac{7\,b}{8\,{\left(a+b\right)}^2}+\frac{b^2}{4\,{\left(a+b\right)}^3}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{b^2}{4\,{\left(a-b\right)}^3}+\frac{7\,b}{8\,{\left(a-b\right)}^2}+\frac{1}{a-b}\right)}{d}-\frac{\frac{2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4-2\,a^2\,b^2+b^4}+\frac{2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{a^4-2\,a^2\,b^2+b^4}+\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a\,b^2-2\,a^3\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(7\,a^2\,b-3\,b^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(15\,a^2\,b-11\,b^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(15\,a^2\,b-11\,b^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(7\,a^2-3\,b^2\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*sin(c + d*x)),x)
```

```
[Out] (a^5*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2)) - (log(tan(c/2 + (d*x)/2) - 1)*(1/(a + b) - (7*
b)/(8*(a + b)^2) + b^2/(4*(a + b)^3)))/d - (log(tan(c/2 + (d*x)/2) + 1)*(b^
2/(4*(a - b)^3) + (7*b)/(8*(a - b)^2) + 1/(a - b)))/d - ((2*a^3*tan(c/2 + (
d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2) + (2*a^3*tan(c/2 + (d*x)/2)^6)/(a^4 + b^
```

```
4 - 2*a^2*b^2) + (4*tan(c/2 + (d*x)/2)^4*(a*b^2 - 2*a^3))/(a^4 + b^4 - 2*a^
2*b^2) - (tan(c/2 + (d*x)/2)^7*(7*a^2*b - 3*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2
)) + (tan(c/2 + (d*x)/2)^3*(15*a^2*b - 11*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2))
 + (tan(c/2 + (d*x)/2)^5*(15*a^2*b - 11*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2)) -
 (b*tan(c/2 + (d*x)/2)*(7*a^2 - 3*b^2))/(4*(a^4 + b^4 - 2*a^2*b^2)))/(d*(6*
tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + ta
n(c/2 + (d*x)/2)^8 + 1))
```

3.171 $\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{(2a+b)\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{(2a-b)\log(1+\sin(c+dx))}{4(a-b)^2d}-\frac{a^3\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d}+\frac{\sec^2(c+dx)(a-b\,\mathrm{s}}{2\,(a^2-b^2)}$$

```
[Out] 1/4*(2*a+b)*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(2*a-b)*ln(1+sin(d*x+c))/(a-b)^2
/d-a^3*ln(a+b*sin(d*x+c))/(a^2-b^2)^2/d+1/2*sec(d*x+c)^2*(a-b*sin(d*x+c))/(
a^2-b^2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2800, 1661, 815}

$$\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\,(a^2-b^2)}-\frac{a^3\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^2}+\frac{(2a+b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(2a-b)\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a + b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((2*a - b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^2*d) - (a^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d
) + (Sec[c + d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2(a^2-b^2)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2(2a^2-b^2)x}{a^2-b^2}}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2(a^2-b^2)d} + \frac{\text{Subst}\left(\int \left(-\frac{b^2(2a+b)}{2(a+b)^2(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)} + \cdots\right.\right.}{2b^2d}$$

$$= \frac{(2a+b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(2a-b)\log(1+\sin(c+dx))}{4(a-b)^2d} - \frac{a^3\log(a+b\sin(\cdots}{(a^2-b^2)^2\cdots}$$

Mathematica [A]

time = 0.34, size = 117, normalized size = 0.93

$$\frac{\frac{(2a+b)\log(1-\sin(c+dx))}{(a+b)^2}+\frac{(2a-b)\log(1+\sin(c+dx))}{(a-b)^2}-\frac{4a^3\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}-\frac{1}{(a+b)(-1+\sin(c+dx))}+\frac{1}{(a-b)(1+\sin(c+dx))}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
[Out] (((2*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^2 + ((2*a - b)*Log[1 + Sin[c + d
*x]])/(a - b)^2 - (4*a^3*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) - 1
/((a + b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/(4*d)
```

Maple [A]

time = 0.32, size = 121, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{(2a+b)\ln(\sin(dx+c)-1)}{4(a+b)^2}+\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(2a-b)\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{a^3\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}}{d}$
default	$\frac{-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{(2a+b)\ln(\sin(dx+c)-1)}{4(a+b)^2}+\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(2a-b)\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{a^3\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}}{d}$
risch	$-\frac{iax}{a^2+2ab+b^2}-\frac{iax}{a^2-2ab+b^2}-\frac{ibx}{2(a^2+2ab+b^2)}+\frac{2ia^3x}{a^4-2a^2b^2+b^4}+\frac{ibx}{2a^2-4ab+2b^2}+\frac{i(-2ia\,e^{2i(dx+c)}+b\,e^{3i(dx+\cdots}}{d(a^2-b^2)(1+e^{2i(\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(4*a+4*b)/(sin(d*x+c)-1)+1/4*(2*a+b)/(a+b)^2*ln(sin(d*x+c)-1)+1/(4*
a-4*b)/(1+sin(d*x+c))+1/4*(2*a-b)/(a-b)^2*ln(1+sin(d*x+c))-a^3/(a+b)^2/(a-b
)^2*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 142, normalized size = 1.13

$$-\frac{\frac{4\,a^{3}\log\left(b\sin\left(dx+c\right)+a\right)}{a^{4}-2\,a^{2}b^{2}+b^{4}}-\frac{\left(2\,a-b\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{2}-2\,ab+b^{2}}-\frac{\left(2\,a+b\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{2}+2\,ab+b^{2}}-\frac{2\,\left(b\sin\left(dx+c\right)-a\right)}{\left(a^{2}-b^{2}\right)\sin\left(dx+c\right)^{2}-a^{2}+b^{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^3*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - (2*a - b)*log
(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (2*a + b)*log(sin(d*x + c) - 1)/(a
^2 + 2*a*b + b^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^2 - a^
2 + b^2))/d
```

Fricas [A]
time = 0.39, size = 157, normalized size = 1.25

$$-\frac{4\,a^{3}\cos\left(dx+c\right)^{2}\log\left(b\sin\left(dx+c\right)+a\right)-\left(2\,a^{3}+3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,a^{3}-3\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)-2\,a^{3}+2\,ab^{2}+2\,\left(a^{2}b-b^{3}\right)\sin\left(dx+c\right)}{4\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^3*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (2*a^3 + 3*a^2*b - b^3
)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a^2*b + b^3)*cos(d*x +
c)^2*log(-sin(d*x + c) + 1) - 2*a^3 + 2*a*b^2 + 2*(a^2*b - b^3)*sin(d*x + c
))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^{3}\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 6.65, size = 177, normalized size = 1.40

$$-\frac{\frac{4\,a^{3}b\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^{4}b-2\,a^{2}b^{3}+b^{5}}-\frac{(2\,a-b)\log\left(|\sin\left(dx+c\right)+1|\right)}{a^{2}-2\,ab+b^{2}}-\frac{(2\,a+b)\log\left(|\sin\left(dx+c\right)-1|\right)}{a^{2}+2\,ab+b^{2}}+\frac{2\left(a^{3}\sin\left(dx+c\right)^{2}-a^{2}b\sin\left(dx+c\right)+b^{3}\sin\left(dx+c\right)-ab^{2}\right)}{(a^{4}-2\,a^{2}b^{2}+b^{4})\left(\sin\left(dx+c\right)^{2}-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*b*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - (2*a
 - b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - (2*a + b)*log(abs(si
n(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 2*(a^3*sin(d*x + c)^2 - a^2*b*sin(d*
x + c) + b^3*sin(d*x + c) - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2
 - 1)))/d
```

Mupad [B]
time = 7.04, size = 217, normalized size = 1.72

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(2\,a-b\right)}{2\,d\,{\left(a-b\right)}^2}-\frac{\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}-\frac{2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{a^2-b^2}+\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{a^2-b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{a^3\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(2\,a+b\right)}{2\,d\,{\left(a+b\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*sin(c + d*x)),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*(2*a - b))/(2*d*(a - b)^2) - ((b*tan(c/2 + (d*
x)/2))/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2) + (b*tan(c/2 +
(d*x)/2)^3)/(a^2 - b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2
+ 1)) - (a^3*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(
a^4 + b^4 - 2*a^2*b^2)) + (log(tan(c/2 + (d*x)/2) - 1)*(2*a + b))/(2*d*(a +
 b)^2)
```

3.172 $\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)d}-\frac{\log(1+\sin(c+dx))}{2(a-b)d}+\frac{a\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d-1/2*ln(1+sin(d*x+c))/(a-b)/d+a*ln(a+b*sin(d*x
+c))/(a^2-b^2)/d
```

Rubi [A]

time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2800, 815}

$$\frac{a\log(a+b\sin(c+dx))}{d\,(a^2-b^2)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d
) + (a*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)}+\frac{a}{(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)(b+x)}\right)dx, x, b\sin(c+dx)\right)}{d}\\
&= -\frac{\log(1-\sin(c+dx))}{2(a+b)d}-\frac{\log(1+\sin(c+dx))}{2(a-b)d}+\frac{a\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 87, normalized size = 1.18

$$\frac{(-a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+a\log(a+b\sin(c+dx))}{(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a + b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - (a + b)*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] + a*Log[a + b*Sin[c + d*x]])/((a - b)*(a + b)*d)
```

Maple [A]
time = 0.24, size = 71, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{\ln(1+\sin(dx+c))}{2a-2b}+\frac{a\ln(a+b\sin(dx+c))}{(a+b)(a-b)}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
default	$\frac{-\frac{\ln(1+\sin(dx+c))}{2a-2b}+\frac{a\ln(a+b\sin(dx+c))}{(a+b)(a-b)}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
risch	$\frac{ix}{a+b}+\frac{ic}{d(a+b)}+\frac{ix}{a-b}+\frac{ic}{d(a-b)}-\frac{2iax}{a^2-b^2}-\frac{2iac}{d(a^2-b^2)}-\frac{\ln(e^{i(dx+c)}-i)}{d(a+b)}-\frac{\ln(e^{i(dx+c)}+i)}{d(a-b)}+\frac{a\ln\left(e^{2i(dx+\cdots\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(2*a-2*b)*ln(1+sin(d*x+c))+a/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/(2*a+
2*b)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 65, normalized size = 0.88

$$\frac{\frac{2\,a\log(b\sin(dx+c)+a)}{a^2-b^2}-\frac{\log(\sin(dx+c)+1)}{a-b}-\frac{\log(\sin(dx+c)-1)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a - b
) - log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [A]
time = 0.35, size = 63, normalized size = 0.85

$$\frac{2\,a\log\left(b\sin\left(dx+c\right)+a\right)-(a+b)\log\left(\sin\left(dx+c\right)+1\right)-(a-b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,(a^2-b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) - (a - b)*
log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 7.25, size = 71, normalized size = 0.96

$$\frac{\frac{2\,ab\log(|b\sin(dx+c)+a|)}{a^2b-b^3} - \frac{\log(|\sin(dx+c)+1|)}{a-b} - \frac{\log(|\sin(dx+c)-1|)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x + c
) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b))/d
```

Mupad [B]
time = 6.73, size = 91, normalized size = 1.23

$$\frac{a\,\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^2-b^2\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{d\,\left(a-b\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d\,\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*sin(c + d*x)),x)
```

```
[Out] (a*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^2 - b^2)
) - log(tan(c/2 + (d*x)/2) + 1)/(d*(a - b)) - log(tan(c/2 + (d*x)/2) - 1)/(
d*(a + b))
```

3.173 $\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-ln(a+b*sin(d*x+c))/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2800, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{ad}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.97

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sin(dx+c))}{a}+\frac{\ln(\sin(dx+c))}{a}}{d}$	33
default	$\frac{-\frac{\ln(a+b\sin(dx+c))}{a}+\frac{\ln(\sin(dx+c))}{a}}{d}$	33
risch	$\frac{\ln\left(e^{2i(dx+c)}-1\right)}{da} - \frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{da}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/a*ln(a+b*sin(d*x+c))+1/a*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 33, normalized size = 0.97

$$-\frac{\frac{\log(b\sin(dx+c)+a)}{a} - \frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(b*sin(d*x + c) + a)/a - log(sin(d*x + c))/a)/d
```

Fricas [A]
time = 0.36, size = 31, normalized size = 0.91

$$-\frac{\log\left(b\sin\left(dx+c\right)+a\right)-\log\left(-\frac{1}{2}\sin\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] -(log(b*sin(d*x + c) + a) - log(-1/2*sin(d*x + c)))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 6.18, size = 35, normalized size = 1.03

$$-\frac{\frac{\log(|b\sin(dx+c)+a|)}{a}-\frac{\log(|\sin(dx+c)|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] -(log(abs(b*sin(d*x + c) + a))/a - log(abs(sin(d*x + c)))/a)/d
```

Mupad [B]
time = 6.36, size = 48, normalized size = 1.41

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*sin(c + d*x)),x)
[Out] (log(tan(c/2 + (d*x)/2)) - log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*
x)/2)^2))/(a*d)
```

3.174 $\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-(a^2-b^2)*ln(sin(d*x+c))/a^3/d+(a^2
-b^2)*ln(a+b*sin(d*x+c))/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2800, 908}

$$\frac{b\csc(c+dx)}{a^2d}-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((a^2 - b^2)*Log[Sin[c
+ d*x]])/(a^3*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^3} - \frac{b^2}{a^2x^2} + \frac{-a^2+b^2}{a^3x} + \frac{a^2-b^2}{a^3(a+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} - \frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d} + \frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}$$

Mathematica [A]
time = 0.11, size = 65, normalized size = 0.77

$$-\frac{-2ab\csc(c+dx) + a^2\csc^2(c+dx) + 2(a^2-b^2)\left(\log(\sin(c+dx)) - \log(a+b\sin(c+dx))\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
[Out] -1/2*(-2*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - b^2)*(Log[Sin[c +
 d*x]] - Log[a + b*Sin[c + d*x]]))/(a^3*d)
```

Maple [A]
time = 0.29, size = 76, normalized size = 0.90

method	result
derivativedivides	$\frac{\frac{\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{a^3} - \frac{1}{2a\sin(dx+c)^2} + \frac{\left(-a^2+b^2\right)\ln(\sin(dx+c))}{a^3} + \frac{b}{a^2\sin(dx+c)}}{d}$
default	$\frac{\frac{\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{a^3} - \frac{1}{2a\sin(dx+c)^2} + \frac{\left(-a^2+b^2\right)\ln(\sin(dx+c))}{a^3} + \frac{b}{a^2\sin(dx+c)}}{d}$
risch	$\frac{2i\left(-ia\,e^{2i(dx+c)}+b\,e^{3i(dx+c)}-b\,e^{i(dx+c)}\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2} + \frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{da} - \frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)b^2}{a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*((a^2-b^2)/a^3*ln(a+b*sin(d*x+c))-1/2/a/sin(d*x+c)^2+1/a^3*(-a^2+b^2)*l
n(sin(d*x+c))+1/a^2*b/sin(d*x+c))
```

Maxima [A]
time = 0.37, size = 77, normalized size = 0.92

$$\frac{\frac{2\,(a^2-b^2)\log(b\sin(dx+c)+a)}{a^3} - \frac{2\,(a^2-b^2)\log(\sin(dx+c))}{a^3} + \frac{2\,b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^2 - b^2)*log(b*sin(d*x + c) + a)/a^3 - 2*(a^2 - b^2)*log(sin(d*x
+ c))/a^3 + (2*b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.36, size = 118, normalized size = 1.40

$$-\frac{2\,ab\sin\left(dx+c\right)-a^2-2\,\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2-a^2+b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)+2\,\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2-a^2+b^2\right)\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)}{2\,\left(a^3d\cos\left(dx+c\right)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) - a^2 - 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)
*log(b*sin(d*x + c) + a) + 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*log(-
1/2*sin(d*x + c)))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^3\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 8.09, size = 114, normalized size = 1.36

$$-\frac{\frac{2\,\left(a^2-b^2\right)\log\left(\left|\sin(dx+c)\right|\right)}{a^3}-\frac{2\,\left(a^2b-b^3\right)\log\left(\left|b\sin(dx+c)+a\right|\right)}{a^3b}-\frac{3\,a^2\sin(dx+c)^2-3\,b^2\sin(dx+c)^2+2\,ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - b^2)*log(abs(sin(d*x + c)))/a^3 - 2*(a^2*b - b^3)*log(abs(b*
sin(d*x + c) + a))/(a^3*b) - (3*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 +
 2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c)^2))/d
```

Mupad [B]
time = 6.63, size = 144, normalized size = 1.71

$$\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(a^2-b^2\right)}{a^3\,d}-\frac{\frac{a}{2}-2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^2\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,\left(a^2-b^2\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*sin(c + d*x)),x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (log(tan(
c/2 + (d*x)/2))*(a^2 - b^2))/(a^3*d) - (a/2 - 2*b*tan(c/2 + (d*x)/2))/(4*a^
2*d*tan(c/2 + (d*x)/2)^2) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (
d*x)/2)^2)*(a^2 - b^2))/(a^3*d)
```

3.175 $\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{b(2a^2-b^2)\csc(c+dx)}{a^4d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2a^3d}+\frac{b\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4ad}+\frac{(a^2-b^2)^2\log(\sin(c+dx))}{a^5d}$$

```
[Out] -b*(2*a^2-b^2)*csc(d*x+c)/a^4/d+1/2*(2*a^2-b^2)*csc(d*x+c)^2/a^3/d+1/3*b*cs
c(d*x+c)^3/a^2/d-1/4*csc(d*x+c)^4/a/d+(a^2-b^2)^2*ln(sin(d*x+c))/a^5/d-(a^2
-b^2)^2*ln(a+b*sin(d*x+c))/a^5/d
```

Rubi [A]
time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 908}

$$\frac{b\csc^3(c+dx)}{3a^2d}+\frac{(a^2-b^2)^2\log(\sin(c+dx))}{a^5d}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^5d}-\frac{b(2a^2-b^2)\csc(c+dx)}{a^4d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2a^3d}-\frac{\csc^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*(2*a^2 - b^2)*Csc[c + d*x])/(a^4*d)) + ((2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^3*d) + (b*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a^2 -
 b^2)^2*Log[Sin[c + d*x]])/(a^5*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]]
)/(a^5*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^5(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{ax^5} - \frac{b^4}{a^2x^4} + \frac{-2a^2b^2+b^4}{a^3x^3} + \frac{2a^2b^2-b^4}{a^4x^2} + \frac{(a^2-b^2)^2}{a^5x} - \frac{(a^2-b^2)^2}{a^5(a+x)}\right) dx, x, b\sin(c+\right.}{d}$$
$$= -\frac{b(2a^2-b^2)\csc(c+dx)}{a^4d} + \frac{(2a^2-b^2)\csc^2(c+dx)}{2a^3d} + \frac{b\csc^3(c+dx)}{3a^2d} - \frac{\csc^4(c+}{4ad}$$

Mathematica [A]
time = 0.70, size = 115, normalized size = 0.78

$$\frac{12ab(-2a^2+b^2)\csc(c+dx)+6a^2(2a^2-b^2)\csc^2(c+dx)+4a^3b\csc^3(c+dx)-3a^4\csc^4(c+dx)+12(a^2-b^2)^2(\log(\sin(c+dx))-\log(a+b\sin(c+dx)))}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
[Out] (12*a*b*(-2*a^2 + b^2)*Csc[c + d*x] + 6*a^2*(2*a^2 - b^2)*Csc[c + d*x]^2 +
4*a^3*b*Csc[c + d*x]^3 - 3*a^4*Csc[c + d*x]^4 + 12*(a^2 - b^2)^2*(Log[Sin[c
 + d*x]] - Log[a + b*Sin[c + d*x]]))/(12*a^5*d)
```

Maple [A]
time = 0.37, size = 137, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{1}{4a\sin(dx+c)^4}-\frac{-2a^2+b^2}{2a^3\sin(dx+c)^2}+\frac{(a^4-2a^2b^2+b^4)\ln(\sin(dx+c))}{a^5}-\frac{(2a^2-b^2)b}{a^4\sin(dx+c)}+\frac{b}{3a^2\sin(dx+c)^3}-\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(}{a^5}}{d}$
default	$\frac{-\frac{1}{4a\sin(dx+c)^4}-\frac{-2a^2+b^2}{2a^3\sin(dx+c)^2}+\frac{(a^4-2a^2b^2+b^4)\ln(\sin(dx+c))}{a^5}-\frac{(2a^2-b^2)b}{a^4\sin(dx+c)}+\frac{b}{3a^2\sin(dx+c)^3}-\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(}{a^5}}{d}$
risch	$\frac{2i(6ia^3e^{6i(dx+c)}-3iab^2e^{6i(dx+c)}-6a^2b\,e^{7i(dx+c)}+3b^3e^{7i(dx+c)}-6ia^3e^{4i(dx+c)}+6iab^2e^{4i(dx+c)}+14a^2b\,e^{5i(dx+c)}-9b^3}{3d\,a^4(e^{2i(dx+c)}-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/4/a/sin(d*x+c)^4-1/2*(-2*a^2+b^2)/a^3/sin(d*x+c)^2+(a^4-2*a^2*b^2+b
^4)/a^5*ln(sin(d*x+c))-(2*a^2-b^2)/a^4*b/sin(d*x+c)+1/3/a^2*b/sin(d*x+c)^3-
(a^4-2*a^2*b^2+b^4)/a^5*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.39, size = 139, normalized size = 0.94

$$-\frac{\frac{12(a^4-2a^2b^2+b^4)\log(b\sin(dx+c)+a)}{a^5}-\frac{12(a^4-2a^2b^2+b^4)\log(\sin(dx+c))}{a^5}-\frac{4a^2b\sin(dx+c)-12(2a^2b-b^3)\sin(dx+c)^3-3a^3+6(2a^3-ab^2)\sin(dx+c)^2}{a^4\sin(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/a^5 - 12*(a^4 - 2
*a^2*b^2 + b^4)*log(sin(d*x + c))/a^5 - (4*a^2*b*sin(d*x + c) - 12*(2*a^2*b
 - b^3)*sin(d*x + c)^3 - 3*a^3 + 6*(2*a^3 - a*b^2)*sin(d*x + c)^2)/(a^4*sin
(d*x + c)^4))/d
```

Fricas [**A**]
time = 0.37, size = 271, normalized size = 1.83

$$\frac{9\,a^{4}-6\,a^{2}b^{2}-6\,(2\,a^{4}-a^{2}b^{2})\cos\left(dx+c\right)^{2}-12\,\left(\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{4}+a^{4}-2\,a^{2}b^{2}+b^{4}-2\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{2}\right)\log\left(b\sin\left(dx+c\right)+a\right)+12\,\left(\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{4}+a^{4}-2\,a^{2}b^{2}+b^{4}-2\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{2}\right)\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\,\left(5\,a^{3}b-3\,ab^{3}-3\,\left(2\,a^{3}b-ab^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{12\,\left(a^{5}d\cos\left(dx+c\right)^{4}-2\,a^{5}d\cos\left(dx+c\right)^{2}+a^{5}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(9*a^4 - 6*a^2*b^2 - 6*(2*a^4 - a^2*b^2)*cos(d*x + c)^2 - 12*((a^4 - 2
*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2
 + b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) + 12*((a^4 - 2*a^2*b^2 + b^
4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d
*x + c)^2)*log(-1/2*sin(d*x + c)) - 4*(5*a^3*b - 3*a*b^3 - 3*(2*a^3*b - a*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x +
 c)^2 + a^5*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{5}\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

Giac [**A**]
time = 8.13, size = 201, normalized size = 1.36

$$\frac{\frac{12\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^{5}}-\frac{12\,\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{5}b}-\frac{25\,a^{4}\sin\left(dx+c\right)^{4}-50\,a^{2}b^{2}\sin\left(dx+c\right)^{4}+25\,b^{4}\sin\left(dx+c\right)^{4}+24\,a^{3}b\sin\left(dx+c\right)^{3}-12\,ab^{3}\sin\left(dx+c\right)^{3}-12\,a^{4}\sin\left(dx+c\right)^{2}+6\,a^{2}b^{2}\sin\left(dx+c\right)^{2}-4\,a^{3}b\sin\left(dx+c\right)+3\,a^{4}}{a^{5}\sin\left(dx+c\right)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(abs(sin(d*x + c)))/a^5 - 12*(a^4*b - 2
*a^2*b^3 + b^5)*log(abs(b*sin(d*x + c) + a))/(a^5*b) - (25*a^4*sin(d*x + c)
^4 - 50*a^2*b^2*sin(d*x + c)^4 + 25*b^4*sin(d*x + c)^4 + 24*a^3*b*sin(d*x +
```

```
c)^3 - 12*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 6*a^2*b^2*sin(d*x
 + c)^2 - 4*a^3*b*sin(d*x + c) + 3*a^4)/(a^5*sin(d*x + c)^4))/d
```

Mupad [B]
time = 6.41, size = 281, normalized size = 1.90

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{3}{16\,a}-\frac{b^2}{8\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{b}{8\,a^2}+\frac{2\,b\,\left(\frac{3}{8\,a}-\frac{b^2}{4\,a^3}\right)}{a}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a\,b^2-3\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(14\,a^2\,b-8\,b^3\right)+\frac{a^3}{4}-\frac{2\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}}{16\,a^4\,d\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}-\frac{\ln\left(a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{a^5\,d}+\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b*sin(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(3/(16*a) - b^2/(8*a^3)))/d - tan(c/2 + (d*x)/2)^4/(6
4*a*d) - (tan(c/2 + (d*x)/2)*(b/(8*a^2) + (2*b*(3/(8*a) - b^2/(4*a^3)))/a))
/d - (tan(c/2 + (d*x)/2)^2*(2*a*b^2 - 3*a^3) + tan(c/2 + (d*x)/2)^3*(14*a^2
*b - 8*b^3) + a^3/4 - (2*a^2*b*tan(c/2 + (d*x)/2))/3)/(16*a^4*d*tan(c/2 + (
d*x)/2)^4) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4
 + b^4 - 2*a^2*b^2))/(a^5*d) + (b*tan(c/2 + (d*x)/2)^3)/(24*a^2*d) + (log(t
an(c/2 + (d*x)/2))*(a^4 + b^4 - 2*a^2*b^2))/(a^5*d)
```

3.176 $\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}d}+\frac{a^2b\sec(c+dx)}{(a^2-b^2)^2d}+\frac{b\sec(c+dx)}{(a^2-b^2)d}-\frac{b\sec^3(c+dx)}{3(a^2-b^2)d}-\frac{a^3\tan(c+dx)}{(a^2-b^2)^2d}+\frac{a\tan^3(c+dx)}{3(a^2-b^2)d}$$

```
[Out] 2*a^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d+a^
2*b*sec(d*x+c)/(a^2-b^2)^2/d+b*sec(d*x+c)/(a^2-b^2)/d-1/3*b*sec(d*x+c)^3/(a
^2-b^2)/d-a^3*tan(d*x+c)/(a^2-b^2)^2/d+1/3*a*tan(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2806, 2687, 30, 2686, 3852, 8, 2739, 632, 210}

$$\frac{a\tan^3(c+dx)}{3d(a^2-b^2)}-\frac{b\sec^3(c+dx)}{3d(a^2-b^2)}+\frac{a^2b\sec(c+dx)}{d(a^2-b^2)^2}+\frac{b\sec(c+dx)}{d(a^2-b^2)}+\frac{2a^4\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}}-\frac{a^3\tan(c+dx)}{d(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) + (a^2*b*Sec[c + d*x])/((a^2 - b^2)^2*d) + (b*Sec[c + d*x])/((a^2 - b^2
)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^3*Tan[c + d*x])/((a^2 - b^
2)^2*d) + (a*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2806

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[b*(g/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[a^2*(g^2/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec(c+dx)\tan^3(c+ $$

$$= -\frac{a^3\int \sec^2(c+dx)\,dx}{(a^2-b^2)^2} + \frac{a^4\int \frac{1}{a+b\sin(c+dx)}\,dx}{(a^2-b^2)^2} + \frac{(a^2b)\int \sec(c+dx)\tan(c+dx)\,dx}{(a^2-b^2)^2} $$

$$= \frac{b\sec(c+dx)}{(a^2-b^2)\,d} - \frac{b\sec^3(c+dx)}{3\,(a^2-b^2)\,d} + \frac{a\tan^3(c+dx)}{3\,(a^2-b^2)\,d} + \frac{a^3\text{Subst}(\int 1\,dx, x, -\tan(c+dx}{(a^2-b^2)^2\,d}$$

$$= \frac{a^2b\sec(c+dx)}{(a^2-b^2)^2\,d} + \frac{b\sec(c+dx)}{(a^2-b^2)\,d} - \frac{b\sec^3(c+dx)}{3\,(a^2-b^2)\,d} - \frac{a^3\tan(c+dx)}{(a^2-b^2)^2\,d} + \frac{a\tan^3(c+dx}{3\,(a^2-b^2)\,d}$$

$$= \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d} + \frac{a^2b\sec(c+dx)}{(a^2-b^2)^2\,d} + \frac{b\sec(c+dx)}{(a^2-b^2)\,d} - \frac{b\sec^3(c+dx)}{3\,(a^2-b^2)\,d} - $$

Mathematica [A]

time = 0.95, size = 195, normalized size = 1.10

$$\frac{\frac{48a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{\sec^3(c+dx)\left(-16a^2b+4b^3+3b(11a^2-5b^2)\cos(c+dx)+12b(-2a^2+b^2)\cos(2(c+dx))+11a^2b\cos(3(c+dx))-5b^3\cos(3(c+dx))+6ab^2\sin(c+dx)+8a^3\sin(3(c+dx))-2ab^2\sin(3(c+dx))\right)}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((48*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
) - (Sec[c + d*x]^3*(-16*a^2*b + 4*b^3 + 3*b*(11*a^2 - 5*b^2)*Cos[c + d*x]
+ 12*b*(-2*a^2 + b^2)*Cos[2*(c + d*x)] + 11*a^2*b*Cos[3*(c + d*x)] - 5*b^3*
Cos[3*(c + d*x)] + 6*a*b^2*Sin[c + d*x] + 8*a^3*Sin[3*(c + d*x)] - 2*a*b^2*
Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(24*d)
```

Maple [A]

time = 0.37, size = 214, normalized size = 1.21

method	result
derivativedivides	$\frac{-\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(32a-32b)}+\frac{16}{(32a-32b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-2a+b}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(32a+32b)}-}{d}$
default	$\frac{-\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(32a-32b)}+\frac{16}{(32a-32b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-2a+b}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(32a+32b)}-}{d}$

risch	$-\frac{2\left(6ia^3e^{4i(dx+c)}-3ia\,b^2e^{4i(dx+c)}-6a^2b\,e^{5i(dx+c)}+3b^3e^{5i(dx+c)}+6ia^3e^{2i(dx+c)}-8a^2b\,e^{3i(dx+c)}+2b^3e^{3i(dx+c)}+4ia^3-\right.}{3(a^4-2a^2b^2+b^4)\left(1+e^{2i(dx+c)}\right)^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-32/3/(tan(1/2*d*x+1/2*c)+1)^3/(32*a-32*b)+16/(32*a-32*b)/(tan(1/2*d*x
+1/2*c)+1)^2-1/2/(a-b)^2*(-2*a+b)/(tan(1/2*d*x+1/2*c)+1)-32/3/(tan(1/2*d*x+
1/2*c)-1)^3/(32*a+32*b)-16/(32*a+32*b)/(tan(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^2
*(-2*a-b)/(tan(1/2*d*x+1/2*c)-1)+2*a^4/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.40, size = 476, normalized size = 2.69

$$\left[-\frac{3\sqrt{-a^2+b^2}\,a^4\cos(dx+c)^3\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^4b-4a^2b^3+2b^5-6(2a^4b-3a^2b^3+b^5)\cos(dx+c)^2-2(a^5-2a^3b^2+ab^4-(4a^5-5a^3b^2+ab^4)\cos(dx+c)^2)\sin(dx+c)}{6(a^6-3a^4b^2+3a^2b^4-b^6)d\cos(dx+c)^3},\ -\frac{3\sqrt{a^2-b^2}\,a^4\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)^3+a^4b-2a^2b^3+b^5-3(2a^4b-3a^2b^3+b^5)\cos(dx+c)^2-(a^5-2a^3b^2+ab^4-(4a^5-5a^3b^2+ab^4)\cos(dx+c)^2)\sin(dx+c)}{3(a^6-3a^4b^2+3a^2b^4-b^6)d\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(2*a^4*b - 3*a^2*b^3 + b^5)*co
s(d*x + c)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3), -1/3*(3*sqrt(a^2 - b^2)*a^4*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2
- b^2)*cos(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(2*a^4*b
 - 3*a^2*b^3 + b^5)*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*
a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 4.79, size = 241, normalized size = 1.36

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-5a^2b+2b^3}{(a^4-2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
+ (3*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 10*a^3*t
an(1/2*d*x + 1/2*c)^3 + 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1/2*d
*x + 1/2*c)^2 - 6*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) -
 5*a^2*b + 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))
/d
```

Mupad [B]
time = 9.55, size = 372, normalized size = 2.10

$$\frac{\frac{2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^4-2a^2b^2+b^4}-\frac{2(5a^2b-2b^3)}{3(a^4-2a^2b^2+b^4)}+\frac{2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a^4-2a^2b^2+b^4}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2ab^2-5a^3)}{3(a^4-2a^2b^2+b^4)}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2b-b^3)}{a^4-2a^2b^2+b^4}-\frac{2a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{a^4-2a^2b^2+b^4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}+\frac{2a^4\,\mathrm{atan}\left(\frac{\frac{a^4(2a^4b-4a^2b^3+2b^5)}{(a+b)^{5/2}(a-b)^{5/2}}+\frac{2a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{5/2}}}{2a^4}\right)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*sin(c + d*x)),x)
```

```
[Out] ((2*a^3*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^2*b^2) - (2*(5*a^2*b - 2*b^3))
/(3*(a^4 + b^4 - 2*a^2*b^2)) + (2*a^3*tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*
a^2*b^2) + (4*tan(c/2 + (d*x)/2)^3*(2*a*b^2 - 5*a^3))/(3*(a^4 + b^4 - 2*a^2
*b^2)) + (4*tan(c/2 + (d*x)/2)^2*(2*a^2*b - b^3))/(a^4 + b^4 - 2*a^2*b^2) -
 (2*a^2*b*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 - 2*a^2*b^2))/(d*(3*tan(c/2 + (d
*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) + (2*a^4*ata
n(((a^4*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/2)) + (2*a
^5*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(5/2)
))/(2*a^4)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.177 $\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{b\sec(c+dx)}{(a^2-b^2)\,d}+\frac{a\tan(c+dx)}{(a^2-b^2)\,d}$$

```
[Out] -2*a^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d-b
*sec(d*x+c)/(a^2-b^2)/d+a*tan(d*x+c)/(a^2-b^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2806, 3852, 8, 2686, 2739, 632, 210}

$$-\frac{2a^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{3/2}}+\frac{a\tan(c+dx)}{d\,(a^2-b^2)}-\frac{b\sec(c+dx)}{d\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (b*Sec[c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2806

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[b*(g/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[a^2*(g^2/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{a\int \sec^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec(c+dx)\tan(c+dx)\,dx}{a^2-b^2}\\
&= -\frac{a\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{(a^2-b^2)\,d} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right.\right.}{(a^2-b^2)\,d}\\
&= -\frac{b\sec(c+dx)}{(a^2-b^2)\,d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d} + \frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}\right.\right.}{(a^2-b^2)\,d}\\
&= -\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} - \frac{b\sec(c+dx)}{(a^2-b^2)\,d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 152, normalized size = 1.58

$$\frac{-2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\cos(c+dx)+\sqrt{a^2-b^2}\,(-b+b\cos(c+dx)+a\sin(c+dx))}{(a-b)(a+b)\sqrt{a^2-b^2}\,d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqr
t[a^2 - b^2]*(-b + b*Cos[c + d*x] + a*Sin[c + d*x]))/((a - b)*(a + b)*Sqrt[
a^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]))
```

Maple [A]
time = 0.27, size = 112, normalized size = 1.17

method	result
derivativedivides	$\frac{-\frac{8}{(8a+8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}-\frac{8}{(8a-8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$
default	$\frac{-\frac{8}{(8a+8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}-\frac{8}{(8a-8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$
risch	$\frac{-2ia+2b\,\mathrm{e}^{i(dx+c)}}{d(-a^2+b^2)\left(1+\mathrm{e}^{2i(dx+c)}\right)}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}+a^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8/(8*a+8*b)/(tan(1/2*d*x+1/2*c)-1)-2*a^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-8/(8*a-8*b)/(tan(1
/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 305, normalized size = 3.18

$$\left[\frac{\sqrt{-a^2+b^2}\,a^2\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^2b+2b^3+2(a^3-ab^2)\sin(dx+c)}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)},\ \frac{\sqrt{a^2-b^2}\,a^2\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)-a^2b+b^3+(a^3-ab^2)\sin(dx+c)}{(a^4-2a^2b^2+b^4)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
 b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(d*x + c) + b)/(s
qrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*si
n(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 8.31, size = 107, normalized size = 1.11

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{(a^2-b^2)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b}{(a^2-b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) -
 b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

Mupad [B]
time = 6.37, size = 148, normalized size = 1.54

$$\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\frac{a^2\,\left(2\,a^2\,b-2\,b^3\right)}{{(a+b)}^{3/2}\,{(a-b)}^{3/2}}+\frac{2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{{(a+b)}^{3/2}\,{(a-b)}^{3/2}}}{2\,a^2}\right)}{d\,{(a+b)}^{3/2}\,{(a-b)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

```
[Out] ((2*b)/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2))/(a^2 - b^2))/(d*(tan(c/2 + (d
*x)/2)^2 - 1)) - (2*a^2*atan(((a^2*(2*a^2*b - 2*b^3))/((a + b)^(3/2)*(a - b
)^(3/2)) + (2*a^3*tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)*(a - b)^(3
/2)))/(2*a^2)))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.178 $\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d-2*arctan((b+a*tan(1/2*d*x+1/2*c)
)/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^2/d
```

Rubi [A]

time = 0.17, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2802, 3135, 3080, 3855, 2739, 632, 210}

$$-\frac{2\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*
d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$
$$= -\frac{\cot(c+dx)}{ad} + \frac{\int \frac{\csc(c+dx)(-b-a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{a}$$
$$= -\frac{\cot(c+dx)}{ad} - \frac{b\int \csc(c+dx)\,dx}{a^2} + \frac{(-a^2+b^2)\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{(2(a^2-b^2))\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\right.\right.}{a^2d}$$
$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} + \frac{(4(a^2-b^2))\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b\right.}{a^2d}$$
$$= -\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad}$$

Mathematica [A]

time = 0.17, size = 108, normalized size = 1.35

$$\frac{-4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) - a\cot\left(\frac{1}{2}(c+dx)\right) + 2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + a\tan\left(\frac{1}{2}(c+dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - a*Co
t[(c + d*x)/2] + 2*b*Log[Cos[(c + d*x)/2]] - 2*b*Log[Sin[(c + d*x)/2]] + a*
Tan[(c + d*x)/2])/(2*a^2*d)
```

Maple [A]

time = 0.31, size = 109, normalized size = 1.36

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a} - \frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2} + \frac{\left(-4a^2+4b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2\sqrt{a^2-b^2}}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a} - \frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2} + \frac{\left(-4a^2+4b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2\sqrt{a^2-b^2}}}{d}$

risch	$-\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{a^2d}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(dx+c)}+\frac{ia+\sqrt{-a^2+b^2}}{b}\right)}{d\,a^2}+\frac{\sqrt{-a^2+b^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a*tan(1/2*d*x+1/2*c)-1/2/a/tan(1/2*d*x+1/2*c)-1/a^2*b*ln(tan(1/2*d
*x+1/2*c))+1/2/a^2*(-4*a^2+4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.41, size = 314, normalized size = 3.92

$$\left[\frac{b\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)\sin\left(dx+c\right)-2a\cos\left(dx+c\right)}{2a^2d\sin\left(dx+c\right)},\frac{b\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\sin\left(dx+c\right)-2a\cos\left(dx+c\right)}{2a^2d\sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c)), 1/2*(b*log(1/2*
cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d
*x + c)))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 8.95, size = 129, normalized size = 1.61

$$-\frac{\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{a^2}-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x + 1/2*c)/a + 4*(
pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/a^2 - (2*b*tan(1/2*d*x + 1/2*c) - a)/(
a^2*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 7.00, size = 204, normalized size = 2.55

$$-\frac{\cot(c+d\,x)}{a\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{\mathrm{atan}\left(\frac{a^3\,\sqrt{b^2-a^2}\,1\mathrm{i}-a\,b^2\,\sqrt{b^2-a^2}\,2\mathrm{i}-b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}+a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}\,3\mathrm{i}}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4-2\,a^3\,b-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b^2+2\,a\,b^3+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^4}\right)\sqrt{b^2-a^2}\,2\mathrm{i}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

```
[Out] (atan((a^3*(b^2 - a^2)^(1/2)*1i - a*b^2*(b^2 - a^2)^(1/2)*2i - b^3*tan(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
)*3i)/(2*a*b^3 - 2*a^3*b + a^4*tan(c/2 + (d*x)/2) + 4*b^4*tan(c/2 + (d*x)/2
) - 5*a^2*b^2*tan(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i)/(a^2*d) - cot(c +
d*x)/(a*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d)
```

3.179 $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d}+\frac{b\cot(c$$

```
[Out] 2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^4/d-1/
2*b*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/a^4/d+1/3*(4*a^2-3*b^2)*cot(d*x+c)/a^
3/d+1/2*b*cot(d*x+c)*csc(d*x+c)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d
```

Rubi [A]
time = 0.29, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2804, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) - (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((4*a^2 - 3*b^2
)*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2804

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m/Sin[e + f*
x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*
m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m - 2)*Cos[e + f*x]*((a + b
*Sin[e + f*x])^(m + 1)/(6*a^2*f*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} - \frac{\int \frac{\csc^2(c+dx)\left(2\left(4a^2-3b^2\right)-ab\sin\right.}{a+b\sin}}{6}$$

$$= \frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} -$$

$$= \frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} +$$

$$= -\frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc}{2a^2d}$$

$$= -\frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc}{2a^2d}$$

$$= \frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} - \frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{(4a^2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 350 vs. 2(154) = 308.
time = 6.13, size = 350, normalized size = 2.27

$$\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} + \frac{\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^3d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24ad} + \frac{(-3a^2b+2b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{(3a^2b-2b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^2\sin\left(\frac{1}{2}(c+dx)\right)+3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^3d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*d) + ((4*a^2*Cos[(c + d*x)/2] - 3*b^2*
Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*a
^2*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a*d) + ((-3*a^2*b + 2*b^3
)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) + ((3*a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2
]])/(2*a^4*d) - (b*Sec[(c + d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-4*a^
2*Sin[(c + d*x)/2] + 3*b^2*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2])/(24*a*d)
```

Maple [A]
time = 0.40, size = 223, normalized size = 1.45

method	result

derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}+\frac{\left(16a^4-32a^2b^2+16b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{8a^4\sqrt{a^2-b^2}}-}{d}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}+\frac{\left(16a^4-32a^2b^2+16b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{8a^4\sqrt{a^2-b^2}}-}{d}$
risch	$-\frac{-12ia^2e^{4i(dx+c)}+6ib^2e^{4i(dx+c)}+3a\,e^{5i(dx+c)}b+12ia^2e^{2i(dx+c)}-12ib^2e^{2i(dx+c)}-8ia^2+6ib^2-3a\,e^{i(dx+c)}b}{3d\,a^3\left(e^{2i(dx+c)}-1\right)^3}-\frac{3b\ln(e^{i}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/a^3*(1/3*tan(1/2*d*x+1/2*c)^3*a^2-a*b*tan(1/2*d*x+1/2*c)^2-5*a^2*t
an(1/2*d*x+1/2*c)+4*b^2*tan(1/2*d*x+1/2*c))+1/8/a^4*(16*a^4-32*a^2*b^2+16*b
^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)-1/24/a/tan(1/2*d*x+1/2*c)^3-1/8*(-5*a^2+4*b^2)/a^3/tan(1/2*d*x+1/2*c)+1/8
/a^2*b/tan(1/2*d*x+1/2*c)^2+1/2/a^4*b*(3*a^2-2*b^2)*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.48, size = 633, normalized size = 4.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 - 3*a*b^2)*cos(d*x + c
)^3 + 6*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a^2 + b^2)*log(((2*a
^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x +
```

```
c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*
b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*
a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) + 12*(a^3 - a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 -
a^4*d)*sin(d*x + c)), -1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 -
 3*a*b^2)*cos(d*x + c)^3 + 12*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*si
n(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2
*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*
cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 12*(a^3 - a*b^2
)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 4.71, size = 273, normalized size = 1.77

$$\frac{\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}+\frac{12\left(3a^2b-2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}+\frac{48\left(a^4-2a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{66a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-44b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^3}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*((a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*t
an(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3 + 12*(3*a^2*b - 2*b^
3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*(pi*floo
r(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt
(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (66*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44
*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan(
1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x +
 1/2*c)^3))/d
```

Mupad [B]
time = 7.14, size = 654, normalized size = 4.25

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24ad}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24ad}+\frac{5\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{8ad}-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8ad}+\frac{3b\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2a^2d}-\frac{b^3\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^4d}+\frac{b\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^2d}-\frac{b^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^3d}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^2d}+\frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^3d}+\frac{\operatorname{atan}\left(\frac{2a^5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}+8b^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}-7a^3b^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}-16a^2b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}+4ab^4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}+7a^4b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}}{2i\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^8-5i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^7b-16i\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^6b^2+14i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^5b^3+34i\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b^4-13i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3b^5-28i\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^6+4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab^7+8i\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^8}\right)\sqrt{-(a+b)^3(a-b)^3}\,2i}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x)),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - cot(c/2 + (d*x)/2)^3/(24*a*d) + (5*cot(c/2
+ (d*x)/2))/(8*a*d) - (5*tan(c/2 + (d*x)/2))/(8*a*d) + (3*b*log(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2)))/(2*a^2*d) - (b^3*log(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)))/(a^4*d) + (b*cot(c/2 + (d*x)/2)^2)/(8*a^2*d) - (b^2*cot(c/2
+ (d*x)/2))/(2*a^3*d) - (b*tan(c/2 + (d*x)/2)^2)/(8*a^2*d) + (b^2*tan(c/2 +
 (d*x)/2))/(2*a^3*d) + (atan((2*a^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b
^4 + 3*a^4*b^2)^(1/2) + 8*b^5*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2) - 7*a^3*b^2*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a
^4*b^2)^(1/2) - 16*a^2*b^3*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^
4*b^2)^(1/2) + 4*a*b^4*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^
2)^(1/2) + 7*a^4*b*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(
1/2))/(a^8*sin(c/2 + (d*x)/2)*2i + b^8*sin(c/2 + (d*x)/2)*8i + a*b^7*cos(c/
2 + (d*x)/2)*4i - a^7*b*cos(c/2 + (d*x)/2)*5i - a^3*b^5*cos(c/2 + (d*x)/2)*
13i + a^5*b^3*cos(c/2 + (d*x)/2)*14i - a^2*b^6*sin(c/2 + (d*x)/2)*28i + a^4
*b^4*sin(c/2 + (d*x)/2)*34i - a^6*b^2*sin(c/2 + (d*x)/2)*16i))*(-(a + b)^3*
(a - b)^3)^(1/2)*2i)/(a^4*d)
```

3.180 $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=307

$$-\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{b(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d}-\frac{(23a^4-35a^2b^2+15}{15a^5d}$$

```
[Out] -2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^6/d+1
/8*b*(15*a^4-20*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^6/d-1/15*(23*a^4-35*a^
2*b^2+15*b^4)*cot(d*x+c)/a^5/d-cot(d*x+c)*csc(d*x+c)/b/d+1/8*(8*a^4-9*a^2*b
^2+4*b^4)*cot(d*x+c)*csc(d*x+c)/a^4/b/d+1/2*a*cot(d*x+c)*csc(d*x+c)^2/b^2/d
-1/30*(15*a^4-22*a^2*b^2+10*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d+1/4*b*co
t(d*x+c)*csc(d*x+c)^3/a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d
```

Rubi [A]
time = 0.72, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2805, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{2(a^2-b^2)^{5/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{(8a^4-9a^2b^2+4b^4)\cot(c+dx)\csc(c+dx)}{8a^4bd}+\frac{b(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d}-\frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d}-\frac{(15a^4-22a^2b^2+10b^4)\cot(c+dx)\csc^2(c+dx)}{30a^3b^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
6*d) + (b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) -
((23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (Cot[c + d*x]*Cs
c[c + d*x])/(b*d) + ((8*a^4 - 9*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x])
/(8*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(2*b^2*d) - ((15*a^4 - 22*a^
2*b^2 + 10*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^3*b^2*d) + (b*Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2805

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(5*a*f*Sin[
e + f*x]^5)), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x]
)^m/Sin[e + f*x]^4)*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m -
 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m
*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x]
, x], x] + Simp[Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Sin[e + f
*x]^2)), x] + Simp[a*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*m*(m
 - 1)*Sin[e + f*x]^3)), x] - Simp[b*(m - 4)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^(m + 1)/(20*a^2*f*Sin[e + f*x]^4)), x]) /; FreeQ[{a, b, e, f, m}, x] &&
 NeQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} + \frac{b\cot(c+dx)\csc^3(c+d}{4a^2d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} - \frac{(15a^4-22a^2b^2+10b^4)}{30a}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{(8a^4-9a^2b^2+4b^4)\cot(c+dx)\csc(c+dx)}{8a^4bd} + \frac{a\cot(}{}$$

$$= -\frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{(8a^4-9a^2b^2+}{}$$

$$= -\frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{(8a^4-9a^2b^2+}{}$$

$$= \frac{b(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx}{15a^5d}$$

$$= \frac{b(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx}{15a^5d}$$

$$= -\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} + \frac{b(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+}{8a^6d}$$

Mathematica [A]
time = 0.95, size = 504, normalized size = 1.64

$$\frac{-1920(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) - 32(23a^5-35a^3b^2+15ab^4)\cot\left(\frac{1}{2}(c+dx)\right) - 270a^4b\csc^2\left(\frac{1}{2}(c+dx)\right) + 120a^2b^3\csc^2\left(\frac{1}{2}(c+dx)\right) + 15a^4b\csc^4\left(\frac{1}{2}(c+dx)\right) + 1800a^4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2400a^2b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 960b^5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 1800a^4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2400a^2b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 960b^5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 270a^4b\sec^2\left(\frac{1}{2}(c+dx)\right) - 120a^2b^3\sec^2\left(\frac{1}{2}(c+dx)\right) - 15a^4b\sec^4\left(\frac{1}{2}(c+dx)\right) - 656a^5\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right) + 320a^3b^2\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right) + 41a^5\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx) - 20a^3b^2\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx) - 3a^5\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx) + 736a^5\tan\left(\frac{1}{2}(c+dx)\right) - 1120a^3b^2\tan\left(\frac{1}{2}(c+dx)\right) + 480ab^4\tan\left(\frac{1}{2}(c+dx)\right) + 6a^5\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{960a^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-1920*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] -
 32*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*Cot[(c + d*x)/2] - 270*a^4*b*Csc[(c +
d*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4 +
1800*a^4*b*Log[Cos[(c + d*x)/2]] - 2400*a^2*b^3*Log[Cos[(c + d*x)/2]] + 960
*b^5*Log[Cos[(c + d*x)/2]] - 1800*a^4*b*Log[Sin[(c + d*x)/2]] + 2400*a^2*b^
3*Log[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 270*a^4*b*Sec[(c
+ d*x)/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4
- 656*a^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Si
```

```
n[(c + d*x)/2]^4 + 41*a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[
(c + d*x)/2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 736*a
^5*Tan[(c + d*x)/2] - 1120*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*
x)/2] + 6*a^5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

Maple [A]
time = 0.50, size = 390, normalized size = 1.27 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32/a^5*(1/5*a^4*tan(1/2*d*x+1/2*c)^5-1/2*b*tan(1/2*d*x+1/2*c)^4*a^3-
7/3*tan(1/2*d*x+1/2*c)^3*a^4+4/3*a^2*b^2*tan(1/2*d*x+1/2*c)^3+8*b*a^3*tan(1
/2*d*x+1/2*c)^2-4*a*b^3*tan(1/2*d*x+1/2*c)^2+22*a^4*tan(1/2*d*x+1/2*c)-36*a
^2*b^2*tan(1/2*d*x+1/2*c)+16*b^4*tan(1/2*d*x+1/2*c))-1/160/a/tan(1/2*d*x+1/
2*c)^5-1/96*(-7*a^2+4*b^2)/a^3/tan(1/2*d*x+1/2*c)^3-1/32*(22*a^4-36*a^2*b^2
+16*b^4)/a^5/tan(1/2*d*x+1/2*c)+1/64*b/a^2/tan(1/2*d*x+1/2*c)^4-1/8/a^4*b*(
2*a^2-b^2)/tan(1/2*d*x+1/2*c)^2-1/8/a^6*b*(15*a^4-20*a^2*b^2+8*b^4)*ln(tan(
1/2*d*x+1/2*c))+1/32/a^6*(-64*a^6+192*a^4*b^2-192*a^2*b^4+64*b^6)/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the ’assume’ command before evaluation *may* h
elp (example of legal syntax is ’assume(4*b^2-4*a^2>0)’, see ‘assume?‘ for
more de
```

Fricas [A]
time = 0.67, size = 1079, normalized size = 3.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(16*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 1
3*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 120*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x
+ c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
```

```
^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 15
*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x +
 c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 2
0*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d
*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2
 + a*b^4)*cos(d*x + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*
b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d
*cos(d*x + c)^2 + a^6*d)*sin(d*x + c)), -1/240*(16*(23*a^5 - 35*a^3*b^2 + 1
5*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 13*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3
- 240*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(
a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
 c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 15*(15*a^4*b - 20*a
^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4
*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5
)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x
 + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*b - 4*a^2*b^3)*co
s(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 +
 a^6*d)*sin(d*x + c))]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**6/(a + b*sin(c + d*x)), x)
```

Giac [**A**]

time = 4.40, size = 490, normalized size = 1.60

$$\frac{\frac{6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-70\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+240\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+660\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1080\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+480\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^5}-\frac{120\,(15\,a^4b-20\,a^2b^3+8\,b^5)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^6}-\frac{1920\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^6}+\frac{4110\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5480\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2192\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-660\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1080\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-480\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-240\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+70\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-40\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^5}{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 70
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*b
*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^4*tan(1/
2*d*x + 1/2*c) - 1080*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x +
```

```
1/2*c))/a^5 - 120*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*d*x + 1/2
*c)))/a^6 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/
(sqrt(a^2 - b^2)*a^6) + (4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 5480*a^2*b^3*t
an(1/2*d*x + 1/2*c)^5 + 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(1/2*d
*x + 1/2*c)^4 + 1080*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 480*a*b^4*tan(1/2*d*x
 + 1/2*c)^4 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x +
1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^
2 + 15*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

Mupad [B]
time = 7.13, size = 1099, normalized size = 3.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + b*sin(c + d*x)),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a*d) + (tan(c/2 + (d*x)/2)^2*(b/(32*a^2) + (b*(7/
(32*a) - b^2/(8*a^3)))/a))/d - (tan(c/2 + (d*x)/2)*(b^2/(8*a^3) - 11/(16*a)
 + (2*b*(b/(16*a^2) + (2*b*(7/(32*a) - b^2/(8*a^3)))/a))/a))/d - (tan(c/2 +
 (d*x)/2)^3*(7/(96*a) - b^2/(24*a^3)))/d - (b*tan(c/2 + (d*x)/2)^4)/(64*a^2
*d) - (log(tan(c/2 + (d*x)/2))*((15*a^4*b)/8 + b^5 - (5*a^2*b^3)/2))/(a^6*d
) + (tan(c/2 + (d*x)/2)^2*((7*a^4)/3 - (4*a^2*b^2)/3) - a^4/5 - tan(c/2 + (
d*x)/2)^4*(22*a^4 + 16*b^4 - 36*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(4*a*b^3 -
8*a^3*b) + (a^3*b*tan(c/2 + (d*x)/2))/2)/(32*a^5*d*tan(c/2 + (d*x)/2)^5) -
(atan((((-(a + b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 3
9*a^10*b^2)/(4*a^10) + ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b
^2))/(4*a^9))*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a
^10*b - 32*a^4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9))*1i)/a^6 + ((-(a + b
)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 39*a^10*b^2)/(4*a
^10) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-
(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a^10*b - 32*a^4*b
^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9))*1i)/a^6)/((15*a^10*b - 8*b^11 + 44*
a^2*b^9 - 99*a^4*b^7 + 113*a^6*b^5 - 65*a^8*b^3)/(2*a^10) + (tan(c/2 + (d*x
)/2)*(16*a^10 - 8*b^10 + 42*a^2*b^8 - 94*a^4*b^6 + 110*a^6*b^4 - 66*a^8*b^2
))/(2*a^9) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b
^4 - 39*a^10*b^2)/(4*a^10) + ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*
a^10*b^2))/(4*a^9))*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)
*(31*a^10*b - 32*a^4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9)))/a^6 + ((-(a
+ b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 39*a^10*b^2)/(
4*a^10) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))
*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a^10*b - 32*a^
4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9)))/a^6))*(-(a + b)^5*(a - b)^5)^(1
/2)*2i)/(a^6*d)
```

3.181 $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=242

$$-\frac{a(4a+b)\log(1-\sin(c+dx))}{8(a+b)^4d}-\frac{a(4a-b)\log(1+\sin(c+dx))}{8(a-b)^4d}+\frac{a^4(a^2+5b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^4\,d}-\frac{}{(a^2}$$

```
[Out] -1/8*a*(4*a+b)*ln(1-sin(d*x+c))/(a+b)^4/d-1/8*a*(4*a-b)*ln(1+sin(d*x+c))/(a
-b)^4/d+a^4*(a^2+5*b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d-a^5/(a^2-b^2)^3/d/
(a+b*sin(d*x+c))+1/4*sec(d*x+c)^4*(a^2+b^2-2*a*b*sin(d*x+c))/(a^2-b^2)^2/d-
1/4*sec(d*x+c)^2*(4*a^4+6*a^2*b^2-2*b^4-a*b*(9*a^2-b^2)*sin(d*x+c))/(a^2-b^
2)^3/d
```

Rubi [A]
time = 0.53, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2800, 1661, 1643}

$$\frac{\sec^4(c+dx)\left(a^2-2ab\sin(c+dx)+b^2\right)}{4d\left(a^2-b^2\right)^2}-\frac{a^5}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{a^4(a^2+5b^2)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{\sec^2(c+dx)\left(2(2a^4+3a^2b^2-b^4)-ab(9a^2-b^2)\sin(c+dx)\right)}{4d\left(a^2-b^2\right)^3}-\frac{a(4a+b)\log(1-\sin(c+dx))}{8d(a+b)^4}-\frac{a(4a-b)\log(\sin(c+dx)+1)}{8d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/8*(a*(4*a + b)*Log[1 - Sin[c + d*x]])/((a + b)^4*d) - (a*(4*a - b)*Log[1
 + Sin[c + d*x]])/(8*(a - b)^4*d) + (a^4*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*
x]])/((a^2 - b^2)^4*d) - a^5/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (Sec[
c + d*x]^4*(a^2 + b^2 - 2*a*b*Sin[c + d*x]))/(4*(a^2 - b^2)^2*d) - (Sec[c +
 d*x]^2*(2*(2*a^4 + 3*a^2*b^2 - b^4) - a*b*(9*a^2 - b^2)*Sin[c + d*x]))/(4*
(a^2 - b^2)^3*d)
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^2(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{4\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^6}{\left(a^2-b^2\right)^2}-\frac{4a^4b^4x}{\left(a^2-b^2\right)^2}-\frac{6ab^6x^2}{\left(a^2-b^2\right)^2}-4}{(a+x)^2\left(b^2-x^2\right)^2}\right.}{4b^2d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{4\left(a^2-b^2\right)^2 d} - \frac{\sec^2(c+dx)\left(2(2a^4+3a^2b^2-b^4)-c\right.}{4\left(a^2-b^2\right)^3 c}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{4\left(a^2-b^2\right)^2 d} - \frac{\sec^2(c+dx)\left(2(2a^4+3a^2b^2-b^4)-c\right.}{4\left(a^2-b^2\right)^3 c}$$

$$= -\frac{a(4a+b)\log(1-\sin(c+dx))}{8(a+b)^4d} - \frac{a(4a-b)\log(1+\sin(c+dx))}{8(a-b)^4d} + \frac{a^4(a^2+5b^2}{}$$

Mathematica [A]
time = 4.25, size = 204, normalized size = 0.84

$$\frac{-\frac{2a(4a+b)\log(1-\sin(c+dx))}{(a+b)^4}-\frac{2a(4a-b)\log(1+\sin(c+dx))}{(a-b)^4}+\frac{16a^4(a^2+5b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^4}+\frac{1}{(a+b)^2(-1+\sin(c+dx))^2}+\frac{7a+3b}{(a+b)^3(-1+\sin(c+dx))}+\frac{1}{(a-b)^2(1+\sin(c+dx))^2}+\frac{-7a+3b}{(a-b)^3(1+\sin(c+dx))}-\frac{16a^5}{(a^2-b^2)^3(a+b\sin(c+dx))}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*a*(4*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^4 - (2*a*(4*a - b)*Log[1 +
Sin[c + d*x]])/(a - b)^4 + (16*a^4*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*x]])/(
a^2 - b^2)^4 + 1/((a + b)^2*(-1 + Sin[c + d*x])^2) + (7*a + 3*b)/((a + b)^3
*(-1 + Sin[c + d*x])) + 1/((a - b)^2*(1 + Sin[c + d*x])^2) + (-7*a + 3*b)/(
```

```
(a - b)^3*(1 + Sin[c + d*x])) - (16*a^5)/((a^2 - b^2)^3*(a + b*Sin[c + d*x]
)))/(16*d)
```

Maple [A]
time = 0.59, size = 205, normalized size = 0.85 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/(a-b)^2/(1+sin(d*x+c))^2-1/16*(-3*b+7*a)/(a-b)^3/(1+sin(d*x+c))-1
/8*a*(4*a-b)/(a-b)^4*ln(1+sin(d*x+c))+1/16/(a+b)^2/(sin(d*x+c)-1)^2-1/16*(-
3*b-7*a)/(a+b)^3/(sin(d*x+c)-1)-1/8*a*(4*a+b)/(a+b)^4*ln(sin(d*x+c)-1)-a^5/
(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))+a^4*(a^2+5*b^2)/(a+b)^4/(a-b)^4*ln(a+b*sin
(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(235) = 470.
time = 0.40, size = 505, normalized size = 2.09

$$\frac{\frac{8\left(a^{6}+5a^{4}b^{2}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^{8}-4a^{6}b^{2}+6a^{4}b^{4}-4a^{2}b^{6}+b^{8}}-\frac{\left(4a^{2}-ab\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{4}-4a^{3}b+6a^{2}b^{2}-4ab^{3}+b^{4}}-\frac{\left(4a^{2}+ab\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{4}+4a^{3}b+6a^{2}b^{2}+4ab^{3}+b^{4}}-\frac{2\left(7a^{5}+6a^{3}b^{2}-ab^{4}+\left(4a^{5}+9a^{3}b^{2}-ab^{4}\right)\sin\left(dx+c\right)^{4}+\left(5a^{4}b-7a^{2}b^{3}+2b^{5}\right)\sin\left(dx+c\right)^{3}-\left(12a^{5}+13a^{3}b^{2}-ab^{4}\right)\sin\left(dx+c\right)^{2}-\left(4a^{4}b-5a^{2}b^{3}+b^{5}\right)\sin\left(dx+c\right)\right)}{a^{7}-3a^{5}b^{2}+3a^{3}b^{4}-ab^{6}+\left(a^{6}b-3a^{4}b^{3}+3a^{2}b^{5}-b^{7}\right)\sin\left(dx+c\right)^{5}+\left(a^{7}-3a^{5}b^{2}+3a^{3}b^{4}-ab^{6}\right)\sin\left(dx+c\right)^{4}-2\left(a^{6}b-3a^{4}b^{3}+3a^{2}b^{5}-b^{7}\right)\sin\left(dx+c\right)^{3}-2\left(a^{7}-3a^{5}b^{2}+3a^{3}b^{4}-ab^{6}\right)\sin\left(dx+c\right)^{2}+\left(a^{6}b-3a^{4}b^{3}+3a^{2}b^{5}-b^{7}\right)\sin\left(dx+c\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(a^6 + 5*a^4*b^2)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8) - (4*a^2 - a*b)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b
+ 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + a*b)*log(sin(d*x + c) - 1)/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(7*a^5 + 6*a^3*b^2 - a*b^4 + (4*a^5
 + 9*a^3*b^2 - a*b^4)*sin(d*x + c)^4 + (5*a^4*b - 7*a^2*b^3 + 2*b^5)*sin(d*
x + c)^3 - (12*a^5 + 13*a^3*b^2 - a*b^4)*sin(d*x + c)^2 - (4*a^4*b - 5*a^2*
b^3 + b^5)*sin(d*x + c))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + (a^6*b - 3*
a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)^5 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 -
a*b^6)*sin(d*x + c)^4 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c
)^3 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^2 + (a^6*b - 3*a
^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(235) = 470.
time = 0.55, size = 555, normalized size = 2.29

$$\frac{2a^{7}-6a^{5}b^{2}+6a^{3}b^{4}-2ab^{6}-2\left(4a^{7}+5a^{5}b^{2}-10a^{3}b^{4}+ab^{6}\right)\cos\left(dx+c\right)^{4}-2\left(4a^{7}-9a^{5}b^{2}+6a^{3}b^{4}-ab^{6}\right)\cos\left(dx+c\right)^{2}+8\left(\left(a^{6}b+5a^{4}b^{3}\right)\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+\left(a^{7}+5a^{5}b^{2}\right)\cos\left(dx+c\right)^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)-\left(\left(4a^{6}b+15a^{5}b^{2}+20a^{4}b^{3}+10a^{3}b^{4}-ab^{6}\right)\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+\left(4a^{7}+15a^{6}b+20a^{5}b^{2}+10a^{4}b^{3}-a^{2}b^{5}\right)\cos\left(dx+c\right)^{4}\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\left(4a^{6}b-15a^{5}b^{2}+20a^{4}b^{3}-10a^{3}b^{4}+ab^{6}\right)\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+\left(4a^{7}-15a^{6}b+20a^{5}b^{2}-10a^{4}b^{3}+a^{2}b^{5}\right)\cos\left(dx+c\right)^{4}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^{6}b-3a^{4}b^{3}+3a^{2}b^{5}-b^{7}-\left(5a^{6}b-12a^{4}b^{3}+9a^{2}b^{5}-2b^{7}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{8\left(\left(a^{8}b-4a^{6}b^{3}+6a^{4}b^{5}-4a^{2}b^{7}+b^{9}\right)d\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+\left(a^{9}-4a^{7}b^{2}+6a^{5}b^{4}-4a^{3}b^{6}+ab^{8}\right)d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 - 2*(4*a^7 + 5*a^5*b^2 - 10*a^
3*b^4 + a*b^6)*cos(d*x + c)^4 - 2*(4*a^7 - 9*a^5*b^2 + 6*a^3*b^4 - a*b^6)*c
```

```
os(d*x + c)^2 + 8*((a^6*b + 5*a^4*b^3)*cos(d*x + c)^4*sin(d*x + c) + (a^7 +
 5*a^5*b^2)*cos(d*x + c)^4)*log(b*sin(d*x + c) + a) - ((4*a^6*b + 15*a^5*b^
2 + 20*a^4*b^3 + 10*a^3*b^4 - a*b^6)*cos(d*x + c)^4*sin(d*x + c) + (4*a^7 +
 15*a^6*b + 20*a^5*b^2 + 10*a^4*b^3 - a^2*b^5)*cos(d*x + c)^4)*log(sin(d*x
+ c) + 1) - ((4*a^6*b - 15*a^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6)*cos(d
*x + c)^4*sin(d*x + c) + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*
b^5)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*
b^5 - b^7 - (5*a^6*b - 12*a^4*b^3 + 9*a^2*b^5 - 2*b^7)*cos(d*x + c)^2)*sin(
d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)
^4*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d
*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(235) = 470.
time = 7.52, size = 494, normalized size = 2.04

$$\frac{\frac{8\left(a^6b+5a^4b^3\right)\log\left(|b\sin(dx+c)+a|\right)}{a^8b-4a^6b^3+6a^4b^5-4a^2b^7+b^9}-\frac{\left(4a^2-ab\right)\log\left(|\sin(dx+c)+1|\right)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{\left(4a^2+ab\right)\log\left(|\sin(dx+c)-1|\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{8\left(a^6b\sin(dx+c)+5a^4b^3\sin(dx+c)+2a^7+4a^5b^2\right)}{\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)\left(b\sin(dx+c)+a\right)}+\frac{2\left(3a^6\sin(dx+c)^4+15a^4b^2\sin(dx+c)^4-9a^5b\sin(dx+c)^3+10a^3b^3\sin(dx+c)^3-ab^5\sin(dx+c)^3-2a^6\sin(dx+c)^2-28a^4b^2\sin(dx+c)^2-8a^2b^4\sin(dx+c)^2+2b^6\sin(dx+c)^2+7a^5b\sin(dx+c)-6a^3b^3\sin(dx+c)-ab^5\sin(dx+c)+12a^4b^2+7a^2b^4-b^6\right)}{\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)\left(\sin(dx+c)^2-1\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*(a^6*b + 5*a^4*b^3)*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3
+ 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (4*a^2 - a*b)*log(abs(sin(d*x + c) + 1))/(
a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + a*b)*log(abs(sin(d*x
+ c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 8*(a^6*b*sin(d*x +
 c) + 5*a^4*b^3*sin(d*x + c) + 2*a^7 + 4*a^5*b^2)/((a^8 - 4*a^6*b^2 + 6*a^4
*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c) + a)) + 2*(3*a^6*sin(d*x + c)^4 + 1
5*a^4*b^2*sin(d*x + c)^4 - 9*a^5*b*sin(d*x + c)^3 + 10*a^3*b^3*sin(d*x + c)
^3 - a*b^5*sin(d*x + c)^3 - 2*a^6*sin(d*x + c)^2 - 28*a^4*b^2*sin(d*x + c)^
2 - 8*a^2*b^4*sin(d*x + c)^2 + 2*b^6*sin(d*x + c)^2 + 7*a^5*b*sin(d*x + c)
- 6*a^3*b^3*sin(d*x + c) - a*b^5*sin(d*x + c) + 12*a^4*b^2 + 7*a^2*b^4 - b^
6)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^2)
)/d
```

Mupad [B]
time = 7.78, size = 755, normalized size = 3.12

$$\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,\left(a^6+5\,a^4\,b^2\right)}{d\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,\left(\frac{1}{{\left(a+b\right)}^2}-\frac{7\,b}{4\,{\left(a+b\right)}^3}+\frac{3\,b^2}{4\,{\left(a+b\right)}^4}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\,\left(\frac{3\,b^2}{4\,{\left(a-b\right)}^4}+\frac{7\,b}{4\,{\left(a-b\right)}^3}+\frac{1}{{\left(a-b\right)}^2}\right)}{d}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,\left(2\,a^3+a\,b^2\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,\left(a\,b^2-2\,a^3\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,\left(a\,b^2-2\,a^3\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\,\left(2\,a^3+a\,b^2\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\,\left(11\,a^4+a^2\,b^2\right)}{2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{8\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,\left(2\,a^4+a^2\,b^2\right)}{\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{8\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\,\left(2\,a^4+a^2\,b^2\right)}{\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\,\left(13\,a^4+31\,a^2\,b^2-8\,b^4\right)}{\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(11\,a^4+a^2\,b^2\right)}{2\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-8\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+12\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-8\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^6 + 5*a^4*b^2)
)/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (log(tan(c/2 + (d*x
)/2) - 1)*(1/(a + b)^2 - (7*b)/(4*(a + b)^3) + (3*b^2)/(4*(a + b)^4)))/d -
(log(tan(c/2 + (d*x)/2) + 1)*((3*b^2)/(4*(a - b)^4) + (7*b)/(4*(a - b)^3) +
 1/(a - b)^2))/d - ((tan(c/2 + (d*x)/2)^2*(a*b^2 + 2*a^3))/(a^4 + b^4 - 2*a
^2*b^2) + (3*tan(c/2 + (d*x)/2)^4*(a*b^2 - 2*a^3))/(a^4 + b^4 - 2*a^2*b^2)
+ (3*tan(c/2 + (d*x)/2)^6*(a*b^2 - 2*a^3))/(a^4 + b^4 - 2*a^2*b^2) + (tan(c
/2 + (d*x)/2)^8*(a*b^2 + 2*a^3))/(a^4 + b^4 - 2*a^2*b^2) - (b*tan(c/2 + (d*
x)/2)^9*(11*a^4 + a^2*b^2))/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (8*b*
tan(c/2 + (d*x)/2)^3*(2*a^4 + a^2*b^2))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2
)) + (8*b*tan(c/2 + (d*x)/2)^7*(2*a^4 + a^2*b^2))/((a^2 - b^2)*(a^4 + b^4 -
 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)^5*(13*a^4 - 8*b^4 + 31*a^2*b^2))/((a^2
 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)*(11*a^4 + a^2*b^2)
)/(2*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) -
 3*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^4 + 2*a*tan(c/2 + (d*x)/
2)^6 - 3*a*tan(c/2 + (d*x)/2)^8 + a*tan(c/2 + (d*x)/2)^10 - 8*b*tan(c/2 + (
d*x)/2)^3 + 12*b*tan(c/2 + (d*x)/2)^5 - 8*b*tan(c/2 + (d*x)/2)^7 + 2*b*tan(
c/2 + (d*x)/2)^9))
```

3.182 $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{a\log(1-\sin(c+dx))}{2(a+b)^3d}+\frac{a\log(1+\sin(c+dx))}{2(a-b)^3d}-\frac{a^2(a^2+3b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}+\frac{a^3}{(a^2-b^2)^2\,d(a+b\sin(c}$$

```
[Out] 1/2*a*ln(1-sin(d*x+c))/(a+b)^3/d+1/2*a*ln(1+sin(d*x+c))/(a-b)^3/d-a^2*(a^2+
3*b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+a^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))+
1/2*sec(d*x+c)^2*(a^2+b^2-2*a*b*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]

time = 0.24, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2800, 1661, 1643}

$$-\frac{a^2(a^2+3b^2)\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^3}+\frac{\sec^2(c+dx)\,(a^2-2ab\sin(c+dx)+b^2)}{2d\,(a^2-b^2)^2}+\frac{a^3}{d\,(a^2-b^2)^2\,(a+b\sin(c+dx))}+\frac{a\log(1-\sin(c+dx))}{2d(a+b)^3}+\frac{a\log(\sin(c+dx)+1)}{2d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*Log[1 - Sin[c + d*x]])/(2*(a + b)^3*d) + (a*Log[1 + Sin[c + d*x]])/(2*(a
 - b)^3*d) - (a^2*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d)
+ a^3/((a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(a^2 + b^2 -
 2*a*b*Sin[c + d*x]))/(2*(a^2 - b^2)^2*d)
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^2(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^4}{\left(a^2-b^2\right)^2}-\frac{2a^2b^2x}{a^2-b^2}-\frac{2ab^4x^2}{\left(a^2-b^2\right)^2}}{(a+x)^2(b^2-x^2)}\,dx\right.}{2b^2d}$$

$$= \frac{\sec^2(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int \left(-\frac{ab^2}{(a+b)^3(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)}\right.\right.}{}$$

$$= \frac{a\log(1-\sin(c+dx))}{2(a+b)^3d} + \frac{a\log(1+\sin(c+dx))}{2(a-b)^3d} - \frac{a^2(a^2+3b^2)\log(a+b\sin(c}{\left(a^2-b^2\right)^3 d}$$

Mathematica [A]
time = 0.53, size = 145, normalized size = 0.90

$$\frac{\frac{2a\log(1-\sin(c+dx))}{(a+b)^3}+\frac{2a\log(1+\sin(c+dx))}{(a-b)^3}-\frac{4a^2(a^2+3b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^3}-\frac{1}{(a+b)^2(-1+\sin(c+dx))}+\frac{1}{(a-b)^2(1+\sin(c+dx))}+\frac{4a^3}{(a^2-b^2)^2(a+b\sin(c+dx))}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a*Log[1 - Sin[c + d*x]])/(a + b)^3 + (2*a*Log[1 + Sin[c + d*x]])/(a - b
)^3 - (4*a^2*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^3 - 1/((a +
 b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)^2*(1 + Sin[c + d*x])) + (4*a^3)/((a
^2 - b^2)^2*(a + b*Sin[c + d*x])))/(4*d)
```

Maple [A]
time = 0.46, size = 143, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{a^3}{(a+b)^2(a-b)^2(a+b\sin(dx+c))}-\frac{a^2\left(a^2+3b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{4(a-b)^2(1+\sin(dx+c))}+\frac{a\ln(1+\sin(dx+c))}{2(a-b)^3}-\frac{1}{4(a+b)^2(\sin(dx+c)}}{d}$
default	$\frac{\frac{a^3}{(a+b)^2(a-b)^2(a+b\sin(dx+c))}-\frac{a^2\left(a^2+3b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{4(a-b)^2(1+\sin(dx+c))}+\frac{a\ln(1+\sin(dx+c))}{2(a-b)^3}-\frac{1}{4(a+b)^2(\sin(dx+c)}}{d}$

risch	$-\frac{iax}{a^3+3a^2b+3a\,b^2+b^3}-\frac{iac}{(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{iax}{a^3-3a^2b+3a\,b^2-b^3}-\frac{iac}{d(a^3-3a^2b+3a\,b^2-b^3)}+\frac{2ia^4x}{a^6-3a^4b^2+3a^2\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))-a^2*(a^2+3*b^2)/(a+b)^3/(a-b)^3*l
n(a+b*sin(d*x+c))+1/4/(a-b)^2/(1+sin(d*x+c))+1/2*a/(a-b)^3*ln(1+sin(d*x+c))
-1/4/(a+b)^2/(sin(d*x+c)-1)+1/2*a/(a+b)^3*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.33, size = 274, normalized size = 1.70

$$-\frac{\frac{2\,(a^4+3\,a^2b^2)\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{a\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{a\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{3\,a^3+ab^2-2\,(a^3+ab^2)\sin(dx+c)^2-(a^2b-b^3)\sin(dx+c)}{a^5-2\,a^3b^2+ab^4-(a^4b-2\,a^2b^3+b^5)\sin(dx+c)^3-(a^5-2\,a^3b^2+ab^4)\sin(dx+c)^2+(a^4b-2\,a^2b^3+b^5)\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^4 + 3*a^2*b^2)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6) - a*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - a*lo
g(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^3 + a*b^2 - 2*(a
^3 + a*b^2)*sin(d*x + c)^2 - (a^2*b - b^3)*sin(d*x + c))/(a^5 - 2*a^3*b^2 +
 a*b^4 - (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)^3 - (a^5 - 2*a^3*b^2 + a*b^
4)*sin(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(159) = 318$.
time = 0.45, size = 388, normalized size = 2.41

$$\frac{a^5-2\,a^3b^2+ab^4+2\,(a^5-ab^4)\cos(dx+c)^2-2\,((a^4b+3\,a^2b^3)\cos(dx+c)^2\sin(dx+c)+(a^5+3\,a^3b^2)\cos(dx+c)^2)\log(b\sin(dx+c)+a)+((a^4b+3\,a^3b^2+3\,a^2b^3+ab^4)\cos(dx+c)^2\sin(dx+c)+(a^5+3\,a^4b+3\,a^3b^2+a^2b^3)\cos(dx+c)^2)\log(\sin(dx+c)+1)+((a^4b-3\,a^3b^2+3\,a^2b^3-ab^4)\cos(dx+c)^2\sin(dx+c)+(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\cos(dx+c)^2)\log(-\sin(dx+c)+1)-(a^4b-2\,a^2b^3+b^5)\sin(dx+c)}{2\,((a^6b-3\,a^4b^3+3\,a^2b^5-b^7)d\cos(dx+c)^2\sin(dx+c)+(a^7-3\,a^5b^2+3\,a^3b^4-ab^6)d\cos(dx+c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^5 - 2*a^3*b^2 + a*b^4 + 2*(a^5 - a*b^4)*cos(d*x + c)^2 - 2*((a^4*b +
 3*a^2*b^3)*cos(d*x + c)^2*sin(d*x + c) + (a^5 + 3*a^3*b^2)*cos(d*x + c)^2)
*log(b*sin(d*x + c) + a) + ((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(d*x
 + c)^2*sin(d*x + c) + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(d*x + c)^2
)*log(sin(d*x + c) + 1) + ((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*cos(d*x
+ c)^2*sin(d*x + c) + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x + c)^2)
*log(-sin(d*x + c) + 1) - (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c))/((a^6*b -
 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)^2*sin(d*x + c) + (a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 6.88, size = 248, normalized size = 1.54

$$-\frac{\frac{2\,(a^4b+3\,a^2b^3)\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{a\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{a\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,a^3\sin(dx+c)^2+2\,ab^2\sin(dx+c)^2+a^2b\sin(dx+c)-b^3\sin(dx+c)-3\,a^3-ab^2}{(a^4-2\,a^2b^2+b^4)\left(b\sin(dx+c)^3+a\sin(dx+c)^2-b\sin(dx+c)-a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^4*b + 3*a^2*b^3)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3
 + 3*a^2*b^5 - b^7) - a*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2
 - b^3) - a*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (2
*a^3*sin(d*x + c)^2 + 2*a*b^2*sin(d*x + c)^2 + a^2*b*sin(d*x + c) - b^3*sin
(d*x + c) - 3*a^3 - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)^3 + a*s
in(d*x + c)^2 - b*sin(d*x + c) - a)))/d
```

Mupad [B]
time = 7.25, size = 351, normalized size = 2.18

$$\frac{\frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^2-b^2}+\frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{a^2-b^2}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^2+b^2)}{(a^2-b^2)^2}-\frac{4\,a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a^4-2\,a^2b^2+b^4}-\frac{4\,a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{(a^2-b^2)^2}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}+\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{d\,(a-b)^3}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)(a^4+3\,a^2b^2)}{d\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)}+\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{d\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*a*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2) + (2*a*tan(c/2 + (d*x)/2)^4)/(a^2 -
 b^2) + (4*b*tan(c/2 + (d*x)/2)^3*(a^2 + b^2))/(a^2 - b^2)^2 - (4*a^2*b*tan
(c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*a^2*b^2) - (4*a^2*b*tan(c/2 + (d*x)/2))/(
a^2 - b^2)^2)/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2 - a*t
an(c/2 + (d*x)/2)^4 + a*tan(c/2 + (d*x)/2)^6 - 4*b*tan(c/2 + (d*x)/2)^3 + 2
*b*tan(c/2 + (d*x)/2)^5)) + (a*log(tan(c/2 + (d*x)/2) + 1))/(d*(a - b)^3) -
 (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4 + 3*a^2*b^2
))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a*log(tan(c/2 + (d*x)/2) - 1)
)/(d*(a + b)^3)
```

3.183 $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)^2d}-\frac{\log(1+\sin(c+dx))}{2(a-b)^2d}+\frac{(a^2+b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d}-\frac{a}{(a^2-b^2)\,d(a+b\sin(c+dx)}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^2/d-1/2*ln(1+sin(d*x+c))/(a-b)^2/d+(a^2+b^2)*ln
(a+b*sin(d*x+c))/(a^2-b^2)^2/d-a/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2800, 815}

$$-\frac{a}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{\left(a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^2}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)^2*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)
^2*d) + ((a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - a/((a^2 -
 b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x}{(a+x)^2(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{2(a+b)^2(b-x)} + \frac{a}{(a-b)(a+b)(a+x)^2} + \frac{a^2+b^2}{(a-b)^2(a+b)^2(a+x)} - \frac{1}{2(a-b)^2(b+x)}\right)\,dx, x\right.}{d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} - \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} + \frac{(a^2+b^2)\log(a+b\sin(c+dx}{(a^2-b^2)^2\,d}$$

Mathematica [A]
time = 0.21, size = 162, normalized size = 1.49

$$-\frac{a((a-b)^2\log(1-\sin(c+dx)) + (a+b)^2\log(1+\sin(c+dx)) - 2(-a^2+b^2+(a^2+b^2)\log(a+b\sin(c+dx)))) + b((a-b)^2\log(1-\sin(c+dx)) + (a+b)^2\log(1+\sin(c+dx)) - 2(a^2+b^2)\log(a+b\sin(c+dx)))\sin(c+dx)}{2(a-b)^2(a+b)^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a*((a - b)^2*Log[1 - Sin[c + d*x]] + (a + b)^2*Log[1 + Sin[c + d*x]]
- 2*(-a^2 + b^2 + (a^2 + b^2)*Log[a + b*Sin[c + d*x]])) + b*((a - b)^2*Log[
1 - Sin[c + d*x]] + (a + b)^2*Log[1 + Sin[c + d*x]] - 2*(a^2 + b^2)*Log[a +
 b*Sin[c + d*x]])*Sin[c + d*x])/((a - b)^2*(a + b)^2*d*(a + b*Sin[c + d*x])
)
```

Maple [A]
time = 0.36, size = 98, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{a}{(a+b)(a-b)(a+b\sin(dx+c))}+\frac{\left(a^2+b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}-\frac{\ln(1+\sin(dx+c))}{2(a-b)^2}-\frac{\ln(\sin(dx+c)-1)}{2(a+b)^2}}{d}$
default	$\frac{-\frac{a}{(a+b)(a-b)(a+b\sin(dx+c))}+\frac{\left(a^2+b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}-\frac{\ln(1+\sin(dx+c))}{2(a-b)^2}-\frac{\ln(\sin(dx+c)-1)}{2(a+b)^2}}{d}$
risch	$\frac{ix}{a^2+2ab+b^2}+\frac{ic}{(a^2+2ab+b^2)d}+\frac{ix}{a^2-2ab+b^2}+\frac{ic}{d(a^2-2ab+b^2)}-\frac{2ia^2x}{a^4-2a^2b^2+b^4}-\frac{2ia^2c}{d(a^4-2a^2b^2+b^4)}-\frac{2ib}{a^4-2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/(a+b)/(a-b)/(a+b*sin(d*x+c))+(a^2+b^2)/(a+b)^2/(a-b)^2*ln(a+b*sin(d
*x+c))-1/2/(a-b)^2*ln(1+sin(d*x+c))-1/2/(a+b)^2*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 124, normalized size = 1.14

$$\frac{\frac{2\,(a^2+b^2)\,\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}-\frac{2\,a}{a^3-ab^2+(a^2b-b^3)\sin(dx+c)}-\frac{\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^2 + b^2)*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - 2*a/(a
^3 - a*b^2 + (a^2*b - b^3)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^2 - 2*a
*b + b^2) - log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

Fricas [A]
time = 0.40, size = 195, normalized size = 1.79

$$-\frac{2\,a^3-2\,ab^2-2\,(a^3+ab^2+(a^2b+b^3)\sin(dx+c))\log(b\sin(dx+c)+a)+(a^3+2\,a^2b+ab^2+(a^2b+2\,ab^2+b^3)\sin(dx+c))\log(\sin(dx+c)+1)+(a^3-2\,a^2b+ab^2+(a^2b-2\,ab^2+b^3)\sin(dx+c))\log(-\sin(dx+c)+1)}{2\,((a^4b-2\,a^2b^3+b^5)d\sin(dx+c)+(a^5-2\,a^3b^2+ab^4)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3 - 2*a*b^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*sin(d*x + c))*log(b*
sin(d*x + c) + a) + (a^3 + 2*a^2*b + a*b^2 + (a^2*b + 2*a*b^2 + b^3)*sin(d*
x + c))*log(sin(d*x + c) + 1) + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 +
 b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/((a^4*b - 2*a^2*b^3 + b^5)*d*si
n(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 5.15, size = 156, normalized size = 1.43

$$\frac{\frac{2\,(a^2b+b^3)\,\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5}-\frac{\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}-\frac{\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}-\frac{2\,(a^2b\sin(dx+c)+b^3\sin(dx+c)+2\,a^3)}{(a^4-2\,a^2b^2+b^4)(b\sin(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^2*b + b^3)*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5)
 - log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - log(abs(sin(d*x + c) -
1))/(a^2 + 2*a*b + b^2) - 2*(a^2*b*sin(d*x + c) + b^3*sin(d*x + c) + 2*a^3)
/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c) + a)))/d
```

Mupad [B]
time = 6.76, size = 158, normalized size = 1.45

$$\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,(a^2+b^2)}{d\,(a^4-2a^2b^2+b^4)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{d\,(a-b)^2}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{d\,(a+b)^2}+\frac{2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,(a^2-b^2)\,\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 + b^2))/(d*(
a^4 + b^4 - 2*a^2*b^2)) - log(tan(c/2 + (d*x)/2) + 1)/(d*(a - b)^2) - log(t
an(c/2 + (d*x)/2) - 1)/(d*(a + b)^2) + (2*b*tan(c/2 + (d*x)/2))/(d*(a^2 - b
^2)*(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))
```

3.184 $\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=53

$$\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(a+b\sin(c+dx))}{a^2d}+\frac{1}{ad(a+b\sin(c+dx))}$$

[Out] ln(sin(d*x+c))/a^2/d-ln(a+b*sin(d*x+c))/a^2/d+1/a/d/(a+b*sin(d*x+c))

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2800, 46}

$$-\frac{\log(a+b\sin(c+dx))}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}+\frac{1}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x])^2,x]

[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[a + b*Sin[c + d*x]]/(a^2*d) + 1/(a*d*(a + b*Sin[c + d*x]))

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx&=\frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx,x,b\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{1}{a(a+x)^2}-\frac{1}{a^2(a+x)}\right)\,dx,x,b\sin(c+dx)\right)}{d}\\&=\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(a+b\sin(c+dx))}{a^2d}+\frac{1}{ad(a+b\sin(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 42, normalized size = 0.79

$$\frac{\log(\sin(c+dx)) - \log(a+b\sin(c+dx)) + \frac{a}{a+b\sin(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]] + a/(a + b*Sin[c + d*x]))/(a^2
*d)
```

Maple [A]
time = 0.18, size = 49, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\frac{\ln(\sin(dx+c))}{a^2}-\frac{\ln(a+b\sin(dx+c))}{a^2}+\frac{1}{a(a+b\sin(dx+c))}}{d}$	49
default	$\frac{\frac{\ln(\sin(dx+c))}{a^2}-\frac{\ln(a+b\sin(dx+c))}{a^2}+\frac{1}{a(a+b\sin(dx+c))}}{d}$	49
risch	$\frac{2i\mathrm{e}^{i(dx+c)}}{da\left(b\,\mathrm{e}^{2i(dx+c)}-b+2ia\,\mathrm{e}^{i(dx+c)}\right)}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{a^2d}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2ia\,\mathrm{e}^{i(dx+c)}}{b}-1\right)}{a^2d}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^2*ln(sin(d*x+c))-1/a^2*ln(a+b*sin(d*x+c))+1/a/(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 47, normalized size = 0.89

$$\frac{\frac{1}{ab\sin(dx+c)+a^2} - \frac{\log(b\sin(dx+c)+a)}{a^2} + \frac{\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a*b*sin(d*x + c) + a^2) - log(b*sin(d*x + c) + a)/a^2 + log(sin(d*x + c
))/a^2)/d
```

Fricas [A]
time = 0.37, size = 69, normalized size = 1.30

$$-\frac{(b\sin(dx+c)+a)\log(b\sin(dx+c)+a) - (b\sin(dx+c)+a)\log\left(-\frac{1}{2}\sin(dx+c)\right) - a}{a^2bd\sin(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((b*sin(d*x + c) + a)*log(b*sin(d*x + c) + a) - (b*sin(d*x + c) + a)*log(-
1/2*sin(d*x + c)) - a)/(a^2*b*d*sin(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 4.41, size = 51, normalized size = 0.96

$$\frac{b\left(\frac{\log\left(\left|-\frac{a}{b\sin(dx+c)+a}+1\right|\right)}{a^2b}+\frac{1}{(b\sin(dx+c)+a)ab}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] b*(log(abs(-a/(b*sin(d*x + c) + a) + 1))/(a^2*b) + 1/((b*sin(d*x + c) + a)*
a*b))/d
```

Mupad [B]
time = 6.38, size = 105, normalized size = 1.98

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2 d}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}{a^2 d}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^3+2\,b\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/
2 + (d*x)/2)^2)/(a^2*d) - (2*b*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d*x)/
2)^2 + a^3 + 2*a^2*b*tan(c/2 + (d*x)/2)))
```

3.185 $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2} dx$

Optimal. Leaf size=114

$$\frac{2b\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^2d}-\frac{(a^2-3b^2)\log(\sin(c+dx))}{a^4d}+\frac{(a^2-3b^2)\log(a+b\sin(c+dx))}{a^4d}-\frac{a^2-}{a^3d(a+b\sin}$$

```
[Out] 2*b*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^2/d-(a^2-3*b^2)*ln(sin(d*x+c))/a^4/
d+(a^2-3*b^2)*ln(a+b*sin(d*x+c))/a^4/d+(-a^2+b^2)/a^3/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 908}

$$\frac{2b\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^2d}-\frac{(a^2-3b^2)\log(\sin(c+dx))}{a^4d}+\frac{(a^2-3b^2)\log(a+b\sin(c+dx))}{a^4d}-\frac{a^2-b^2}{a^3d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^2*d) - ((a^2 - 3*b^2)*Log[
Sin[c + d*x]])/(a^4*d) + ((a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^4*d) -
(a^2 - b^2)/(a^3*d*(a + b*Sin[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2x^3} - \frac{2b^2}{a^3x^2} + \frac{-a^2+3b^2}{a^4x} + \frac{a^2-b^2}{a^3(a+x)^2} + \frac{a^2-3b^2}{a^4(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{2b\csc(c+dx)}{a^3d} - \frac{\csc^2(c+dx)}{2a^2d} - \frac{(a^2-3b^2)\log(\sin(c+dx))}{a^4d} + \frac{(a^2-3b^2)\log(a}{a^4}$$

Mathematica [A]
time = 0.42, size = 96, normalized size = 0.84

$$-\frac{-4ab\csc(c+dx) + a^2\csc^2(c+dx) + 2(a^2-3b^2)\log(\sin(c+dx)) - 2(a^2-3b^2)\log(a+b\sin(c+dx)) + \frac{2a(a-b)(a+b)}{a+b\sin(c+dx)}}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(-4*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - 3*b^2)*Log[Sin[c
+ d*x]] - 2*(a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]] + (2*a*(a - b)*(a + b))/(
a + b*Sin[c + d*x]))/(a^4*d)
```

Maple [A]
time = 0.36, size = 105, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{(a^2-3b^2)\ln(a+b\sin(dx+c))}{a^4} - \frac{a^2-b^2}{a^3(a+b\sin(dx+c))} - \frac{1}{2a^2\sin(dx+c)^2} + \frac{(-a^2+3b^2)\ln(\sin(dx+c))}{a^4} + \frac{2b}{a^3\sin(dx+c)}}{d}$
default	$\frac{\frac{(a^2-3b^2)\ln(a+b\sin(dx+c))}{a^4} - \frac{a^2-b^2}{a^3(a+b\sin(dx+c))} - \frac{1}{2a^2\sin(dx+c)^2} + \frac{(-a^2+3b^2)\ln(\sin(dx+c))}{a^4} + \frac{2b}{a^3\sin(dx+c)}}{d}$
risch	$-\frac{2i(-3iab\,e^{4i(dx+c)} - 3b^2e^{5i(dx+c)} + 3iab\,e^{2i(dx+c)} - 4a^2e^{3i(dx+c)} + 6b^2e^{3i(dx+c)} - 3\,e^{i(dx+c)}b^2 + a^2e^{5i(dx+c)} + a^2e^{i(dx+c)})}{(e^{2i(dx+c)}-1)^2(b\,e^{2i(dx+c)} - b + 2ia\,e^{i(dx+c)})\,d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((a^2-3*b^2)/a^4*ln(a+b*sin(d*x+c))-(a^2-b^2)/a^3/(a+b*sin(d*x+c))-1/2/
a^2/sin(d*x+c)^2+(-a^2+3*b^2)/a^4*ln(sin(d*x+c))+2/a^3*b/sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 116, normalized size = 1.02

$$\frac{\frac{3\,ab\sin(dx+c) - 2\,(a^2-3\,b^2)\sin(dx+c)^2 - a^2}{a^3b\sin(dx+c)^3 + a^4\sin(dx+c)^2} + \frac{2\,(a^2-3\,b^2)\log(b\sin(dx+c)+a)}{a^4} - \frac{2\,(a^2-3\,b^2)\log(\sin(dx+c))}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((3*a*b*sin(d*x + c) - 2*(a^2 - 3*b^2)*sin(d*x + c)^2 - a^2)/(a^3*b*sin
(d*x + c)^3 + a^4*sin(d*x + c)^2) + 2*(a^2 - 3*b^2)*log(b*sin(d*x + c) + a)
/a^4 - 2*(a^2 - 3*b^2)*log(sin(d*x + c))/a^4)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(112) = 224$.
time = 0.39, size = 259, normalized size = 2.27

$$-\frac{3\,a^{2}b\sin\left(dx+c\right)-3\,a^{3}+6\,ab^{2}+2\,\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}+2\,\left(a^{3}-3\,ab^{2}-\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}+\left(a^{2}b-3\,b^{3}-\left(a^{2}b-3\,b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)-2\,\left(a^{3}-3\,ab^{2}-\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}+\left(a^{2}b-3\,b^{3}-\left(a^{2}b-3\,b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\sin\left(dx+c\right)\right)}{2\,\left(a^{5}d\cos\left(dx+c\right)^{2}-a^{5}d+\left(a^{4}bd\cos\left(dx+c\right)^{2}-a^{4}bd\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^2*b*sin(d*x + c) - 3*a^3 + 6*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x +
c)^2 + 2*(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - 3*b^3 -
 (a^2*b - 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) - 2*
(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - 3*b^3 - (a^2*b -
 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x + c)))/(a^5*d*cos(d*
x + c)^2 - a^5*d + (a^4*b*d*cos(d*x + c)^2 - a^4*b*d)*sin(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^{3}\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

Giac [**A**]
time = 5.94, size = 165, normalized size = 1.45

$$-\frac{\frac{2\,\left(a^{2}-3\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^{4}}-\frac{2\,\left(a^{2}b-3\,b^{3}\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{4}b}+\frac{2\,\left(a^{2}b\sin\left(dx+c\right)-3\,b^{3}\sin\left(dx+c\right)+2\,a^{3}-4\,ab^{2}\right)}{\left(b\sin\left(dx+c\right)+a\right)a^{4}}-\frac{3\,a^{2}\sin\left(dx+c\right)^{2}-9\,b^{2}\sin\left(dx+c\right)^{2}+4\,ab\sin\left(dx+c\right)-a^{2}}{a^{4}\sin\left(dx+c\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - 3*b^2)*log(abs(sin(d*x + c)))/a^4 - 2*(a^2*b - 3*b^3)*log(ab
s(b*sin(d*x + c) + a))/(a^4*b) + 2*(a^2*b*sin(d*x + c) - 3*b^3*sin(d*x + c)
 + 2*a^3 - 4*a*b^2)/((b*sin(d*x + c) + a)*a^4) - (3*a^2*sin(d*x + c)^2 - 9*
b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) - a^2)/(a^4*sin(d*x + c)^2))/d
```

Mupad [B]
time = 6.38, size = 235, normalized size = 2.06

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{2}-8\,b^2\right)+\frac{a^2}{2}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,a^2\,b-2\,b^3\right)}{a}-3\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\,b\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-3\,b^2\right)}{a^4\,d}+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^2-3\,b^2\right)}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(a^3*d) - (tan(c/2 + (d*x)/2)^2*(a^2/2 - 8*b^2) + a^
2/2 - (4*tan(c/2 + (d*x)/2)^3*(3*a^2*b - 2*b^3))/a - 3*a*b*tan(c/2 + (d*x)/
2))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 + 4*a^4*tan(c/2 + (d*x)/2)^4 + 8*a^3*b*t
an(c/2 + (d*x)/2)^3)) - tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (log(tan(c/2 + (d*
x)/2))*(a^2 - 3*b^2))/(a^4*d) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2
 + (d*x)/2)^2)*(a^2 - 3*b^2))/(a^4*d)
```

3.186 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$-\frac{4b(a^2-b^2)\csc(c+dx)}{a^5d}+\frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d}+\frac{2b\csc^3(c+dx)}{3a^3d}-\frac{\csc^4(c+dx)}{4a^2d}+\frac{(a^4-6a^2b^2+5b^4)\log}{a^6d}$$

```
[Out] -4*b*(a^2-b^2)*csc(d*x+c)/a^5/d+1/2*(2*a^2-3*b^2)*csc(d*x+c)^2/a^4/d+2/3*b*
csc(d*x+c)^3/a^3/d-1/4*csc(d*x+c)^4/a^2/d+(a^4-6*a^2*b^2+5*b^4)*ln(sin(d*x+
c))/a^6/d-(a^4-6*a^2*b^2+5*b^4)*ln(a+b*sin(d*x+c))/a^6/d+(a^2-b^2)^2/a^5/d/
(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 908}

$$\frac{2b\csc^3(c+dx)}{3a^3d}-\frac{\csc^4(c+dx)}{4a^2d}+\frac{(a^2-b^2)^2}{a^5d(a+b\sin(c+dx))}-\frac{4b(a^2-b^2)\csc(c+dx)}{a^5d}+\frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d}+\frac{(a^4-6a^2b^2+5b^4)\log(\sin(c+dx))}{a^6d}-\frac{(a^4-6a^2b^2+5b^4)\log(a+b\sin(c+dx))}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*b*(a^2 - b^2)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) + ((a
^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2 + 5*
b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin[c +
 d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^5(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^5} - \frac{2b^4}{a^3x^4} + \frac{-2a^2b^2+3b^4}{a^4x^3} + \frac{4b^2(a^2-b^2)}{a^5x^2} + \frac{a^4-6a^2b^2+5b^4}{a^6x} - \frac{(a^2-b^2)^2}{a^5(a+x)^2} + \frac{-a^4+6}{a^6(}\right.\right.}{d}$$
$$= -\frac{4b(a^2-b^2)\csc(c+dx)}{a^5d} + \frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c}{4a}$$

Mathematica [A]
time = 6.12, size = 187, normalized size = 0.99

$$-\frac{4(a-b)b(a+b)\csc(c+dx)}{a^5d} + \frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c+dx)}{4a^2d} + \frac{(a^4-6a^2b^2+5b^4)\log(\sin(c+dx))}{a^6d} - \frac{(a^4-6a^2b^2+5b^4)\log(a+b\sin(c+dx))}{a^6d} + \frac{(a^2-b^2)^2}{a^5d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*(a - b)*b*(a + b)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]
^2)/(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) +
 ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2
+ 5*b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin
[c + d*x]))
```

Maple [A]
time = 0.43, size = 172, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{\left(a^4-6a^2b^2+5b^4\right)\ln(a+b\sin(dx+c))}{a^6}+\frac{a^4-2a^2b^2+b^4}{a^5(a+b\sin(dx+c))}-\frac{1}{4a^2\sin(dx+c)^4}-\frac{-2a^2+3b^2}{2a^4\sin(dx+c)^2}+\frac{\left(a^4-6a^2b^2+5b^4\right)\ln(\sin(dx+c))}{a^6}+\frac{}{3}}{d}$
default	$\frac{-\frac{\left(a^4-6a^2b^2+5b^4\right)\ln(a+b\sin(dx+c))}{a^6}+\frac{a^4-2a^2b^2+b^4}{a^5(a+b\sin(dx+c))}-\frac{1}{4a^2\sin(dx+c)^4}-\frac{-2a^2+3b^2}{2a^4\sin(dx+c)^2}+\frac{\left(a^4-6a^2b^2+5b^4\right)\ln(\sin(dx+c))}{a^6}+\frac{}{3}}{d}$
risch	$2i\left(-18\,e^{i(dx+c)}b^2a^2+15\,e^{i(dx+c)}b^4+3\,e^{i(dx+c)}a^4-18a^2b^2e^{9i(dx+c)}+82a^2b^2e^{7i(dx+c)}-128a^2b^2e^{5i(dx+c)}+82a^2b^2e^{3i(dx+}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(a^4-6*a^2*b^2+5*b^4)/a^6*ln(a+b*sin(d*x+c))+(a^4-2*a^2*b^2+b^4)/a^5/
(a+b*sin(d*x+c))-1/4/a^2/sin(d*x+c)^4-1/2*(-2*a^2+3*b^2)/a^4/sin(d*x+c)^2+(
a^4-6*a^2*b^2+5*b^4)/a^6*ln(sin(d*x+c))+2/3/a^3*b/sin(d*x+c)^3-4*b*(a^2-b^2
)/a^5/sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 189, normalized size = 1.01

$$\frac{\frac{5\,a^{3}b\sin\left(dx+c\right)+12\,\left(a^{4}-6\,a^{2}b^{2}+5\,b^{4}\right)\sin\left(dx+c\right)^{4}-3\,a^{4}-6\,\left(6\,a^{3}b-5\,ab^{3}\right)\sin\left(dx+c\right)^{3}+2\,\left(6\,a^{4}-5\,a^{2}b^{2}\right)\sin\left(dx+c\right)^{2}}{a^{5}b\sin\left(dx+c\right)^{5}+a^{6}\sin\left(dx+c\right)^{4}}-\frac{12\,\left(a^{4}-6\,a^{2}b^{2}+5\,b^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}}+\frac{12\,\left(a^{4}-6\,a^{2}b^{2}+5\,b^{4}\right)\log\left(\sin\left(dx+c\right)\right)}{a^{6}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*((5*a^3*b*sin(d*x + c) + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*sin(d*x + c)^4 -
 3*a^4 - 6*(6*a^3*b - 5*a*b^3)*sin(d*x + c)^3 + 2*(6*a^4 - 5*a^2*b^2)*sin(d
*x + c)^2)/(a^5*b*sin(d*x + c)^5 + a^6*sin(d*x + c)^4) - 12*(a^4 - 6*a^2*b^
2 + 5*b^4)*log(b*sin(d*x + c) + a)/a^6 + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(s
in(d*x + c))/a^6)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(182) = 364.
time = 0.39, size = 542, normalized size = 2.88

$$\frac{21\,a^{5}-82\,a^{3}b^{2}+60\,ab^{4}+12\,\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{4}-2\,\left(18\,a^{5}-77\,a^{3}b^{2}+60\,ab^{4}\right)\cos\left(dx+c\right)^{2}-12\,\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}+\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{4}-2\,\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{2}+\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}+\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}\right)\cos\left(dx+c\right)^{4}-2\,\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)+12\,\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}+\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{4}-2\,\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{2}+\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}+\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}\right)\cos\left(dx+c\right)^{4}-2\,\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(31\,a^{4}b-30\,a^{2}b^{3}-6\,\left(6\,a^{4}b-5\,a^{2}b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{12\,\left(a^{7}d\cos\left(dx+c\right)^{4}-2\,a^{7}d\cos\left(dx+c\right)^{2}+a^{7}d+\left(a^{6}bd\cos\left(dx+c\right)^{4}-2\,a^{6}bd\cos\left(dx+c\right)^{2}+a^{6}bd\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(21*a^5 - 82*a^3*b^2 + 60*a*b^4 + 12*(a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d
*x + c)^4 - 2*(18*a^5 - 77*a^3*b^2 + 60*a*b^4)*cos(d*x + c)^2 - 12*(a^5 - 6
*a^3*b^2 + 5*a*b^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 -
6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b -
 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x
+ c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 12*(a^5 - 6*a^3*b^2 + 5*a*b
^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 6*a^3*b^2 + 5*a*
b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b - 6*a^2*b^3 + 5*b
^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-1/2*sin(d*x + c)) - (31*a^4*b - 30*a^2*b^3 - 6*(6*a^4*b - 5*a^2*
b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*d*cos(d*x
+ c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)^2 + a^6*b
*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{5}{\left(c + dx \right)}}{\left(a + b \sin{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 6.49, size = 278, normalized size = 1.48

$$\frac{\frac{12\,(a^4-6\,a^2b^2+5\,b^4)\log(|\sin(dx+c)|)}{a^6}-\frac{12\,(a^4b-6\,a^2b^3+5\,b^5)\log(|b\sin(dx+c)+a|)}{a^6b}+\frac{12\,(a^4b\sin(dx+c)-6\,a^2b^3\sin(dx+c)+5\,b^5\sin(dx+c)+2\,a^5-8\,a^3b^2+6\,ab^4)}{(b\sin(dx+c)+a)a^6}-\frac{25\,a^4\sin(dx+c)^4-150\,a^2b^2\sin(dx+c)^4+125\,b^4\sin(dx+c)^4+48\,a^3b\sin(dx+c)^3-48\,ab^3\sin(dx+c)^3-12\,a^4\sin(dx+c)^2+18\,a^2b^2\sin(dx+c)^2-8\,a^3b\sin(dx+c)+3\,a^4}{a^6\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(abs(sin(d*x + c)))/a^6 - 12*(a^4*b -
 6*a^2*b^3 + 5*b^5)*log(abs(b*sin(d*x + c) + a))/(a^6*b) + 12*(a^4*b*sin(d*
x + c) - 6*a^2*b^3*sin(d*x + c) + 5*b^5*sin(d*x + c) + 2*a^5 - 8*a^3*b^2 +
6*a*b^4)/((b*sin(d*x + c) + a)*a^6) - (25*a^4*sin(d*x + c)^4 - 150*a^2*b^2*
sin(d*x + c)^4 + 125*b^4*sin(d*x + c)^4 + 48*a^3*b*sin(d*x + c)^3 - 48*a*b^
3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 18*a^2*b^2*sin(d*x + c)^2 - 8*a^
3*b*sin(d*x + c) + 3*a^4)/(a^6*sin(d*x + c)^4))/d
```

Mupad [B]
time = 6.90, size = 439, normalized size = 2.34

$$\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(3\,a^4-62\,a^2\,b^2+64\,b^4\right)-\frac{a^4}{4}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{11\,a^4}{4}-\frac{10\,a^2\,b^2}{3}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(20\,a\,b^3-\frac{62\,a^3\,b}{3}\right)-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(60\,a^4\,b-96\,a^2\,b^3+32\,b^5\right)}{a}+\frac{5\,a^3\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6}}{d\,\left(16\,a^6\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+16\,a^6\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+32\,b\,a^5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\right)}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,a^2\,d}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{\frac{a^2}{16}+\frac{b^2}{8}}{a^4}+\frac{1}{8\,a^2}-\frac{b^2}{2\,a^4}\right)}{d}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{b\,\left(32\,a^2+64\,b^2\right)}{64\,a^5}-\frac{b}{4\,a^3}+\frac{4\,b\,\left(\frac{\frac{a^2}{8}+\frac{b^2}{4}}{a^4}+\frac{1}{4\,a^2}-\frac{b^2}{a^4}\right)}{a}\right)}{d}+\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^4-6\,a^2\,b^2+5\,b^4\right)}{a^6\,d}+\frac{b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{12\,a^3\,d}-\frac{\ln\left(a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^4-6\,a^2\,b^2+5\,b^4\right)}{a^6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(3*a^4 + 64*b^4 - 62*a^2*b^2) - a^4/4 + tan(c/2 + (d*
x)/2)^2*((11*a^4)/4 - (10*a^2*b^2)/3) + tan(c/2 + (d*x)/2)^3*(20*a*b^3 - (6
2*a^3*b)/3) - (tan(c/2 + (d*x)/2)^5*(60*a^4*b + 32*b^5 - 96*a^2*b^3))/a + (
5*a^3*b*tan(c/2 + (d*x)/2))/6)/(d*(16*a^6*tan(c/2 + (d*x)/2)^4 + 16*a^6*tan
(c/2 + (d*x)/2)^6 + 32*a^5*b*tan(c/2 + (d*x)/2)^5)) - tan(c/2 + (d*x)/2)^4/
(64*a^2*d) + (tan(c/2 + (d*x)/2)^2*((a^2/16 + b^2/8)/a^4 + 1/(8*a^2) - b^2/
(2*a^4)))/d - (tan(c/2 + (d*x)/2)*((b*(32*a^2 + 64*b^2))/(64*a^5) - b/(4*a^
3) + (4*b*((a^2/8 + b^2/4)/a^4 + 1/(4*a^2) - b^2/a^4))/a))/d + (log(tan(c/2
 + (d*x)/2))*(a^4 + 5*b^4 - 6*a^2*b^2))/(a^6*d) + (b*tan(c/2 + (d*x)/2)^3)/
(12*a^3*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4
 + 5*b^4 - 6*a^2*b^2))/(a^6*d)
```

3.187 $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=333

$$\frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}+\frac{8a^3b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}+\frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2}+\frac{\cos(c}{12(a+b)^2d(1}$$

```
[Out] 2*a^5*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/d+8*
a^3*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/d+
1/12*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))^2+1/12*cos(d*x+c)/(a+b)^2/d/(1-sin
(d*x+c))-1/4*(3*a+b)*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1/12*cos(d*x+c)/(a
-b)^2/d/(1+sin(d*x+c))^2-1/12*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))+1/4*(3*a-
b)*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))+a^4*b*cos(d*x+c)/(a^2-b^2)^3/d/(a+b*
sin(d*x+c))
```

Rubi [A]
time = 0.47, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2810, 2729, 2727, 2743, 12, 2739, 632, 210}

$$\frac{2a^5\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{a^4b\cos(c+dx)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{8a^3b^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{(3a+b)\cos(c+dx)}{4d(a+b)^3(1-\sin(c+dx))}+\frac{\cos(c+dx)}{12d(a+b)^2(1-\sin(c+dx))}+\frac{(3a-b)\cos(c+dx)}{4d(a-b)^3(\sin(c+dx)+1)}-\frac{\cos(c+dx)}{12d(a-b)^2(\sin(c+dx)+1)}+\frac{\cos(c+dx)}{12d(a+b)^2(1-\sin(c+dx))^2}-\frac{\cos(c+dx)}{12d(a-b)^2(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)
*d) + (8*a^3*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 -
b^2)^(7/2)*d) + Cos[c + d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])^2) + Cos[c
+ d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])) - ((3*a + b)*Cos[c + d*x])/(4*(a
 + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d
*x])^2) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d*x])) + ((3*a - b)*Cos
[c + d*x])/(4*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4*b*Cos[c + d*x])/((a^2
- b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(\frac{1}{4(a+b)^2(-1+\sin(c+dx))^2} + \frac{3a+b}{4(a+b)^3(-1+\sin(c+dx))} + \frac{\quad}{4(a-b)^2(\cdots}\right.$$

$$= \frac{\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{4(a-b)^2} - \frac{(3a-b)\int \frac{1}{1+\sin(c+dx)}\,dx}{4(a-b)^3} + \frac{\int \frac{1}{(-1+\sin(c+dx))^2}\,dx}{4(a+b)^2} + \frac{(3a+b)\cdots}{4\cdots}$$

$$= \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} - \frac{(3a+b)\cos(c+dx)}{4(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c\cdots}{12(a-b)^2d(1\cdots}$$

$$= \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))} - \frac{(3a+b)\,c\cdots}{4(a+b)^3d(1\cdots}$$

$$= \frac{8a^3b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} + \frac{\cos(\cdots}{12(a+b)^2d(\cdots}$$

$$= \frac{8a^3b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} + \frac{\cos(\cdots}{12(a+b)^2d(\cdots}$$

$$= \frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{8a^3b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c\cdots}{12(a+b)^2d(1\cdots}$$

Mathematica [A]
time = 1.24, size = 341, normalized size = 1.02

$$\frac{\frac{24a^3(a^2+4b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{4(4a+b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{1}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{4(-4a+b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{12a^4b\cos(c+dx)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((24*a^3*(a^2 + 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(7/2) + 1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
2*Sin[(c + d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) - (
4*(4*a + b)*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])) + (2*Sin[(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3) - 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(-4*a + b
)*Sin[(c + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12
*a^4*b*Cos[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])))/(12*d)
```

Maple [A]
time = 0.52, size = 248, normalized size = 0.74

method	result
derivativedivides	$\frac{\frac{2a^3\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(a^2+4b^2)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^3(a+b)^3}-\frac{1}{3(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{}{2(a-b)^2\left(\tan\right.}}{d}$
default	$\frac{\frac{2a^3\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(a^2+4b^2)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^3(a+b)^3}-\frac{1}{3(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{}{2(a-b)^2\left(\tan\right.}}{d}$
risch	$\frac{\frac{14ia^2b^3e^{4i(dx+c)}}{3}+\frac{2ib^5e^{2i(dx+c)}}{3}+8a^3b^2e^{7i(dx+c)}+\frac{44a^3b^2e^{5i(dx+c)}}{3}+\frac{4ab^4e^{5i(dx+c)}}{3}+\frac{70ia^4be^{2i(dx+c)}}{3}+\frac{82ia^4be^{4i(dx+c)}}{3}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^3/(a-b)^3/(a+b)^3*((b^2*tan(1/2*d*x+1/2*c)+a*b)/(a*tan(1/2*d*x+1/2
*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(a^2+4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/3/(a-b)^2/(tan(1/2*d*x+1/2*c)+
1)^3+1/2/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^2+a/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)-
1/3/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^2+a
/(a+b)^3/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 0.45, size = 815, normalized size = 2.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - 2*(7*a^6*b + 2*a^4*b^3 - 1
0*a^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(7*a^6*b - 16*a^4*b^3 + 11*a^2*b^5 - 2*
b^7)*cos(d*x + c)^2 - 3*((a^5*b + 4*a^3*b^3)*cos(d*x + c)^3*sin(d*x + c) +
(a^6 + 4*a^4*b^2)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)) - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (4*a^7 - 7*a^5
*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3
 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9 - 4*a^
7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3), -1/3*(a^6*b - 3*a
^4*b^3 + 3*a^2*b^5 - b^7 - (7*a^6*b + 2*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x
 + c)^4 - (7*a^6*b - 16*a^4*b^3 + 11*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*((
a^5*b + 4*a^3*b^3)*cos(d*x + c)^3*sin(d*x + c) + (a^6 + 4*a^4*b^2)*cos(d*x
+ c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d
*x + c))) - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (4*a^7 - 7*a^5*b^2 + 2*a
^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b
^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 6.59, size = 406, normalized size = 1.22

$$\frac{2\left(\frac{3\,(a^5+4\,a^3b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{3\,\left(a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4b\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}+\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-18\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-10\,a^3b-2\,ab^3}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*(a^5 + 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*sqrt(a^2 - b^2)) + 3*(a^3*b^2*tan(1/2*d*x + 1/2*c) + a^4*b)/((a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x
+ 1/2*c) + a)) + (3*a^4*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b^2*tan(1/2*d*x + 1/
2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 6*a*b^3*tan(1/2*d*x + 1/2*c)^4 -
```

```
10*a^4*tan(1/2*d*x + 1/2*c)^3 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^4*t
an(1/2*d*x + 1/2*c)^3 + 24*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x
 + 1/2*c) + 9*a^2*b^2*tan(1/2*d*x + 1/2*c) - 10*a^3*b - 2*a*b^3)/((a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

Mupad [B]
time = 9.90, size = 722, normalized size = 2.17

$$\frac{\frac{2\,\left(13\,a^4\,b+2\,a^2\,b^3\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-3\,a^5+14\,a^3\,b^2+4\,a\,b^4\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(a^4\,b+4\,a^2\,b^3\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(29\,a^4\,b+16\,a^2\,b^3\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(7\,a^5+30\,a^3\,b^2+8\,a\,b^4\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(-7\,a^5+48\,a^3\,b^2+4\,a\,b^4\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(11\,a^4\,b+42\,a^2\,b^3-8\,b^5\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{2\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(a^2+4\,b^2\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{d\,\left(-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5-6\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}+\frac{2\,a^3\,\mathrm{atan}\left(\frac{\frac{a^3\,\left(a^2+4\,b^2\right)\,\left(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}+\frac{2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+4\,b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}}{2\,a^5+8\,a^3\,b^2}\right)\,\left(a^2+4\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*(13*a^4*b + 2*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*ta
n(c/2 + (d*x)/2)*(4*a*b^4 - 3*a^5 + 14*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4
- 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^6*(a^4*b + 4*a^2*b^3))/(a^6 - b^6 + 3
*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2 + (d*x)/2)^2*(29*a^4*b + 16*a^2*b^3))/(3
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^5*(8*a*b^4 +
7*a^5 + 30*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 +
 (d*x)/2)^3*(4*a*b^4 - 7*a^5 + 48*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2)) + (2*tan(c/2 + (d*x)/2)^4*(11*a^4*b - 8*b^5 + 42*a^2*b^3))/(3*(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*a^3*tan(c/2 + (d*x)/2)^7*(a^2 + 4*b^2
))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - 2*
a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^6 - a*tan(c/2 + (d*x)/2)^8
- 6*b*tan(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 - 2*b*tan(c/2 + (d*x)
/2)^7)) + (2*a^3*atan(((a^3*(a^2 + 4*b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*
a^4*b^3))/((a + b)^(7/2)*(a - b)^(7/2)) + (2*a^4*tan(c/2 + (d*x)/2)*(a^2 +
4*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(7/2)))/
(2*a^5 + 8*a^3*b^2))*(a^2 + 4*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.188 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}-\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))}$$

```
[Out] -2*a^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d-4
*a*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d+1
/2*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))-1/2*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+
c))-a^2*b*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.22, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2810, 2727, 2743, 12, 2739, 632, 210}

$$-\frac{4ab^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{2a^3\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2d(a-b)^2(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*d) - (4*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b
^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c + d*
x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^2*b*Cos[c + d*x])/((a^2 - b^2)^2
*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{1}{2(a+b)^2(-1+\sin(c+dx))} + \frac{1}{2(a-b)^2(1+\sin(c+dx))} - \frac{}{(a^2-b^2)\,(}\right.$$

$$= \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} - \frac{(2ab^2)\int \frac{1}{a+b\sin(c+dx)}\,dx}{(a^2-b^2)^2} - \frac{a^2\int \frac{1}{(a+b\sin(c+}}{a^2-b^2}$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{a^2b\cos(c}{(a^2-b^2)^2\,d(a+}$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{a^2b\cos(c}{(a^2-b^2)^2\,d(a+}$$

$$= -\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c}{2(a-b)^2d(1}$$

$$= -\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c}{2(a-b)^2d(1}$$

$$= -\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d} - \frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d} + \frac{\cos(c}{2(a+b)^2d(1}$$

Mathematica [A]
time = 0.72, size = 169, normalized size = 0.84

$$\frac{-\frac{2a(a^2+2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{1}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right) - \frac{a^2b\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*a*(a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(5/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])) + 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (a^2*b*
Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

Maple [A]
time = 0.38, size = 162, normalized size = 0.81

method	result

derivativedivides	$\frac{-\frac{1}{(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(a^2+2b^2)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{(a+b)^2\left(\tan\left(\ldots\right.\right.}}{d}$
default	$\frac{-\frac{1}{(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(a^2+2b^2)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{(a+b)^2\left(\tan\left(\ldots\right.\right.}}{d}$
risch	$\frac{2i\left(3a^3e^{i(dx+c)}+4ia^2b\,e^{2i(dx+c)}-ib^3e^{2i(dx+c)}+2a\,b^2e^{3i(dx+c)}+2ia^2b+ib^3+a^3e^{3i(dx+c)}\right)}{\left(1+e^{2i(dx+c)}\right)\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)(a^2-b^2)^2d}+\frac{a^3\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2\ldots}}{b\sqrt{-\ldots}}\right.}{\sqrt{-a^2+b^2}\,(a+b\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-2*a/(a-b)^2/(a+b)^2*((b^2*tan(1/2*d*
x+1/2*c)+a*b)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(a^2+2*b^2)
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-
1/(a+b)^2/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.40, size = 569, normalized size = 2.84

$$\left[-\frac{2a^4b-4a^2b^3+2b^5+2(2a^4b-a^2b^3-b^5)\cos(dx+c)^2+((a^3b+2ab^3)\cos(dx+c)\sin(dx+c)+(a^4+2a^2b^2)\cos(dx+c))\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2(a^5-2a^3b^2+ab^4)\sin(dx+c)}{2((a^6b-3a^4b^3+3a^2b^5-b^7)d\cos(dx+c)\sin(dx+c)+(a^7-3a^5b^2+3a^3b^4-ab^6)d\cos(dx+c))},-\frac{a^4b-2a^2b^3+b^5+(2a^4b-a^2b^3-b^5)\cos(dx+c)^2-((a^3b+2ab^3)\cos(dx+c)\sin(dx+c)+(a^4+2a^2b^2)\cos(dx+c))\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-(a^5-2a^3b^2+ab^4)\sin(dx+c)}{(a^6b-3a^4b^3+3a^2b^5-b^7)d\cos(dx+c)\sin(dx+c)+(a^7-3a^5b^2+3a^3b^4-ab^6)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x +
c)^2 + ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*cos
(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(
a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*co
s(d*x + c)), -(a^4*b - 2*a^2*b^3 + b^5 + (2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*c
os(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*
cos(d*x + c))) - (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^
3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 4.17, size = 251, normalized size = 1.26

$$-\frac{2\left(\frac{(a^3+2ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^2b}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)(a^4-2a^2b^2+b^4)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*ta
n(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2
 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a
^2*b*tan(1/2*d*x + 1/2*c)^2 + 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*
x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c) - 3*a^2*b)/((a*tan(1/2*d*x + 1/2*
c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)*(a^4 - 2*
a^2*b^2 + b^4)))/d
```

Mupad [B]
time = 8.68, size = 313, normalized size = 1.56

$$-\frac{\frac{6a^2b}{(a^2-b^2)^2}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(4ab^2-a^3)}{(a^2-b^2)^2}-\frac{2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2+2b^2)}{a^4-2a^2b^2+b^4}-\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^2+2b^2)}{(a^2-b^2)^2}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{2a\,\mathrm{atan}\left(\frac{\frac{a(a^2+2b^2)(2a^4b-4a^2b^3+2b^5)}{(a+b)^{5/2}(a-b)^{5/2}}+\frac{2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2+2b^2)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{5/2}}}{2a^3+4ab^2}\right)(a^2+2b^2)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*sin(c + d*x))^2,x)
```

```
[Out] - ((6*a^2*b)/(a^2 - b^2)^2 + (2*tan(c/2 + (d*x)/2)*(4*a*b^2 - a^3))/(a^2 -
b^2)^2 - (2*b*tan(c/2 + (d*x)/2)^2*(a^2 + 2*b^2))/(a^4 + b^4 - 2*a^2*b^2) -
 (2*a*tan(c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a^2 - b^2)^2)/(d*(a + 2*b*tan(c/
2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (d*x)/2)^3)) - (2*a*a
tan(((a*(a^2 + 2*b^2)*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)
^(5/2)) + (2*a^2*tan(c/2 + (d*x)/2)*(a^2 + 2*b^2)*(a^4 + b^4 - 2*a^2*b^2))/
((a + b)^(5/2)*(a - b)^(5/2)))/(4*a*b^2 + 2*a^3))*(a^2 + 2*b^2))/(d*(a + b)
^(5/2)*(a - b)^(5/2))
```

3.189 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$-\frac{2(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}\,d}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

```
[Out] 2*b*arctanh(cos(d*x+c))/a^3/d-2*cot(d*x+c)/a^2/d+cot(d*x+c)/a/d/(a+b*sin(d*
x+c))-2*(a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/d/
(a^2-b^2)^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2802, 3135, 3080, 3855, 2739, 632, 210}

$$\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}-\frac{2(a^2-2b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\sqrt{a^2-b^2}}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*Sq
rt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*Cot[c + d*x])/(
a^2*d) + Cot[c + d*x]/(a*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(2\left(a^2-b^2\right)-\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(-2b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\sin(c+dx)\right.}{a+b\sin(c+dx)}}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\int\csc(c+dx)\,dx}{a^3} - \frac{\left(a^2-2b^2\right)}{}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2(a^2-2b}{}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{(4(a^2-2b}{}$$

$$= -\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}\,d} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+d}{a^2d}$$

Mathematica [A]
time = 0.52, size = 139, normalized size = 1.21

$$-\frac{\frac{4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a\cot\left(\frac{1}{2}(c+dx)\right) - 4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2ab\cos(c+dx)}{a+b\sin(c+dx)} - a\tan\left(\frac{1}{2}(c+dx)\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*((4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + a*Cot[(c + d*x)/2] - 4*b*Log[Cos[(c + d*x)/2]] + 4*b*Log[Si
n[(c + d*x)/2]] + (2*a*b*Cos[c + d*x])/(a + b*Sin[c + d*x]) - a*Tan[(c + d*
x)/2])/(a^3*d)
```

Maple [A]
time = 0.42, size = 156, normalized size = 1.36

method	result

derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{1}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(a^2-2b^2)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{2\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}}\right.}{a^3}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{1}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(a^2-2b^2)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{2\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}}\right.}{a^3}}{d}$
risch	$-\frac{2\left(-3a\,e^{i(dx+c)}+2ib\,e^{2i(dx+c)}-2ib+a\,e^{3i(dx+c)}\right)}{\left(e^{2i(dx+c)}-1\right)\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)a^2d}-\frac{2b\ln\left(e^{i(dx+c)}-1\right)}{a^3d}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a^2*tan(1/2*d*x+1/2*c)-1/2/a^2/tan(1/2*d*x+1/2*c)-2/a^3*b*ln(tan(1
/2*d*x+1/2*c))-2/a^3*((b^2*tan(1/2*d*x+1/2*c)+a*b)/(a*tan(1/2*d*x+1/2*c)^2+
2*b*tan(1/2*d*x+1/2*c)+a)+(a^2-2*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1
/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(110) = 220$.
time = 0.45, size = 768, normalized size = 6.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*b^3 - (a^2*b
 - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*l
og(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a^4 - a^2*b^2)*cos(d*x + c) -
2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x
 + c))*log(1/2*cos(d*x + c) + 1/2) + 2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos
(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(
(a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^2)*d*sin(d*x + c) - (a^5*
b - a^3*b^3)*d), (2*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*
b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (a
^4 - a^2*b^2)*cos(d*x + c) - (a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^
2 + (a^3*b - a*b^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^2*b^2 -
b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2))/((a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^
2)*d*sin(d*x + c) - (a^5*b - a^3*b^3)*d)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A]

time = 21.82, size = 218, normalized size = 1.90

$$-\frac{\frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)(a^2-2\,b^2)}{\sqrt{a^2-b^2}\,a^3}-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-14\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^2
+ 12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*(a^2 - 2*b^2)/(sqrt(a^2 - b^2)*a^3) - (4*a*b*tan
(1/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 4*b^2*tan(1/2*d*x + 1/
2*c)^2 - 14*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/((a*tan(1/2*d*x + 1/2*c)^3 +
2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3))/d
```

Mupad [B]
time = 8.32, size = 1616, normalized size = 14.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*sin(c + d*x))^2,x)
```

```
[Out] -(a^4*cos(c + d*x) - b^4/2 - b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
 + (a^2*b^2)/2 + (b^4*cos(2*c + 2*d*x))/2 - a^2*b^2*cos(c + d*x) - a*b^3*si
n(2*c + 2*d*x) + a^3*b*sin(2*c + 2*d*x) + a^2*b^2*log(sin(c/2 + (d*x)/2)/co
s(c/2 + (d*x)/2)) + b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c
+ 2*d*x) + b^3*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(
c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^
(1/2)*4i + a^2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d
*x)/2) + 8*b^4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*co
s(c/2 + (d*x)/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - (a
^2*b^2*cos(2*c + 2*d*x))/2 - a*b^3*sin(c + d*x) + a^3*b*sin(c + d*x) - a^2*
b*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^
2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^
4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)
/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i - a^3*sin(c + d*x
)*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^
2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^
4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)
/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - a^2*b^2*log(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x) - b^3*cos(2*c + 2*d*x)
*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2
*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4
*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/
2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - 2*a*b^3*sin(c +
d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 2*a^3*b*sin(c + d*x)*log(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + a*b^2*sin(c + d*x)*atan((a^3*cos(c
/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4*sin(c/2 + (d*x)
/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/2) - 8*a^2*b^2*s
in(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*4i + a^2*b*cos(2*c + 2*d*x)*atan((a^3
*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)*(b^2 - a^
2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*sin(c/2
 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4*sin(c/2 +
 (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/2) - 8*a^2
```

```
*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i)/(2*a^3*d*(a^2 - b^2)*(b/4 +
 (a*sin(c + d*x))/2 - (b*cos(2*c + 2*d*x))/4))
```

3.190 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=238

$$\frac{2(a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d}-\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d}-($$

```
[Out] -b*(3*a^2-4*b^2)*arctanh(cos(d*x+c))/a^5/d+1/3*(7*a^2-12*b^2)*cot(d*x+c)/a^
4/d-(a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d+1/3*(3*a^2-4*b^2)*cot(d*x+c)*
csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+b*si
n(d*x+c))+2*(a^4-5*a^2*b^2+4*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))/a^5/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2803, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d}-\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd}+\frac{2(a^4-5a^2b^2+4b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^5*Sqrt[a^2 - b^2]*d) - (b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^
5*d) + ((7*a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) - ((a^2 - 2*b^2)*Cot[c + d
*x]*Csc[c + d*x])/(a^3*b*d) + ((3*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(
3*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a +
 b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2803

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m
 + 1)/Sin[e + f*x]^3)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e
+ f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[(3*a^2 +
 b^2*(m - 2))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(3*a^2*b*f*(m + 1)
*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))} + \frac{\int \csc^3(c+dx)(6(a \cdots}{\cdots}$$

$$= -\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\mathrm{co}\cdots}{3\cdots}$$

$$= \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot\cdots}{3a^2bd(a+\cdots}$$

$$= \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot\cdots}{3a^2bd(a+\cdots}$$

$$= -\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot\cdots}{\cdots}$$

$$= -\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot\cdots}{\cdots}$$

$$= \frac{2(a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} - \frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx)\cdots}{a^5d}$$

Mathematica [A]
time = 6.17, size = 403, normalized size = 1.69

$$\frac{2(a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} + \frac{\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-9b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^4d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} + \frac{(-3a^2b+4b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{(3a^2b-4b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^4d} + \frac{a^2b\cos(c+dx)-b^3\cos(c+dx)}{a^4d(a+b\sin(c+dx))} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] +
 a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + ((4*a^2*C
os[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (b*
Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a
^2*d) + ((-3*a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d) + ((3*a^2*b - 4*
b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^2)/(4*a^3*d) + (S
ec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2]))/(6*a^4*
d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(a^4*d*(a + b*Sin[c + d*x])) +
 (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)
```

Maple [A]
time = 0.49, size = 287, normalized size = 1.21

method	result

derivativedivides	$\frac{\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+12b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4}+\frac{\frac{2\left(b^2\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab\left(a^2-b^2\right)\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{2\left(a^4-\cdots\right.}{\cdots}}{a^5}}{\cdots}$
default	$\frac{\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+12b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4}+\frac{\frac{2\left(b^2\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab\left(a^2-b^2\right)\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{2\left(a^4-\cdots\right.}{\cdots}}{a^5}}{\cdots}$
risch	$\frac{12b^2a\,e^{i(dx+c)}+\frac{50ia^2b\,e^{2i(dx+c)}}{3}-\frac{22a^3e^{i(dx+c)}}{3}-28a\,b^2e^{3i(dx+c)}+24ib^3e^{4i(dx+c)}-\frac{14ia^2b}{3}-4a\,b^2e^{7i(dx+c)}+20a\,b^2e^{5i(dx\cdots}}{\left(e^{2i(dx+c)}-1\right)^3\left(b\,e^{2i(d\cdots}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/a^4*(1/3*tan(1/2*d*x+1/2*c)^3*a^2-2*a*b*tan(1/2*d*x+1/2*c)^2-5*a^2
*tan(1/2*d*x+1/2*c)+12*b^2*tan(1/2*d*x+1/2*c))+2/a^5*((b^2*(a^2-b^2)*tan(1/
2*d*x+1/2*c)+a*b*(a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+
a)+(a^4-5*a^2*b^2+4*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2)))-1/24/a^2/tan(1/2*d*x+1/2*c)^3-1/8*(-5*a^2+12*b^2)/a
^4/tan(1/2*d*x+1/2*c)+1/4/a^3*b/tan(1/2*d*x+1/2*c)^2+1/a^5*b*(3*a^2-4*b^2)*
ln(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(227) = 454.
time = 0.46, size = 1149, normalized size = 4.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(4*(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 3*((a^2*b - 4*b^3)*cos(d*x +
c)^4 + a^2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 -
(a^3 - 4*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
- b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*
sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2)) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) + 3*((3*a^2
*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*co
s(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c
)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 + (3*a^3*b -
 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^3*b - 4*a*
b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*
x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c)), -1/6*(4*
(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 6*((a^2*b - 4*b^3)*cos(d*x + c)^4 + a^
2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 - (a^3 - 4*
a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c
) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) +
 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 -
 4*b^4)*cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x +
 c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*c
os(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 +
(3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(
-1/2*cos(d*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^
3*b - 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*
b*d*cos(d*x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c))
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 17.41, size = 356, normalized size = 1.50

$$\frac{\frac{24\,(3\,a^2b-4\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5}+\frac{48\,(a^4-5\,a^2b^2+4\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}+\frac{48\,(a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3b-ab^3)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)a^5}-\frac{132\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-176\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/24*(24*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + 48*(a^4 - 5
*a^2*b^2 + 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(
1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + (a^4*tan(1/
2*d*x + 1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*tan(1/2*d*x + 1/
2*c) + 36*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 + 48*(a^2*b^2*tan(1/2*d*x + 1/2
*c) - b^4*tan(1/2*d*x + 1/2*c) + a^3*b - a*b^3)/((a*tan(1/2*d*x + 1/2*c)^2
+ 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5) - (132*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
176*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b^2*t
an(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^5*tan(1/2*d*
x + 1/2*c)^3))/d

Mupad [B]
time = 6.83, size = 973, normalized size = 4.09

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(28a^2b-40b^3)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(8ab^2-\frac{14a^3}{3}\right)-\frac{a^3}{3}+\frac{4a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(5a^4+4a^2b^2-16b^4)}{a}}{d\left(8a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+16ba^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24a^2d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{16a^2+32b^2}{64a^4}+\frac{3}{8a^2}-\frac{2b^2}{a^4}\right)}{d}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4a^3d}+\frac{b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(3a^2-4b^2)}{a^5d}+\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{b^2-a^2}(a^2-4b^2)\left(\frac{2(a^9-8a^7b^2+8a^5b^4)}{a^8}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(5a^7b-20a^5b^3+16a^3b^5)}{a^7}+\frac{\left(2a^2b-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3a^{10}-4a^8b^2)}{a^7}\right)\sqrt{b^2-a^2}(a^2-4b^2)}{a^5}\right)1i}{a^5}+\frac{\sqrt{b^2-a^2}(a^2-4b^2)\left(\frac{2(a^9-8a^7b^2+8a^5b^4)}{a^8}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(5a^7b-20a^5b^3+16a^3b^5)}{a^7}-\frac{\left(2a^2b-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3a^{10}-4a^8b^2)}{a^7}\right)\sqrt{b^2-a^2}(a^2-4b^2)}{a^5}\right)1i}{a^5}}{\frac{4(3a^6b-19a^4b^3+32a^2b^5-16b^7)}{a^8}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a^6-14a^4b^2+28a^2b^4-16b^6)}{a^7}-\frac{\sqrt{b^2-a^2}(a^2-4b^2)\left(\frac{2(a^9-8a^7b^2+8a^5b^4)}{a^8}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(5a^7b-20a^5b^3+16a^3b^5)}{a^7}+\frac{\left(2a^2b-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3a^{10}-4a^8b^2)}{a^7}\right)\sqrt{b^2-a^2}(a^2-4b^2)}{a^5}\right)}{a^5}+\frac{\sqrt{b^2-a^2}(a^2-4b^2)\left(\frac{2(a^9-8a^7b^2+8a^5b^4)}{a^8}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(5a^7b-20a^5b^3+16a^3b^5)}{a^7}-\frac{\left(2a^2b-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3a^{10}-4a^8b^2)}{a^7}\right)\sqrt{b^2-a^2}(a^2-4b^2)}{a^5}\right)}{a^5}}\right)\sqrt{b^2-a^2}(a^2-4b^2)\,2i}{a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^2,x)

[Out] (tan(c/2 + (d*x)/2)^3*(28*a^2*b - 40*b^3) - tan(c/2 + (d*x)/2)^2*(8*a*b^2 -
 (14*a^3)/3) - a^3/3 + (4*a^2*b*tan(c/2 + (d*x)/2))/3 + (tan(c/2 + (d*x)/2)
^4*(5*a^4 - 16*b^4 + 4*a^2*b^2))/a)/(d*(8*a^5*tan(c/2 + (d*x)/2)^3 + 8*a^5*
tan(c/2 + (d*x)/2)^5 + 16*a^4*b*tan(c/2 + (d*x)/2)^4)) + tan(c/2 + (d*x)/2)
^3/(24*a^2*d) - (tan(c/2 + (d*x)/2)*((16*a^2 + 32*b^2)/(64*a^4) + 3/(8*a^2)
 - (2*b^2)/a^4))/d - (b*tan(c/2 + (d*x)/2)^2)/(4*a^3*d) + (b*log(tan(c/2 +
(d*x)/2))*(3*a^2 - 4*b^2))/(a^5*d) + (atan((((b^2 - a^2)^(1/2)*(a^2 - 4*b^2
)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(5*a^7*b +
 16*a^3*b^5 - 20*a^5*b^3))/a^7 + ((2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10
- 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/a^5)*1i)/a^5 + ((b^2 -
a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/
2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 - ((2*a^2*b - (2*tan(
c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/
a^5)*1i)/a^5)/((4*(3*a^6*b - 16*b^7 + 32*a^2*b^5 - 19*a^4*b^3))/a^8 + (4*ta
n(c/2 + (d*x)/2)*(2*a^6 - 16*b^6 + 28*a^2*b^4 - 14*a^4*b^2))/a^7 - ((b^2 -
a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/
2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 + ((2*a^2*b - (2*tan(
c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/
a^5))/a^5 + ((b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b
^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 -
 ((2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(
1/2)*(a^2 - 4*b^2))/a^5))/a^5))*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*2i)/(a^5*d)

3.191 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=424

$$-\frac{2(a^2-6b^2)(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{b(15a^4-40a^2b^2+24b^4)\tanh^{-1}(\cos(c+dx))}{4a^7d}-\frac{(38a^4-1}{}$$

```
[Out] -2*(a^2-6*b^2)*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))/a^7/d+1/4*b*(15*a^4-40*a^2*b^2+24*b^4)*arctanh(cos(d*x+c))/a^7/d-1/15*
(38*a^4-135*a^2*b^2+90*b^4)*cot(d*x+c)/a^6/d+1/4*(4*a^4-17*a^2*b^2+12*b^4)*
cot(d*x+c)*csc(d*x+c)/a^5/b/d-1/30*(15*a^4-82*a^2*b^2+60*b^4)*cot(d*x+c)*cs
c(d*x+c)^2/a^4/b^2/d-1/2*cot(d*x+c)*csc(d*x+c)/b/d/(a+b*sin(d*x+c))+1/6*a*c
ot(d*x+c)*csc(d*x+c)^2/b^2/d/(a+b*sin(d*x+c))+1/6*(2*a^4-12*a^2*b^2+9*b^4)*
cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d/(a+b*sin(d*x+c))+3/10*b*cot(d*x+c)*csc(d*
x+c)^3/a^2/d/(a+b*sin(d*x+c))-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d/(a+b*sin(d*x+
c))
```

Rubi [A]
time = 1.00, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2805, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}-\frac{2(a^2-6b^2)(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}-\frac{(15a^4-82a^2b^2+60b^4)\cot(c+dx)\csc^2(c+dx)}{30a^4b^2d}+\frac{b(15a^4-40a^2b^2+24b^4)\tanh^{-1}(\cos(c+dx))}{4a^7d}-\frac{(38a^4-135a^2b^2+90b^4)\cot(c+dx)}{15a^6d}+\frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{4a^5bd}+\frac{(2a^4-12a^2b^2+9b^4)\cot(c+dx)\csc^2(c+dx)}{6a^3b^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*d) + (b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*ArcTanh[Cos[c + d*x]
])/(4*a^7*d) - ((38*a^4 - 135*a^2*b^2 + 90*b^4)*Cot[c + d*x])/(15*a^6*d) +
((4*a^4 - 17*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(4*a^5*b*d) - ((1
5*a^4 - 82*a^2*b^2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^4*b^2*d) -
(Cot[c + d*x]*Csc[c + d*x])/(2*b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d*x]*
Csc[c + d*x]^2)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((2*a^4 - 12*a^2*b^2 + 9*b
^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^3*b^2*d*(a + b*Sin[c + d*x])) + (3*b*
Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x
]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2805

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(5*a*f*Sin[
e + f*x]^5)), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x]
)^m/Sin[e + f*x]^4)*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m -
 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m
*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x]
, x], x] + Simp[Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Sin[e + f
*x]^2)), x] + Simp[a*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*m*(m
 - 1)*Sin[e + f*x]^3)), x] - Simp[b*(m - 4)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^(m + 1)/(20*a^2*f*Sin[e + f*x]^4)), x]) /; FreeQ[{a, b, e, f, m}, x] &&
 NeQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
```

```
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{3b\cot(c+dx)\csc^3(c+}{10a^2d(a+b\sin(c+d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{(2a^4-12a^2b^2+9b^4)\,c}{6a^3b^2d(a+}$$

$$= -\frac{(15a^4-82a^2b^2+60b^4)\cot(c+dx)\csc^2(c+dx)}{30a^4b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} +$$

$$= \frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{(15a^4-82a^2b^2+60b^4)\cot(c+}{30a^4b^2d}$$

$$= -\frac{(38a^4-135a^2b^2+90b^4)\cot(c+dx)}{15a^6d} + \frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(}{4a^5bd}$$

$$= -\frac{(38a^4-135a^2b^2+90b^4)\cot(c+dx)}{15a^6d} + \frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(}{4a^5bd}$$

$$= \frac{b(15a^4-40a^2b^2+24b^4)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{(38a^4-135a^2b^2+90b^4)\cot(c+}{15a^6d}$$

$$= \frac{b(15a^4-40a^2b^2+24b^4)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{(38a^4-135a^2b^2+90b^4)\cot(c+}{15a^6d}$$

$$= -\frac{2(a^2-6b^2)(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} + \frac{b(15a^4-40a^2b^2+24b^4)\,\mathrm{ta}}{4a^7d}$$

Mathematica [A]
time = 1.04, size = 361, normalized size = 0.85

$$-\frac{1920(a^2-6b^2)(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) - 240b(15a^4-40a^2b^2+24b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 240b(15a^4-40a^2b^2+24b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2a\cot(c+dx)\csc^5(c+dx)\left(196a^5-735a^3b^2+540ab^4-12(16a^5-85a^3b^2+60ab^4)\cos(2(c+dx))+(92a^5-285a^3b^2+180ab^4)\cos(4(c+dx))+1162a^4b\sin(c+dx)-3060a^2b^3\sin(c+dx)+1800b^5\sin(c+dx)-562a^4b\sin(3(c+dx))+1470a^2b^3\sin(3(c+dx))-900b^5\sin(3(c+dx))+76a^4b\sin(5(c+dx))-270a^2b^3\sin(5(c+dx))+180b^5\sin(5(c+dx))\right)}{b+a\csc(c+dx)}}{960a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/960*(1920*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]] - 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Cos[(c + d*x)
/2]] + 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Co
t[c + d*x]*Csc[c + d*x]^5*(196*a^5 - 735*a^3*b^2 + 540*a*b^4 - 12*(16*a^5 -
 85*a^3*b^2 + 60*a*b^4)*Cos[2*(c + d*x)] + (92*a^5 - 285*a^3*b^2 + 180*a*b^
4)*Cos[4*(c + d*x)] + 1162*a^4*b*Sin[c + d*x] - 3060*a^2*b^3*Sin[c + d*x] +
 1800*b^5*Sin[c + d*x] - 562*a^4*b*Sin[3*(c + d*x)] + 1470*a^2*b^3*Sin[3*(c
 + d*x)] - 900*b^5*Sin[3*(c + d*x)] + 76*a^4*b*Sin[5*(c + d*x)] - 270*a^2*b
^3*Sin[5*(c + d*x)] + 180*b^5*Sin[5*(c + d*x)]))/(b + a*Csc[c + d*x]))/(a^7
*d)
```

Maple [**A**]
time = 0.63, size = 465, normalized size = 1.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32/a^6*(1/5*a^4*tan(1/2*d*x+1/2*c)^5-b*tan(1/2*d*x+1/2*c)^4*a^3-7/3*
tan(1/2*d*x+1/2*c)^3*a^4+4*a^2*b^2*tan(1/2*d*x+1/2*c)^3+16*b*a^3*tan(1/2*d*
x+1/2*c)^2-16*a*b^3*tan(1/2*d*x+1/2*c)^2+22*a^4*tan(1/2*d*x+1/2*c)-108*a^2*
b^2*tan(1/2*d*x+1/2*c)+80*b^4*tan(1/2*d*x+1/2*c))-2/a^7*((b^2*(a^4-2*a^2*b^
2+b^4)*tan(1/2*d*x+1/2*c)+b*a*(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*d*x+1/2*c)^2+
2*b*tan(1/2*d*x+1/2*c)+a)+(a^6-8*a^4*b^2+13*a^2*b^4-6*b^6)/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/160/a^2/tan(1/2
*d*x+1/2*c)^5-1/96*(-7*a^2+12*b^2)/a^4/tan(1/2*d*x+1/2*c)^3-1/32*(22*a^4-10
8*a^2*b^2+80*b^4)/a^6/tan(1/2*d*x+1/2*c)+1/32*b/a^3/tan(1/2*d*x+1/2*c)^4-1/
2/a^5*b*(a^2-b^2)/tan(1/2*d*x+1/2*c)^2-1/4/a^7*b*(15*a^4-40*a^2*b^2+24*b^4)
*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 964 vs. 2(401) = 802.

time = 0.68, size = 2011, normalized size = 4.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/120*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 -
 27*a^4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 60*((a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^
3*b^2 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a
^3*b^2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^
2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c
)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2)) + 30*(4*a^6 - 17*a^4*b^2 + 12*a^2*b^4)*cos(d
*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 - 15*a^4*b^
2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5*b - 40*a
^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^4 - 2*(
15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos
(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 -
15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5
*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c
)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*lo
g(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 90*a*b^5)*cos(d
*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x + c)^3 + 15*(15*
a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^7*b*d*cos(d*x
 + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2 - a^7*b*d - (
a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*x + c)), 1/120
*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 - 27*a^
4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 120*((a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^3*b^2
 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a^3*b^
2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d
*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(4*a^6 - 17*a^4*b^2 + 12*
a^2*b^4)*cos(d*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6
)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (1
5*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*
x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4
```

```
 + 24*b^6)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x +
c)^2 - (15*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^
5)*cos(d*x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 9
0*a*b^5)*cos(d*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x +
c)^3 + 15*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a
^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2
 - a^7*b*d - (a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*
x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**6/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 9.68, size = 596, normalized size = 1.41

$$-\frac{\frac{120\,(15\,a^4b-40\,a^2b^3+24\,b^5)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^7}+\frac{960\,(a^6-8\,a^4b^2+13\,a^2b^4-6\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^7}+\frac{960\,(a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^5b-2\,a^3b^3+ab^5)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)a^7}-\frac{3\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^7b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+60\,a^6b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+240\,a^7b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-240\,a^5b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+330\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1620\,a^6b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1200\,a^4b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{10}}-\frac{4110\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-10960\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6576\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-330\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1620\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1200\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-240\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+240\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+35\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-60\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^5}{a^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/480*(120*(15*a^4*b - 40*a^2*b^3 + 24*b^5)*log(abs(tan(1/2*d*x + 1/2*c)))
/a^7 + 960*(a^6 - 8*a^4*b^2 + 13*a^2*b^4 - 6*b^6)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sq
rt(a^2 - b^2)*a^7) + 960*(a^4*b^2*tan(1/2*d*x + 1/2*c) - 2*a^2*b^4*tan(1/2*
d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c) + a^5*b - 2*a^3*b^3 + a*b^5)/((a*ta
n(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^7) - (3*a^8*tan(1/2*
d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^8*tan(1/2*d*x + 1/2
*c)^3 + 60*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^7*b*tan(1/2*d*x + 1/2*c)^
2 - 240*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 330*a^8*tan(1/2*d*x + 1/2*c) - 162
0*a^6*b^2*tan(1/2*d*x + 1/2*c) + 1200*a^4*b^4*tan(1/2*d*x + 1/2*c))/a^10 -
(4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 10960*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 +
 6576*b^5*tan(1/2*d*x + 1/2*c)^5 - 330*a^5*tan(1/2*d*x + 1/2*c)^4 + 1620*a^
3*b^2*tan(1/2*d*x + 1/2*c)^4 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 240*a^4*
b*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 35*a^5*tan(
1/2*d*x + 1/2*c)^2 - 60*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b*tan(1/2*d
*x + 1/2*c) - 3*a^5)/(a^7*tan(1/2*d*x + 1/2*c)^5))/d
```

Mupad [B]
time = 6.91, size = 1424, normalized size = 3.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + b*sin(c + d*x))^2,x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a^2*d) + (tan(c/2 + (d*x)/2)*(1/(4*a^2) + b^2/(2*
a^4) - (4*b*((b*(64*a^2 + 128*b^2))/(256*a^5) - b/(8*a^3) + (4*b*((64*a^2 +
 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/a))/a + ((64*a^2 + 128*b^
2)*((64*a^2 + 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/(32*a^2)))/d
 - (tan(c/2 + (d*x)/2)^3*((64*a^2 + 128*b^2)/(3072*a^4) + 5/(96*a^2) - b^2/
(6*a^4)))/d - (tan(c/2 + (d*x)/2)^3*((31*a^4*b)/3 - 8*a^2*b^3) + tan(c/2 +
(d*x)/2)^4*(48*a*b^4 + (59*a^5)/3 - 72*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(124
*a^4*b + 224*b^5 - 360*a^2*b^3) + a^5/5 - tan(c/2 + (d*x)/2)^2*((32*a^5)/15
 - 2*a^3*b^2) - (3*a^4*b*tan(c/2 + (d*x)/2))/5 + (2*tan(c/2 + (d*x)/2)^6*(1
1*a^6 + 32*b^6 - 24*a^2*b^4 - 22*a^4*b^2))/a)/(d*(32*a^7*tan(c/2 + (d*x)/2)
^5 + 32*a^7*tan(c/2 + (d*x)/2)^7 + 64*a^6*b*tan(c/2 + (d*x)/2)^6)) + (tan(c
/2 + (d*x)/2)^2*((b*(64*a^2 + 128*b^2))/(512*a^5) - b/(16*a^3) + (2*b*((64*
a^2 + 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/a))/d - (log(tan(c/2
 + (d*x)/2))*(15*a^4*b + 24*b^5 - 40*a^2*b^3))/(4*a^7*d) - (b*tan(c/2 + (d*
x)/2)^4)/(32*a^3*d) - (atan((((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((
4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/2 + (d*x)
/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11) + ((2*a^
2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2 - 6*b^2)*
(-(a + b)^3*(a - b)^3)^(1/2))/a^7)*1i)/a^7 + ((a^2 - 6*b^2)*(-(a + b)^3*(a
- b)^3)^(1/2)*((4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) +
(tan(c/2 + (d*x)/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(
2*a^11) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11)
)*(a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^7)*1i)/a^7)/((15*a^10*b - 1
44*b^11 + 552*a^2*b^9 - 802*a^4*b^7 + 539*a^6*b^5 - 160*a^8*b^3)/a^12 + (ta
n(c/2 + (d*x)/2)*(8*a^10 - 144*b^10 + 516*a^2*b^8 - 682*a^4*b^6 + 400*a^6*b
^4 - 98*a^8*b^2))/a^11 - ((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((4*a^
13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/2 + (d*x)/2)*
(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11) + ((2*a^2*b
- (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2 - 6*b^2)*(-(a
 + b)^3*(a - b)^3)^(1/2))/a^7))/a^7 + ((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)
^(1/2)*((4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/
2 + (d*x)/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11)
 - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2
- 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^7))/a^7))*(a^2 - 6*b^2)*(-(a + b)^
3*(a - b)^3)^(1/2)*2i)/(a^7*d)
```

3.192 $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=321

$$-\frac{(8a^2-5ab-b^2)\log(1-\sin(c+dx))}{16(a+b)^5d}-\frac{(8a^2+5ab-b^2)\log(1+\sin(c+dx))}{16(a-b)^5d}+\frac{a^3(a^4+13a^2b^2+10b^4)\log}{(a^2-b^2)^5}$$

```
[Out] -1/16*(8*a^2-5*a*b-b^2)*ln(1-sin(d*x+c))/(a+b)^5/d-1/16*(8*a^2+5*a*b-b^2)*l
n(1+sin(d*x+c))/(a-b)^5/d+a^3*(a^4+13*a^2*b^2+10*b^4)*ln(a+b*sin(d*x+c))/(a
^2-b^2)^5/d-1/2*a^5/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^2-a^4*(a^2+5*b^2)/(a^2-b
^2)^4/d/(a+b*sin(d*x+c))+1/4*sec(d*x+c)^4*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*sin(
d*x+c))/(a^2-b^2)^3/d-1/8*sec(d*x+c)^2*(8*a^3*(a^2+5*b^2)-b*(27*a^4+22*a^2*
b^2-b^4)*sin(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A]

time = 0.80, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2800, 1661, 1643}

$$-\frac{(8a^2-5ab-b^2)\log(1-\sin(c+dx))}{16d(a+b)^5}-\frac{(8a^2+5ab-b^2)\log(\sin(c+dx)+1)}{16d(a-b)^5}+\frac{\sec^4(c+dx)\left(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx)\right)}{4d\left(a^2-b^2\right)^3}-\frac{a^5}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2}-\frac{a^4(a^2+5b^2)}{d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}+\frac{a^3(a^4+13a^2b^2+10b^4)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^5}-\frac{\sec^2(c+dx)\left(8a^3(a^2+5b^2)-b(27a^4+22a^2b^2-b^4)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/16*((8*a^2 - 5*a*b - b^2)*Log[1 - Sin[c + d*x]])/((a + b)^5*d) - ((8*a^2
 + 5*a*b - b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) + (a^3*(a^4 + 13*a^
2*b^2 + 10*b^4)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) - a^5/(2*(a^2 -
b^2)^3*d*(a + b*Sin[c + d*x])^2) - (a^4*(a^2 + 5*b^2))/((a^2 - b^2)^4*d*(a
+ b*Sin[c + d*x])) + (Sec[c + d*x]^4*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Sin
[c + d*x]))/(4*(a^2 - b^2)^3*d) - (Sec[c + d*x]^2*(8*a^3*(a^2 + 5*b^2) - b*
(27*a^4 + 22*a^2*b^2 - b^4)*Sin[c + d*x]))/(8*(a^2 - b^2)^4*d)
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
```

```
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^3(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx)\right)}{4\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^3b^6\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3} - \frac{a^2b^4}{}}{}\right.}{}$$

$$= \frac{\sec^4(c+dx)\left(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx)\right)}{4\left(a^2-b^2\right)^3 d} - \frac{\sec^2(c+dx)\left(8a^3(a^2+5b\right.}{}$$

$$= \frac{\sec^4(c+dx)\left(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx)\right)}{4\left(a^2-b^2\right)^3 d} - \frac{\sec^2(c+dx)\left(8a^3(a^2+5b\right.}{}$$

$$= -\frac{(8a^2-5ab-b^2)\log(1-\sin(c+dx))}{16(a+b)^5d} - \frac{(8a^2+5ab-b^2)\log(1+\sin(c+dx))}{16(a-b)^5d}$$

Mathematica [A]
time = 6.24, size = 304, normalized size = 0.95

$$-\frac{(8a^2-5ab-b^2)\log(1-\sin(c+dx))}{16(a+b)^5d} - \frac{(8a^2+5ab-b^2)\log(1+\sin(c+dx))}{16(a-b)^5d} + \frac{a^3(a^4+13a^2b^2+10b^4)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^5 d} + \frac{1}{16(a+b)^3d(1-\sin(c+dx))^2} - \frac{7a+b}{16(a+b)^4d(1-\sin(c+dx))} + \frac{1}{16(a-b)^3d(1+\sin(c+dx))^2} - \frac{7a-b}{16(a-b)^4d(1+\sin(c+dx))} - \frac{a^5}{2\left(a^2-b^2\right)^3 d(a+b\sin(c+dx))^2} - \frac{a^4(a^2+5b^2)}{\left(a^2-b^2\right)^4 d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/16*((8*a^2 - 5*a*b - b^2)*Log[1 - Sin[c + d*x]])/((a + b)^5*d) - ((8*a^2
 + 5*a*b - b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) + (a^3*(a^4 + 13*a^
2*b^2 + 10*b^4)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) + 1/(16*(a + b)^
```

```
3*d*(1 - Sin[c + d*x])^2) - (7*a + b)/(16*(a + b)^4*d*(1 - Sin[c + d*x])) +
 1/(16*(a - b)^3*d*(1 + Sin[c + d*x])^2) - (7*a - b)/(16*(a - b)^4*d*(1 + S
in[c + d*x])) - a^5/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) - (a^4*(a^2
+ 5*b^2))/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
```

Maple [A]
time = 0.98, size = 263, normalized size = 0.82

method	result
derivativedivides	$\frac{1}{16(a+b)^3(\sin(dx+c)-1)^2}-\frac{-b-7a}{16(a+b)^4(\sin(dx+c)-1)}+\frac{\left(-8a^2+5ab+b^2\right)\ln(\sin(dx+c)-1)}{16(a+b)^5}-\frac{a^5}{2(a+b)^3(a-b)^3(a+b\sin(dx+c))^2}+\frac{a^3\left(a\right.}{}$
default	$\frac{1}{16(a+b)^3(\sin(dx+c)-1)^2}-\frac{-b-7a}{16(a+b)^4(\sin(dx+c)-1)}+\frac{\left(-8a^2+5ab+b^2\right)\ln(\sin(dx+c)-1)}{16(a+b)^5}-\frac{a^5}{2(a+b)^3(a-b)^3(a+b\sin(dx+c))^2}+\frac{a^3\left(a\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/(a+b)^3/(sin(d*x+c)-1)^2-1/16*(-b-7*a)/(a+b)^4/(sin(d*x+c)-1)+1/1
6/(a+b)^5*(-8*a^2+5*a*b+b^2)*ln(sin(d*x+c)-1)-1/2*a^5/(a+b)^3/(a-b)^3/(a+b*
sin(d*x+c))^2+a^3*(a^4+13*a^2*b^2+10*b^4)/(a+b)^5/(a-b)^5*ln(a+b*sin(d*x+c)
)-a^4*(a^2+5*b^2)/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))+1/16/(a-b)^3/(1+sin(d*x+
c))^2-1/16*(-b+7*a)/(a-b)^4/(1+sin(d*x+c))+1/16/(a-b)^5*(-8*a^2-5*a*b+b^2)*
ln(1+sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(311) = 622$.
time = 0.63, size = 730, normalized size = 2.27

$$\frac{\frac{16\left(a^7+13a^5b^2+10a^3b^4\right)\log(b\sin(dx+c)+a)}{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}}-\frac{\left(8a^2+5ab-b^2\right)\log(\sin(dx+c)+1)}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5}-\frac{\left(8a^2-5ab-b^2\right)\log(\sin(dx+c)-1)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5}-\frac{2\left(18a^7+72a^5b^2+6a^3b^4+\left(8a^6b+67a^4b^3+22a^2b^5-b^7\right)\sin(dx+c)^5+2\left(6a^7+41a^5b^2+2a^3b^4-ab^6\right)\sin(dx+c)^4-\left(5a^6b+159a^4b^3+27a^2b^5+b^7\right)\sin(dx+c)^3-4\left(8a^7+37a^5b^2+4a^3b^4-ab^6\right)\sin(dx+c)^2-\left(a^6b-86a^4b^3-11a^2b^5\right)\sin(dx+c)\right)}{a^{10}-4a^8b^2+6a^6b^4-4a^4b^6+a^2b^8+\left(a^8b^2-4a^6b^4+6a^4b^6-4a^2b^8+b^{10}\right)\sin(dx+c)^6+2\left(a^9b-4a^7b^3+6a^5b^5-4a^3b^7+ab^9\right)\sin(dx+c)^5+\left(a^{10}-6a^8b^2+14a^6b^4-16a^4b^6+9a^2b^8-2b^{10}\right)\sin(dx+c)^4-4\left(a^9b-4a^7b^3+6a^5b^5-4a^3b^7+ab^9\right)\sin(dx+c)^3-\left(2a^{10}-9a^8b^2+16a^6b^4-14a^4b^6+6a^2b^8-b^{10}\right)\sin(dx+c)^2+2\left(a^9b-4a^7b^3+6a^5b^5-4a^3b^7+ab^9\right)\sin(dx+c)}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(a^7 + 13*a^5*b^2 + 10*a^3*b^4)*log(b*sin(d*x + c) + a)/(a^10 - 5*
a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - (8*a^2 + 5*a*b - b^
2)*log(sin(d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5) - (8*a^2 - 5*a*b - b^2)*log(sin(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a
^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*(18*a^7 + 72*a^5*b^2 + 6*a^3*b^4 +
 (8*a^6*b + 67*a^4*b^3 + 22*a^2*b^5 - b^7)*sin(d*x + c)^5 + 2*(6*a^7 + 41*a
^5*b^2 + 2*a^3*b^4 - a*b^6)*sin(d*x + c)^4 - (5*a^6*b + 159*a^4*b^3 + 27*a^
2*b^5 + b^7)*sin(d*x + c)^3 - 4*(8*a^7 + 37*a^5*b^2 + 4*a^3*b^4 - a*b^6)*si
n(d*x + c)^2 - (a^6*b - 86*a^4*b^3 - 11*a^2*b^5)*sin(d*x + c))/(a^10 - 4*a^
8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8 + (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6
- 4*a^2*b^8 + b^10)*sin(d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a
```

```
^3*b^7 + a*b^9)*sin(d*x + c)^5 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4 - 16*a^4*b^
6 + 9*a^2*b^8 - 2*b^10)*sin(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 -
 4*a^3*b^7 + a*b^9)*sin(d*x + c)^3 - (2*a^10 - 9*a^8*b^2 + 16*a^6*b^4 - 14*
a^4*b^6 + 6*a^2*b^8 - b^10)*sin(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b
^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(311) = 622$.
time = 0.75, size = 981, normalized size = 3.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^9 - 16*a^7*b^2 + 24*a^5*b^4 - 16*a^3*b^6 + 4*a*b^8 - 4*(6*a^9 +
35*a^7*b^2 - 39*a^5*b^4 - 3*a^3*b^6 + a*b^8)*cos(d*x + c)^4 - 16*(a^9 - 3*a
^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^2 - 16*((a^7*b^2 + 13*a^5*b^4 +
10*a^3*b^6)*cos(d*x + c)^6 - 2*(a^8*b + 13*a^6*b^3 + 10*a^4*b^5)*cos(d*x +
c)^4*sin(d*x + c) - (a^9 + 14*a^7*b^2 + 23*a^5*b^4 + 10*a^3*b^6)*cos(d*x +
c)^4)*log(b*sin(d*x + c) + a) + ((8*a^7*b^2 + 45*a^6*b^3 + 104*a^5*b^4 + 12
5*a^4*b^5 + 80*a^3*b^6 + 23*a^2*b^7 - b^9)*cos(d*x + c)^6 - 2*(8*a^8*b + 45
*a^7*b^2 + 104*a^6*b^3 + 125*a^5*b^4 + 80*a^4*b^5 + 23*a^3*b^6 - a*b^8)*cos
(d*x + c)^4*sin(d*x + c) - (8*a^9 + 45*a^8*b + 112*a^7*b^2 + 170*a^6*b^3 +
184*a^5*b^4 + 148*a^4*b^5 + 80*a^3*b^6 + 22*a^2*b^7 - b^9)*cos(d*x + c)^4)*
log(sin(d*x + c) + 1) + ((8*a^7*b^2 - 45*a^6*b^3 + 104*a^5*b^4 - 125*a^4*b^
5 + 80*a^3*b^6 - 23*a^2*b^7 + b^9)*cos(d*x + c)^6 - 2*(8*a^8*b - 45*a^7*b^2
 + 104*a^6*b^3 - 125*a^5*b^4 + 80*a^4*b^5 - 23*a^3*b^6 + a*b^8)*cos(d*x + c
)^4*sin(d*x + c) - (8*a^9 - 45*a^8*b + 112*a^7*b^2 - 170*a^6*b^3 + 184*a^5*
b^4 - 148*a^4*b^5 + 80*a^3*b^6 - 22*a^2*b^7 + b^9)*cos(d*x + c)^4)*log(-sin
(d*x + c) + 1) - 2*(2*a^8*b - 8*a^6*b^3 + 12*a^4*b^5 - 8*a^2*b^7 + 2*b^9 +
(8*a^8*b + 59*a^6*b^3 - 45*a^4*b^5 - 23*a^2*b^7 + b^9)*cos(d*x + c)^4 - (11
*a^8*b - 36*a^6*b^3 + 42*a^4*b^5 - 20*a^2*b^7 + 3*b^9)*cos(d*x + c)^2)*sin(
d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b
^12)*d*cos(d*x + c)^6 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5
*a^3*b^9 - a*b^11)*d*cos(d*x + c)^4*sin(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a
^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 14.52, size = 585, normalized size = 1.82

$$\frac{\frac{16\left(a^7b+13a^5b^3+10a^3b^5\right)\log\left(|b\sin(dx+c)+a|\right)}{a^{10}b-5a^8b^3+10a^6b^5-10a^4b^7+5a^2b^9-b^{11}}-\frac{\left(8a^2+5ab-b^2\right)\log\left(|\sin(dx+c)+1|\right)}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5}-\frac{\left(8a^2-5ab-b^2\right)\log\left(|\sin(dx+c)-1|\right)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5}-\frac{2\left(8a^6b\sin(dx+c)^5+67a^4b^3\sin(dx+c)^5+22a^2b^5\sin(dx+c)^5-b^7\sin(dx+c)^5+12a^7\sin(dx+c)^4+82a^5b^2\sin(dx+c)^4+4a^3b^4\sin(dx+c)^4-2ab^6\sin(dx+c)^4-5a^6b\sin(dx+c)^3-159a^4b^3\sin(dx+c)^3-27a^2b^5\sin(dx+c)^3-b^7\sin(dx+c)^3-32a^7\sin(dx+c)^2-148a^5b^2\sin(dx+c)^2-16a^3b^4\sin(dx+c)^2+4ab^6\sin(dx+c)^2-a^6b\sin(dx+c)+86a^4b^3\sin(dx+c)+11a^2b^5\sin(dx+c)+18a^7+72a^5b^2+6a^3b^4\right)}{\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)\left(b\sin(dx+c)^3+a\sin(dx+c)^2-b\sin(dx+c)-a\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(16*(a^7*b + 13*a^5*b^3 + 10*a^3*b^5)*log(abs(b*sin(d*x + c) + a))/(a^
10*b - 5*a^8*b^3 + 10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11) - (8*a^2 + 5
*a*b - b^2)*log(abs(sin(d*x + c) + 1))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2
*b^3 + 5*a*b^4 - b^5) - (8*a^2 - 5*a*b - b^2)*log(abs(sin(d*x + c) - 1))/(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*(8*a^6*b*sin(d*
x + c)^5 + 67*a^4*b^3*sin(d*x + c)^5 + 22*a^2*b^5*sin(d*x + c)^5 - b^7*sin(
d*x + c)^5 + 12*a^7*sin(d*x + c)^4 + 82*a^5*b^2*sin(d*x + c)^4 + 4*a^3*b^4*
sin(d*x + c)^4 - 2*a*b^6*sin(d*x + c)^4 - 5*a^6*b*sin(d*x + c)^3 - 159*a^4*
b^3*sin(d*x + c)^3 - 27*a^2*b^5*sin(d*x + c)^3 - b^7*sin(d*x + c)^3 - 32*a^
7*sin(d*x + c)^2 - 148*a^5*b^2*sin(d*x + c)^2 - 16*a^3*b^4*sin(d*x + c)^2 +
 4*a*b^6*sin(d*x + c)^2 - a^6*b*sin(d*x + c) + 86*a^4*b^3*sin(d*x + c) + 11
*a^2*b^5*sin(d*x + c) + 18*a^7 + 72*a^5*b^2 + 6*a^3*b^4)/((a^8 - 4*a^6*b^2
+ 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c)^3 + a*sin(d*x + c)^2 - b*sin
(d*x + c) - a)^2))/d
```

Mupad [B]
time = 10.74, size = 1229, normalized size = 3.83

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-2a^7+11a^5b^2+38a^3b^4+ab^6\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(-a^7+12a^5b^2+33a^3b^4+4ab^6\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(-a^7+12a^5b^2+33a^3b^4+4ab^6\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(-2a^7+11a^5b^2+38a^3b^4+ab^6\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(6a^7+13a^5b^2+118a^3b^4+7ab^6\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(37a^6+58a^4b^2+a^2b^4\right)}{4\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(7a^6+132a^4b^2-57a^2b^4+14b^6\right)}{2\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(7a^6+132a^4b^2-57a^2b^4+14b^6\right)}{2\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(83a^6+226a^4b^2-17a^2b^4-4b^6\right)}{4\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(83a^6+226a^4b^2-17a^2b^4-4b^6\right)}{4\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(37a^6+58a^4b^2+a^2b^4\right)}{4\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(4a^2+24b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(2a^2-4b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^2-4b^2\right)+a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+a^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^2+16b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(a^2+16b^2\right)-12ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+8ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-12ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\left(\frac{3b^2}{2(a-b)^5}+\frac{21b}{8(a-b)^4}+\frac{1}{(a-b)^3}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\left(\frac{1}{(a+b)^3}-\frac{21b}{8(a+b)^4}+\frac{3b^2}{2(a+b)^5}\right)}{d}+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\left(a^7+13a^5b^2+10a^3b^4\right)}{d\left(a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^2*(a*b^6 - 2*a^7 + 38*a^3*b^4 + 11*a^5*b^2))/(a^8 + b^
8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) - (4*tan(c/2 + (d*x)/2)^4*(4*a*b^6 -
 a^7 + 33*a^3*b^4 + 12*a^5*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6
*b^2) - (4*tan(c/2 + (d*x)/2)^8*(4*a*b^6 - a^7 + 33*a^3*b^4 + 12*a^5*b^2))/
(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) + (tan(c/2 + (d*x)/2)^10*(a
*b^6 - 2*a^7 + 38*a^3*b^4 + 11*a^5*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4
 - 4*a^6*b^2) + (2*tan(c/2 + (d*x)/2)^6*(7*a*b^6 + 6*a^7 + 118*a^3*b^4 + 13
*a^5*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) + (b*tan(c/2 + (
d*x)/2)^11*(37*a^6 + a^2*b^4 + 58*a^4*b^2))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*a
^4*b^4 - 4*a^6*b^2)) + (b*tan(c/2 + (d*x)/2)^5*(7*a^6 + 14*b^6 - 57*a^2*b^4
 + 132*a^4*b^2))/(2*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (b*t
an(c/2 + (d*x)/2)^7*(7*a^6 + 14*b^6 - 57*a^2*b^4 + 132*a^4*b^2))/(2*(a^8 +
b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (b*tan(c/2 + (d*x)/2)^3*(83*a^6
```

```
 - 4*b^6 - 17*a^2*b^4 + 226*a^4*b^2))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4
 - 4*a^6*b^2)) - (b*tan(c/2 + (d*x)/2)^9*(83*a^6 - 4*b^6 - 17*a^2*b^4 + 226
*a^4*b^2))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (b*tan(c/2
 + (d*x)/2)*(37*a^6 + a^2*b^4 + 58*a^4*b^2))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*
a^4*b^4 - 4*a^6*b^2)))/(d*(tan(c/2 + (d*x)/2)^6*(4*a^2 + 24*b^2) - tan(c/2
+ (d*x)/2)^10*(2*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2*
tan(c/2 + (d*x)/2)^12 + a^2 - tan(c/2 + (d*x)/2)^4*(a^2 + 16*b^2) - tan(c/2
 + (d*x)/2)^8*(a^2 + 16*b^2) - 12*a*b*tan(c/2 + (d*x)/2)^3 + 8*a*b*tan(c/2
+ (d*x)/2)^5 + 8*a*b*tan(c/2 + (d*x)/2)^7 - 12*a*b*tan(c/2 + (d*x)/2)^9 + 4
*a*b*tan(c/2 + (d*x)/2)^11 + 4*a*b*tan(c/2 + (d*x)/2))) - (log(tan(c/2 + (d
*x)/2) + 1)*((3*b^2)/(2*(a - b)^5) + (21*b)/(8*(a - b)^4) + 1/(a - b)^3))/d
 - (log(tan(c/2 + (d*x)/2) - 1)*(1/(a + b)^3 - (21*b)/(8*(a + b)^4) + (3*b^
2)/(2*(a + b)^5)))/d + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/
2)^2)*(a^7 + 10*a^3*b^4 + 13*a^5*b^2))/(d*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4
*b^6 + 10*a^6*b^4 - 5*a^8*b^2))
```

3.193 $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$\frac{(2a-b)\log(1-\sin(c+dx))}{4(a+b)^4d}+\frac{(2a+b)\log(1+\sin(c+dx))}{4(a-b)^4d}-\frac{a(a^4+8a^2b^2+3b^4)\log(a+b\sin(c+dx))}{(a^2-b^2)^4\,d}+\frac{}{2}$$

```
[Out] 1/4*(2*a-b)*ln(1-sin(d*x+c))/(a+b)^4/d+1/4*(2*a+b)*ln(1+sin(d*x+c))/(a-b)^4
/d-a*(a^4+8*a^2*b^2+3*b^4)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d+1/2*a^3/(a^2-b^
2)^2/d/(a+b*sin(d*x+c))^2+a^2*(a^2+3*b^2)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))+1/
2*sec(d*x+c)^2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A]

time = 0.40, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2800, 1661, 1643}

$$\frac{a^2(a^2+3b^2)}{d\,(a^2-b^2)^3\,(a+b\sin(c+dx))}+\frac{\sec^2(c+dx)\,(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx))}{2d\,(a^2-b^2)^3}-\frac{a(a^4+8a^2b^2+3b^4)\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^4}+\frac{a^3}{2d\,(a^2-b^2)^2\,(a+b\sin(c+dx))^2}+\frac{(2a-b)\log(1-\sin(c+dx))}{4d(a+b)^4}+\frac{(2a+b)\log(\sin(c+dx)+1)}{4d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a - b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^4*d) + ((2*a + b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^4*d) - (a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c +
 d*x]])/((a^2 - b^2)^4*d) + a^3/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2)
+ (a^2*(a^2 + 3*b^2))/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x
]^2*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Sin[c + d*x]))/(2*(a^2 - b^2)^3*d)
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^3(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)\left(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^3b^4\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3} - \frac{a^2b^2}{}}{}\right.}{}$$

$$= \frac{\sec^2(c+dx)\left(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int \left(\frac{b^2(-2a+b)}{2(a+b)^4(b-x)} - \right.\right.}{}$$

$$= \frac{(2a-b)\log(1-\sin(c+dx))}{4(a+b)^4 d} + \frac{(2a+b)\log(1+\sin(c+dx))}{4(a-b)^4 d} - \frac{a(a^4+8a^2b^2+}{}$$

Mathematica [A]
time = 1.47, size = 196, normalized size = 0.84

$$\frac{\frac{(2a-b)\log(1-\sin(c+dx))}{(a+b)^4} + \frac{(2a+b)\log(1+\sin(c+dx))}{(a-b)^4} - \frac{4a(a^4+8a^2b^2+3b^4)\log(a+b\sin(c+dx))}{(a^2-b^2)^4} - \frac{1}{(a+b)^3(-1+\sin(c+dx))} + \frac{1}{(a-b)^3(1+\sin(c+dx))} + \frac{2a^3}{(a^2-b^2)^2(a+b\sin(c+dx))^2} + \frac{4a^2(a^2+3b^2)}{(a^2-b^2)^3(a+b\sin(c+dx))}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (((2*a - b)*Log[1 - Sin[c + d*x]])/(a + b)^4 + ((2*a + b)*Log[1 + Sin[c + d
*x]])/(a - b)^4 - (4*a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c + d*x]])/(
a^2 - b^2)^4 - 1/((a + b)^3*(-1 + Sin[c + d*x])) + 1/((a - b)^3*(1 + Sin[c
+ d*x])) + (2*a^3)/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) + (4*a^2*(a^2 + 3
*b^2))/((a^2 - b^2)^3*(a + b*Sin[c + d*x])))/(4*d)
```

Maple [A]
time = 0.62, size = 197, normalized size = 0.85

method	result

derivativedivides	$\frac{-\frac{1}{4(a+b)^3(\sin(dx+c)-1)}+\frac{(2a-b)\ln(\sin(dx+c)-1)}{4(a+b)^4}+\frac{1}{4(a-b)^3(1+\sin(dx+c))}+\frac{(2a+b)\ln(1+\sin(dx+c))}{4(a-b)^4}+\frac{a^3}{2(a+b)^2(a-b)^2(a+b\sin(d}}{d}$
default	$\frac{-\frac{1}{4(a+b)^3(\sin(dx+c)-1)}+\frac{(2a-b)\ln(\sin(dx+c)-1)}{4(a+b)^4}+\frac{1}{4(a-b)^3(1+\sin(dx+c))}+\frac{(2a+b)\ln(1+\sin(dx+c))}{4(a-b)^4}+\frac{a^3}{2(a+b)^2(a-b)^2(a+b\sin(d}}{d}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/(a+b)^3/(sin(d*x+c)-1)+1/4*(2*a-b)/(a+b)^4*ln(sin(d*x+c)-1)+1/4/(
a-b)^3/(1+sin(d*x+c))+1/4*(2*a+b)/(a-b)^4*ln(1+sin(d*x+c))+1/2*a^3/(a+b)^2/
(a-b)^2/(a+b*sin(d*x+c))^2-a*(a^4+8*a^2*b^2+3*b^4)/(a+b)^4/(a-b)^4*ln(a+b*s
in(d*x+c))+a^2*(a^2+3*b^2)/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.31, size = 441, normalized size = 1.90

$$-\frac{\frac{4\left(a^5+8a^3b^2+3ab^4\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{(2a+b)\log\left(\sin\left(dx+c\right)+1\right)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{(2a-b)\log\left(\sin\left(dx+c\right)-1\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{2\left(4a^5+8a^3b^2-\left(2a^4b+9a^2b^3+b^5\right)\sin\left(dx+c\right)^3-\left(3a^5+10a^3b^2-ab^4\right)\sin\left(dx+c\right)^2+\left(a^4b+11a^2b^3\right)\sin\left(dx+c\right)\right)}{a^8-3a^6b^2+3a^4b^4-a^2b^6-\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)\sin\left(dx+c\right)^4-2\left(a^7b-3a^5b^3+3a^3b^5-ab^7\right)\sin\left(dx+c\right)^3-\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)\sin\left(dx+c\right)^2+2\left(a^7b-3a^5b^3+3a^3b^5-ab^7\right)\sin\left(dx+c\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^
2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (2*a + b)*log(sin(d*x + c) + 1)/(a^4 - 4
*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (2*a - b)*log(sin(d*x + c) - 1)/(a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(4*a^5 + 8*a^3*b^2 - (2*a^4*b +
9*a^2*b^3 + b^5)*sin(d*x + c)^3 - (3*a^5 + 10*a^3*b^2 - a*b^4)*sin(d*x + c)
^2 + (a^4*b + 11*a^2*b^3)*sin(d*x + c))/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*
b^6 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*sin(d*x + c)^4 - 2*(a^7*b - 3
*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*sin(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7)*sin(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(224) = 448$.
time = 0.53, size = 788, normalized size = 3.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + 2*(3*a^7 + 7*a^5*b^2 - 11*a
^3*b^4 + a*b^6)*cos(d*x + c)^2 + 4*((a^5*b^2 + 8*a^3*b^4 + 3*a*b^6)*cos(d*x
```

```
 + c)^4 - 2*(a^6*b + 8*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^2*sin(d*x + c) - (
a^7 + 9*a^5*b^2 + 11*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2)*log(b*sin(d*x + c)
+ a) - ((2*a^5*b^2 + 9*a^4*b^3 + 16*a^3*b^4 + 14*a^2*b^5 + 6*a*b^6 + b^7)*c
os(d*x + c)^4 - 2*(2*a^6*b + 9*a^5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*a^2*b^
5 + a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (2*a^7 + 9*a^6*b + 18*a^5*b^2 + 23
*a^4*b^3 + 22*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*cos(d*x + c)^2)*log(sin
(d*x + c) + 1) - ((2*a^5*b^2 - 9*a^4*b^3 + 16*a^3*b^4 - 14*a^2*b^5 + 6*a*b^
6 - b^7)*cos(d*x + c)^4 - 2*(2*a^6*b - 9*a^5*b^2 + 16*a^4*b^3 - 14*a^3*b^4
+ 6*a^2*b^5 - a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (2*a^7 - 9*a^6*b + 18*a^
5*b^2 - 23*a^4*b^3 + 22*a^3*b^4 - 15*a^2*b^5 + 6*a*b^6 - b^7)*cos(d*x + c)^
2)*log(-sin(d*x + c) + 1) - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (2*a^6
*b + 7*a^4*b^3 - 8*a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 -
 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^4 - 2*(a^9*b - 4*
a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)^2*sin(d*x + c) - (a
^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)^
2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. 2(224) = 448.
time = 12.51, size = 464, normalized size = 2.00

$$-\frac{\frac{4\,(a^5b+8\,a^3b^3+3\,ab^5)\log(|b\sin(dx+c)+a|)}{a^8b-4\,a^6b^3+6\,a^4b^5-4\,a^2b^7+b^9}-\frac{(2\,a+b)\log(|\sin(dx+c)+1|)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{(2\,a-b)\log(|\sin(dx+c)-1|)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(a^5\sin(dx+c)^2+8\,a^3b^2\sin(dx+c)^2+3\,ab^4\sin(dx+c)^2-3\,a^4b\sin(dx+c)+2\,a^2b^3\sin(dx+c)+b^5\sin(dx+c)-6\,a^3b^2-6\,ab^4\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\left(\sin(dx+c)^2-1\right)}-\frac{2\left(3\,a^5b^2\sin(dx+c)^2+24\,a^3b^4\sin(dx+c)^2+9\,ab^6\sin(dx+c)^2+8\,a^6b\sin(dx+c)+52\,a^4b^3\sin(dx+c)+12\,a^2b^5\sin(dx+c)+6\,a^7+26\,a^5b^2+4\,a^3b^4\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)(b\sin(dx+c)+a)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*(a^5*b + 8*a^3*b^3 + 3*a*b^5)*log(abs(b*sin(d*x + c) + a))/(a^8*b -
 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (2*a + b)*log(abs(sin(d*x + c)
+ 1))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (2*a - b)*log(abs(sin(d
*x + c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(a^5*sin(d*x
+ c)^2 + 8*a^3*b^2*sin(d*x + c)^2 + 3*a*b^4*sin(d*x + c)^2 - 3*a^4*b*sin(d*
x + c) + 2*a^2*b^3*sin(d*x + c) + b^5*sin(d*x + c) - 6*a^3*b^2 - 6*a*b^4)/(
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)) - 2*(
3*a^5*b^2*sin(d*x + c)^2 + 24*a^3*b^4*sin(d*x + c)^2 + 9*a*b^6*sin(d*x + c)
^2 + 8*a^6*b*sin(d*x + c) + 52*a^4*b^3*sin(d*x + c) + 12*a^2*b^5*sin(d*x +
```

```
c) + 6*a^7 + 26*a^5*b^2 + 4*a^3*b^4)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*
b^6 + b^8)*(b*sin(d*x + c) + a)^2))/d
```

Mupad [B]
time = 7.46, size = 690, normalized size = 2.97

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,(2a-b)}{2d\,(a+b)^4}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,(-a^5+6a^3b^2+7ab^4)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(a^5+2a^3b^2+9ab^4)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\,(3a^4b+13a^2b^3-4b^5)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,(-a^5+6a^3b^2+7ab^4)}{(a^2-b^2)\,(a^4-2a^2b^2+b^4)}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\,(7a^4+5a^2b^2)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,(3a^4+13a^2b^2-4b^4)}{(a^2-b^2)\,(a^4-2a^2b^2+b^4)}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(7a^3b+5ab^3)}{(a^2-b^2)\,(a^4-2a^2b^2+b^4)}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(2a^2+8b^2)+4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+a^2-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\,(2a+b)}{2d\,(a-b)^4}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,(a^5+8a^3b^2+3ab^4)}{d\,(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) - 1)*(2*a - b))/(2*d*(a + b)^4) - ((2*tan(c/2 + (d*
x)/2)^6*(7*a*b^4 - a^5 + 6*a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) -
(4*tan(c/2 + (d*x)/2)^4*(9*a*b^4 + a^5 + 2*a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2) - (tan(c/2 + (d*x)/2)^5*(3*a^4*b - 4*b^5 + 13*a^2*b^3))/(a^6
- b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (2*tan(c/2 + (d*x)/2)^2*(7*a*b^4 - a^5 + 6
*a^3*b^2))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(c/2 + (d*x)/2)^7*
(7*a^4 + 5*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (b*tan(c/2 + (d*
x)/2)^3*(3*a^4 - 4*b^4 + 13*a^2*b^2))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2))
 + (a*tan(c/2 + (d*x)/2)*(5*a*b^3 + 7*a^3*b))/((a^2 - b^2)*(a^4 + b^4 - 2*a
^2*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^8 - tan(c/2 + (d*x)/2)^4*(2*a^2 + 8*b^
2) + 4*b^2*tan(c/2 + (d*x)/2)^2 + 4*b^2*tan(c/2 + (d*x)/2)^6 + a^2 - 4*a*b*
tan(c/2 + (d*x)/2)^3 - 4*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2
)^7 + 4*a*b*tan(c/2 + (d*x)/2))) + (log(tan(c/2 + (d*x)/2) + 1)*(2*a + b))/
(2*d*(a - b)^4) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)
*(3*a*b^4 + a^5 + 8*a^3*b^2))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6
*b^2))
```

3.194 $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=149

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)^3d}-\frac{\log(1+\sin(c+dx))}{2(a-b)^3d}+\frac{a(a^2+3b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}-\frac{a}{2\,(a^2-b^2)\,d(a+b\sin(c+$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^3/d-1/2*ln(1+sin(d*x+c))/(a-b)^3/d+a*(a^2+3*b^2
)*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/2*a/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+(-a
^2-b^2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2800, 815}

$$-\frac{a}{2d\,(a^2-b^2)\,(a+b\sin(c+dx))^2}-\frac{a^2+b^2}{d\,(a^2-b^2)^2\,(a+b\sin(c+dx))}+\frac{a(a^2+3b^2)\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^3}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^3}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)^3*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)
^3*d) + (a*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) - a/(2*
(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) - (a^2 + b^2)/((a^2 - b^2)^2*d*(a + b
*Sin[c + d*x]))
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+x)^3(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)^3(b-x)} + \frac{a}{(a-b)(a+b)(a+x)^3} + \frac{a^2+b^2}{(a-b)^2(a+b)^2(a+x)^2} + \frac{a^3+3ab^2}{(a-b)^3(a+b)^3(a+x)} - \right.\right.}{d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^3d} - \frac{\log(1+\sin(c+dx))}{2(a-b)^3d} + \frac{a(a^2+3b^2)\log(a+b\sin(c+}{(a^2-b^2)^3\,d}$$

Mathematica [A]
time = 1.46, size = 213, normalized size = 1.43

$$\frac{-\frac{\log(1-\sin(c+dx))}{(a+b)^2} + \frac{\log(1+\sin(c+dx))}{(a-b)^2} - \frac{4ab\log(a+b\sin(c+dx))}{(a^2-b^2)^2} + \frac{2b}{(a^2-b^2)(a+b\sin(c+dx))} + a\left(\frac{\log(1-\sin(c+dx))}{(a+b)^3} - \frac{\log(1+\sin(c+dx))}{(a-b)^3} + \frac{b\left(2(3a^2+b^2)\log(a+b\sin(c+dx)) + \frac{(a^2-b^2)(-5a^2+b^2-4ab\sin(c+dx))}{(a+b\sin(c+dx))^2}\right)}{(a^2-b^2)^3}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-(Log[1 - Sin[c + d*x]]/(a + b)^2) + Log[1 + Sin[c + d*x]]/(a - b)^2 - (4*
a*b*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^2 + (2*b)/((a^2 - b^2)*(a + b*Sin[
c + d*x])) + a*(Log[1 - Sin[c + d*x]]/(a + b)^3 - Log[1 + Sin[c + d*x]]/(a
- b)^3 + (b*(2*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]] + ((a^2 - b^2)*(-5*a^2
 + b^2 - 4*a*b*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2))/(a^2 - b^2)^3))/(2*b
*d)
```

Maple [A]
time = 0.52, size = 134, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{a}{2(a+b)(a-b)(a+b\sin(dx+c))^2} - \frac{a^2+b^2}{(a+b)^2(a-b)^2(a+b\sin(dx+c))} + \frac{a\left(a^2+3b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3} - \frac{\ln(1+\sin(dx+c))}{2(a-b)^3} - \frac{\ln(\sin(dx+c)}{2(a+b)^3}}{d}$
default	$\frac{-\frac{a}{2(a+b)(a-b)(a+b\sin(dx+c))^2} - \frac{a^2+b^2}{(a+b)^2(a-b)^2(a+b\sin(dx+c))} + \frac{a\left(a^2+3b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3} - \frac{\ln(1+\sin(dx+c))}{2(a-b)^3} - \frac{\ln(\sin(dx+c)}{2(a+b)^3}}{d}$
risch	$\frac{ix}{a^3-3a^2b+3a\,b^2-b^3} + \frac{ic}{d(a^3-3a^2b+3a\,b^2-b^3)} + \frac{ix}{a^3+3a^2b+3a\,b^2+b^3} + \frac{ic}{(a^3+3a^2b+3a\,b^2+b^3)d} - \frac{2ia^3x}{a^6-3a^4b^2+3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a/(a+b)/(a-b)/(a+b*sin(d*x+c))^2-(a^2+b^2)/(a+b)^2/(a-b)^2/(a+b*s
in(d*x+c))+a*(a^2+3*b^2)/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-1/2/(a-b)^3*ln(
1+sin(d*x+c))-1/2/(a+b)^3*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.50, size = 228, normalized size = 1.53

$$\frac{\frac{2\,\left(a^3+3\,ab^2\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{3\,a^3+ab^2+2\,\left(a^2b+b^3\right)\sin\left(dx+c\right)}{a^6-2\,a^4b^2+a^2b^4+\left(a^4b^2-2\,a^2b^4+b^6\right)\sin\left(dx+c\right)^2+2\,\left(a^5b-2\,a^3b^3+ab^5\right)\sin\left(dx+c\right)}-\frac{\log\left(\sin\left(dx+c\right)+1\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\log\left(\sin\left(dx+c\right)-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^3 + 3*a*b^2)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) - (3*a^3 + a*b^2 + 2*(a^2*b + b^3)*sin(d*x + c))/(a^6 - 2*a^4*b^2 +
 a^2*b^4 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^
3 + a*b^5)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) - log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(143) = 286.
time = 0.42, size = 462, normalized size = 3.10

$$\frac{3\,a^5-2\,a^3b^2-ab^4-2\,\left(a^5+4\,a^3b^2+3\,ab^4-\left(a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^2+2\,\left(a^4b+3\,a^2b^3\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)+\left(a^5+3\,a^4b+4\,a^3b^2+4\,a^2b^3+3\,ab^4+b^5-\left(a^3b^2+3\,a^2b^3+3\,ab^4+b^5\right)\cos\left(dx+c\right)^2+2\,\left(a^4b+3\,a^3b^2+3\,a^2b^3+ab^4\right)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^5-3\,a^4b+4\,a^3b^2-4\,a^2b^3+3\,ab^4-b^5-\left(a^3b^2-3\,a^2b^3+3\,ab^4-b^5\right)\cos\left(dx+c\right)^2+2\,\left(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(a^4b-b^5\right)\sin\left(dx+c\right)}{2\,\left(\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)d\cos\left(dx+c\right)^2-2\,\left(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7\right)d\sin\left(dx+c\right)-\left(a^8-2\,a^6b^2+2\,a^2b^6-b^8\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^5 - 2*a^3*b^2 - a*b^4 - 2*(a^5 + 4*a^3*b^2 + 3*a*b^4 - (a^3*b^2 +
3*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^2*b^3)*sin(d*x + c))*log(b*sin(d*x
 + c) + a) + (a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*
b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*
a^2*b^3 + a*b^4)*sin(d*x + c))*log(sin(d*x + c) + 1) + (a^5 - 3*a^4*b + 4*a
^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*
cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*sin(d*x + c))*lo
g(-sin(d*x + c) + 1) + 2*(a^4*b - b^5)*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4
+ 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*
b^7)*d*sin(d*x + c) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 13.36, size = 257, normalized size = 1.72

$$\frac{\frac{2\,(a^3b+3\,ab^3)\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7} - \frac{\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{3\,a^3b^2\sin(dx+c)^2+9\,ab^4\sin(dx+c)^2+8\,a^4b\sin(dx+c)+18\,a^2b^3\sin(dx+c)-2\,b^5\sin(dx+c)+6\,a^5+7\,a^3b^2-ab^4}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)(b\sin(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3*b + 3*a*b^3)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 +
3*a^2*b^5 - b^7) - log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3) - log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^3*b^
2*sin(d*x + c)^2 + 9*a*b^4*sin(d*x + c)^2 + 8*a^4*b*sin(d*x + c) + 18*a^2*b
^3*sin(d*x + c) - 2*b^5*sin(d*x + c) + 6*a^5 + 7*a^3*b^2 - a*b^4)/((a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*(b*sin(d*x + c) + a)^2))/d
```

Mupad [B]
time = 6.85, size = 304, normalized size = 2.04

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(3\,a^2\,b^2+b^4)}{a\,(a^4-2\,a^2\,b^2+b^4)}+\frac{4\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{4\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^4-2\,a^2\,b^2+b^4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2+4\,b^2)+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d\,(a+b)^3}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{d\,(a-b)^3}+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,(a^3+3\,a\,b^2)}{d\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^2*(b^4 + 3*a^2*b^2))/(a*(a^4 + b^4 - 2*a^2*b^2)) + (
4*a^2*b*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^2*b^2) + (4*a^2*b*tan(c/2 + (d
*x)/2)^3)/(a^4 + b^4 - 2*a^2*b^2))/(d*(tan(c/2 + (d*x)/2)^2*(2*a^2 + 4*b^2)
 + a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(
c/2 + (d*x)/2))) - log(tan(c/2 + (d*x)/2) - 1)/(d*(a + b)^3) - log(tan(c/2
+ (d*x)/2) + 1)/(d*(a - b)^3) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2
 + (d*x)/2)^2)*(3*a*b^2 + a^3))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
```

3.195 $\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3} dx$

Optimal. Leaf size=75

$$\frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(a+b\sin(c+dx))}{a^3d}+\frac{1}{2ad(a+b\sin(c+dx))^2}+\frac{1}{a^2d(a+b\sin(c+dx))}$$

```
[Out] ln(sin(d*x+c))/a^3/d-ln(a+b*sin(d*x+c))/a^3/d+1/2/a/d/(a+b*sin(d*x+c))^2+1/
a^2/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2800, 46}

$$-\frac{\log(a+b\sin(c+dx))}{a^3d}+\frac{\log(\sin(c+dx))}{a^3d}+\frac{1}{a^2d(a+b\sin(c+dx))}+\frac{1}{2ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[a + b*Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a +
 b*Sin[c + d*x])^2) + 1/(a^2*d*(a + b*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{1}{a(a+x)^3} - \frac{1}{a^2(a+x)^2} - \frac{1}{a^3(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(a+b\sin(c+dx))}{a^3d} + \frac{1}{2ad(a+b\sin(c+dx))^2} + \frac{}{a^2d(a-}$$

Mathematica [A]
time = 0.18, size = 60, normalized size = 0.80

$$\frac{2\log(\sin(c+dx)) - 2\log(a+b\sin(c+dx)) + \frac{a(3a+2b\sin(c+dx))}{(a+b\sin(c+dx))^2}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*Log[Sin[c + d*x]] - 2*Log[a + b*Sin[c + d*x]] + (a*(3*a + 2*b*Sin[c + d*
x]))/(a + b*Sin[c + d*x])^2)/(2*a^3*d)
```

Maple [A]
time = 0.28, size = 66, normalized size = 0.88

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sin(dx+c))}{a^3}+\frac{1}{a^2(a+b\sin(dx+c))}+\frac{1}{2a(a+b\sin(dx+c))^2}+\frac{\ln(\sin(dx+c))}{a^3}}{d}$	66
default	$\frac{-\frac{\ln(a+b\sin(dx+c))}{a^3}+\frac{1}{a^2(a+b\sin(dx+c))}+\frac{1}{2a(a+b\sin(dx+c))^2}+\frac{\ln(\sin(dx+c))}{a^3}}{d}$	66
risch	$\frac{2i\left(3ia\,e^{2i(dx+c)}+b\,e^{3i(dx+c)}-b\,e^{i(dx+c)}\right)}{\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)^2a^2d}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{d\,a^3}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{d\,a^3}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^3*ln(a+b*sin(d*x+c))+1/a^2/(a+b*sin(d*x+c))+1/2/a/(a+b*sin(d*x+c)
)^2+1/a^3*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.55, size = 81, normalized size = 1.08

$$\frac{\frac{2\,b\sin(dx+c)+3\,a}{a^2b^2\sin(dx+c)^2+2\,a^3b\sin(dx+c)+a^4} - \frac{2\,\log(b\sin(dx+c)+a)}{a^3} + \frac{2\,\log(\sin(dx+c))}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((2*b*sin(d*x + c) + 3*a)/(a^2*b^2*sin(d*x + c)^2 + 2*a^3*b*sin(d*x + c
) + a^4) - 2*log(b*sin(d*x + c) + a)/a^3 + 2*log(sin(d*x + c))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(73) = 146$.
time = 0.37, size = 154, normalized size = 2.05

$$-\frac{2\,ab\sin\left(dx+c\right)+3\,a^2+2\,\left(b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)-2\,\left(b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2\right)\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)}{2\,\left(a^3b^2d\cos\left(dx+c\right)^2-2\,a^4bd\sin\left(dx+c\right)-\left(a^5+a^3b^2\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) + 3*a^2 + 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
c) - a^2 - b^2)*log(b*sin(d*x + c) + a) - 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2)*log(-1/2*sin(d*x + c)))/(a^3*b^2*d*cos(d*x + c)^2 -
2*a^4*b*d*sin(d*x + c) - (a^5 + a^3*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 13.90, size = 69, normalized size = 0.92

$$-\frac{\frac{2\,\log\left(\left|b\sin(dx+c)+a\right|\right)}{a^3}-\frac{2\,\log\left(\left|\sin(dx+c)\right|\right)}{a^3}-\frac{2\,ab\sin(dx+c)+3\,a^2}{\left(b\sin(dx+c)+a\right)^2a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(b*sin(d*x + c) + a))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 - (
2*a*b*sin(d*x + c) + 3*a^2)/((b*sin(d*x + c) + a)^2*a^3))/d
```

Mupad [B]
time = 6.54, size = 369, normalized size = 4.92

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}{a^3\,d}-\frac{6\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{d\,\left(a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\,a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^5+4\,a^4\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\,a^4\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+4\,a^3\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}-\frac{4\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^4+4\,a^3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\,a^3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+4\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}-\frac{4\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^4+4\,a^3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\,a^3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+4\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/
2 + (d*x)/2)^2)/(a^3*d) - (6*b^2*tan(c/2 + (d*x)/2)^2)/(d*(2*a^5*tan(c/2 +
(d*x)/2)^2 + a^5*tan(c/2 + (d*x)/2)^4 + a^5 + 4*a^3*b^2*tan(c/2 + (d*x)/2)^
2 + 4*a^4*b*tan(c/2 + (d*x)/2) + 4*a^4*b*tan(c/2 + (d*x)/2)^3)) - (4*b*tan(
c/2 + (d*x)/2))/(d*(2*a^4*tan(c/2 + (d*x)/2)^2 + a^4*tan(c/2 + (d*x)/2)^4 +
 a^4 + 4*a^2*b^2*tan(c/2 + (d*x)/2)^2 + 4*a^3*b*tan(c/2 + (d*x)/2) + 4*a^3*
b*tan(c/2 + (d*x)/2)^3)) - (4*b*tan(c/2 + (d*x)/2)^3)/(d*(2*a^4*tan(c/2 + (
d*x)/2)^2 + a^4*tan(c/2 + (d*x)/2)^4 + a^4 + 4*a^2*b^2*tan(c/2 + (d*x)/2)^2
 + 4*a^3*b*tan(c/2 + (d*x)/2) + 4*a^3*b*tan(c/2 + (d*x)/2)^3))
```

3.196 $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{3b\csc(c+dx)}{a^4d}-\frac{\csc^2(c+dx)}{2a^3d}-\frac{(a^2-6b^2)\log(\sin(c+dx))}{a^5d}+\frac{(a^2-6b^2)\log(a+b\sin(c+dx))}{a^5d}-\frac{a^2-b}{2a^3d(a+b\sin}$$

```
[Out] 3*b*csc(d*x+c)/a^4/d-1/2*csc(d*x+c)^2/a^3/d-(a^2-6*b^2)*ln(sin(d*x+c))/a^5/
d+(a^2-6*b^2)*ln(a+b*sin(d*x+c))/a^5/d+1/2*(-a^2+b^2)/a^3/d/(a+b*sin(d*x+c)
)^2+(-a^2+3*b^2)/a^4/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 908}

$$\frac{3b\csc(c+dx)}{a^4d}-\frac{\csc^2(c+dx)}{2a^3d}-\frac{(a^2-6b^2)\log(\sin(c+dx))}{a^5d}+\frac{(a^2-6b^2)\log(a+b\sin(c+dx))}{a^5d}-\frac{a^2-3b^2}{a^4d(a+b\sin(c+dx))}-\frac{a^2-b^2}{2a^3d(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^3*d) - ((a^2 - 6*b^2)*Log[
Sin[c + d*x]])/(a^5*d) + ((a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]])/(a^5*d) -
(a^2 - b^2)/(2*a^3*d*(a + b*Sin[c + d*x])^2) - (a^2 - 3*b^2)/(a^4*d*(a + b*
Sin[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{a^3x^3} - \frac{3b^2}{a^4x^2} + \frac{-a^2+6b^2}{a^5x} + \frac{a^2-b^2}{a^3(a+x)^3} + \frac{a^2-3b^2}{a^4(a+x)^2} + \frac{a^2-6b^2}{a^5(a+x)}\right)dx, x, b\sin(c-\right.}{d}$$
$$= \frac{3b\csc(c+dx)}{a^4d} - \frac{\csc^2(c+dx)}{2a^3d} - \frac{(a^2-6b^2)\log(\sin(c+dx))}{a^5d} + \frac{(a^2-6b^2)\log(\ldots}{\ldots}$$

Mathematica [A]
time = 0.63, size = 121, normalized size = 0.83

$$-\frac{-6ab\csc(c+dx) + a^2\csc^2(c+dx) + 2(a^2-6b^2)\log(\sin(c+dx)) - 2(a^2-6b^2)\log(a+b\sin(c+dx)) + \frac{a^2(a-b)(a+b)}{(a+b\sin(c+dx))^2} + \frac{2a(a^2-3b^2)}{a+b\sin(c+dx)}}{2a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
[Out] -1/2*(-6*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - 6*b^2)*Log[Sin[c
+ d*x]] - 2*(a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]] + (a^2*(a - b)*(a + b))/(
a + b*Sin[c + d*x])^2 + (2*a*(a^2 - 3*b^2))/(a + b*Sin[c + d*x]))/(a^5*d)
```

Maple [A]
time = 0.45, size = 131, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{1}{2a^3\sin(dx+c)^2} + \frac{(-a^2+6b^2)\ln(\sin(dx+c))}{a^5} + \frac{3b}{a^4\sin(dx+c)} + \frac{(a^2-6b^2)\ln(a+b\sin(dx+c))}{a^5} - \frac{a^2-3b^2}{a^4(a+b\sin(dx+c))} - \frac{a^2-b^2}{2a^3(a+b\sin(dx\ldots}}{d}$
default	$\frac{-\frac{1}{2a^3\sin(dx+c)^2} + \frac{(-a^2+6b^2)\ln(\sin(dx+c))}{a^5} + \frac{3b}{a^4\sin(dx+c)} + \frac{(a^2-6b^2)\ln(a+b\sin(dx+c))}{a^5} - \frac{a^2-3b^2}{a^4(a+b\sin(dx+c))} - \frac{a^2-b^2}{2a^3(a+b\sin(dx\ldots}}{d}$
risch	$-\frac{2i(3ia^3e^{6i(dx+c)} - 18iab^2e^{6i(dx+c)} + a^2be^{7i(dx+c)} - 6b^3e^{7i(dx+c)} - 10ia^3e^{4i(dx+c)} + 36iab^2e^{4i(dx+c)} + 5a^2be^{5i(dx+c)} + \ldots}{(e^{2i(dx+c)}-1)^2(be^{2i(dx+c)}-\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/2/a^3/sin(d*x+c)^2+(-a^2+6*b^2)/a^5*ln(sin(d*x+c))+3/a^4*b/sin(d*x+
c)+(a^2-6*b^2)/a^5*ln(a+b*sin(d*x+c))-(a^2-3*b^2)/a^4/(a+b*sin(d*x+c))-1/2*
(a^2-b^2)/a^3/(a+b*sin(d*x+c))^2)
```

Maxima [A]
time = 0.50, size = 156, normalized size = 1.08

$$\frac{\frac{4a^2b\sin(dx+c) - 2(a^2b-6b^3)\sin(dx+c)^3 - a^3 - 3(a^3-6ab^2)\sin(dx+c)^2}{a^4b^2\sin(dx+c)^4 + 2a^5b\sin(dx+c)^3 + a^6\sin(dx+c)^2} + \frac{2(a^2-6b^2)\log(b\sin(dx+c)+a)}{a^5} - \frac{2(a^2-6b^2)\log(\sin(dx+c))}{a^5}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((4*a^2*b*sin(d*x + c) - 2*(a^2*b - 6*b^3)*sin(d*x + c)^3 - a^3 - 3*(a^
3 - 6*a*b^2)*sin(d*x + c)^2)/(a^4*b^2*sin(d*x + c)^4 + 2*a^5*b*sin(d*x + c)
^3 + a^6*sin(d*x + c)^2) + 2*(a^2 - 6*b^2)*log(b*sin(d*x + c) + a)/a^5 - 2*
(a^2 - 6*b^2)*log(sin(d*x + c))/a^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(141) = 282$.
time = 0.40, size = 404, normalized size = 2.79

$$-\frac{4\,a^{4}-18\,a^{2}b^{2}-3\,(a^{4}-6\,a^{2}b^{2})\cos\left(dx+c\right)^{2}-2\,\left((a^{2}b^{2}-6\,b^{4})\cos\left(dx+c\right)^{4}+a^{4}-5\,a^{2}b^{2}-6\,b^{4}-(a^{4}-4\,a^{2}b^{2}-12\,b^{4})\cos\left(dx+c\right)^{2}+2\,\left(a^{3}b-6\,ab^{3}-(a^{3}b-6\,ab^{3})\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)+2\,\left((a^{2}b^{2}-6\,b^{4})\cos\left(dx+c\right)^{4}+a^{4}-5\,a^{2}b^{2}-6\,b^{4}-(a^{4}-4\,a^{2}b^{2}-12\,b^{4})\cos\left(dx+c\right)^{2}+2\,\left(a^{3}b-6\,ab^{3}-(a^{3}b-6\,ab^{3})\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)-2\,\left(a^{3}b+6\,ab^{3}+(a^{3}b-6\,ab^{3})\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{2\,\left(a^{5}b^{2}d\cos\left(dx+c\right)^{4}-(a^{7}+2\,a^{5}b^{2})d\cos\left(dx+c\right)^{2}+(a^{7}+a^{5}b^{2})d-2\,\left(a^{6}bd\cos\left(dx+c\right)^{2}-a^{6}bd\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^4 - 18*a^2*b^2 - 3*(a^4 - 6*a^2*b^2)*cos(d*x + c)^2 - 2*((a^2*b^2
 - 6*b^4)*cos(d*x + c)^4 + a^4 - 5*a^2*b^2 - 6*b^4 - (a^4 - 4*a^2*b^2 - 12*
b^4)*cos(d*x + c)^2 + 2*(a^3*b - 6*a*b^3 - (a^3*b - 6*a*b^3)*cos(d*x + c)^2
)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 2*((a^2*b^2 - 6*b^4)*cos(d*x + c)
^4 + a^4 - 5*a^2*b^2 - 6*b^4 - (a^4 - 4*a^2*b^2 - 12*b^4)*cos(d*x + c)^2 +
2*(a^3*b - 6*a*b^3 - (a^3*b - 6*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1
/2*sin(d*x + c)) - 2*(a^3*b + 6*a*b^3 + (a^3*b - 6*a*b^3)*cos(d*x + c)^2)*s
in(d*x + c))/(a^5*b^2*d*cos(d*x + c)^4 - (a^7 + 2*a^5*b^2)*d*cos(d*x + c)^2
 + (a^7 + a^5*b^2)*d - 2*(a^6*b*d*cos(d*x + c)^2 - a^6*b*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{3}\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 7.39, size = 154, normalized size = 1.06

$$-\frac{\frac{2\,\left(a^{2}-6\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^{5}}-\frac{2\,\left(a^{2}b-6\,b^{3}\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{5}b}+\frac{2\,a^{2}b\sin\left(dx+c\right)^{3}-12\,b^{3}\sin\left(dx+c\right)^{3}+3\,a^{3}\sin\left(dx+c\right)^{2}-18\,ab^{2}\sin\left(dx+c\right)^{2}-4\,a^{2}b\sin\left(dx+c\right)+a^{3}}{\left(b\sin\left(dx+c\right)^{2}+a\sin\left(dx+c\right)\right)^{2}a^{4}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - 6*b^2)*log(abs(sin(d*x + c)))/a^5 - 2*(a^2*b - 6*b^3)*log(ab
s(b*sin(d*x + c) + a))/(a^5*b) + (2*a^2*b*sin(d*x + c)^3 - 12*b^3*sin(d*x +
 c)^3 + 3*a^3*sin(d*x + c)^2 - 18*a*b^2*sin(d*x + c)^2 - 4*a^2*b*sin(d*x +
c) + a^3)/((b*sin(d*x + c)^2 + a*sin(d*x + c))^2*a^4))/d
```

Mupad [B]
time = 6.79, size = 334, normalized size = 2.30

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(22\,a\,b^2-a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(26\,a^2\,b-8\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(22\,a^2\,b-32\,b^3\right)-\frac{a^3}{2}+4\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^4-96\,a^2\,b^2+112\,b^4\right)}{2\,a}}{d\,\left(4\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(8\,a^6+16\,a^4\,b^2\right)+16\,a^5\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+16\,a^5\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}+\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^4\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-6\,b^2\right)}{a^5\,d}+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^2-6\,b^2\right)}{a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(22*a*b^2 - a^3) + tan(c/2 + (d*x)/2)^3*(26*a^2*b - 8
*b^3) + tan(c/2 + (d*x)/2)^5*(22*a^2*b - 32*b^3) - a^3/2 + 4*a^2*b*tan(c/2
+ (d*x)/2) - (tan(c/2 + (d*x)/2)^4*(a^4 + 112*b^4 - 96*a^2*b^2))/(2*a))/(d*
(4*a^6*tan(c/2 + (d*x)/2)^2 + 4*a^6*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/
2)^4*(8*a^6 + 16*a^4*b^2) + 16*a^5*b*tan(c/2 + (d*x)/2)^3 + 16*a^5*b*tan(c/
2 + (d*x)/2)^5)) - tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (3*b*tan(c/2 + (d*x)/2)
)/(2*a^4*d) - (log(tan(c/2 + (d*x)/2))*(a^2 - 6*b^2))/(a^5*d) + (log(a + 2*
b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 - 6*b^2))/(a^5*d)
```

3.197 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{2b(3a^2-5b^2)\csc(c+dx)}{a^6d}+\frac{(a^2-3b^2)\csc^2(c+dx)}{a^5d}+\frac{b\csc^3(c+dx)}{a^4d}-\frac{\csc^4(c+dx)}{4a^3d}+\frac{(a^4-12a^2b^2+15b^4)\,\mathrm{lo}}{a^7d}$$

```
[Out] -2*b*(3*a^2-5*b^2)*csc(d*x+c)/a^6/d+(a^2-3*b^2)*csc(d*x+c)^2/a^5/d+b*csc(d*
x+c)^3/a^4/d-1/4*csc(d*x+c)^4/a^3/d+(a^4-12*a^2*b^2+15*b^4)*ln(sin(d*x+c))/
a^7/d-(a^4-12*a^2*b^2+15*b^4)*ln(a+b*sin(d*x+c))/a^7/d+1/2*(a^2-b^2)^2/a^5/
d/(a+b*sin(d*x+c))^2+(a^4-6*a^2*b^2+5*b^4)/a^6/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 908}

$$\frac{b\csc^3(c+dx)}{a^4d}-\frac{\csc^4(c+dx)}{4a^3d}-\frac{2b(3a^2-5b^2)\csc(c+dx)}{a^6d}+\frac{(a^2-b^2)^2}{2a^5d(a+b\sin(c+dx))^2}+\frac{(a^2-3b^2)\csc^2(c+dx)}{a^5d}+\frac{(a^4-12a^2b^2+15b^4)\log(\sin(c+dx))}{a^7d}-\frac{(a^4-12a^2b^2+15b^4)\log(a+b\sin(c+dx))}{a^7d}+\frac{a^4-6a^2b^2+5b^4}{a^6d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(3*a^2 - 5*b^2)*Csc[c + d*x])/(a^6*d) + ((a^2 - 3*b^2)*Csc[c + d*x]^2
)/(a^5*d) + (b*Csc[c + d*x]^3)/(a^4*d) - Csc[c + d*x]^4/(4*a^3*d) + ((a^4 -
 12*a^2*b^2 + 15*b^4)*Log[Sin[c + d*x]])/(a^7*d) - ((a^4 - 12*a^2*b^2 + 15*
b^4)*Log[a + b*Sin[c + d*x]])/(a^7*d) + (a^2 - b^2)^2/(2*a^5*d*(a + b*Sin[c
 + d*x])^2) + (a^4 - 6*a^2*b^2 + 5*b^4)/(a^6*d*(a + b*Sin[c + d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^5(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^3x^5} - \frac{3b^4}{a^4x^4} + \frac{2b^2(-a^2+3b^2)}{a^5x^3} + \frac{2(3a^2b^2-5b^4)}{a^6x^2} + \frac{a^4-12a^2b^2+15b^4}{a^7x} - \frac{(a^2-b^2)^2}{a^5(a+x)^3} + \right.\right.}{d}$$

$$= -\frac{2b(3a^2-5b^2)\csc(c+dx)}{a^6d} + \frac{(a^2-3b^2)\csc^2(c+dx)}{a^5d} + \frac{b\csc^3(c+dx)}{a^4d} - \frac{\csc^4(}{4}$$

Mathematica [A]

time = 4.89, size = 195, normalized size = 0.88

$$\frac{-8ab(3a^2-5b^2)\csc(c+dx) + 4a^2(a^2-3b^2)\csc^2(c+dx) + 4a^3b\csc^3(c+dx) - a^4\csc^4(c+dx) + 4(a^4-12a^2b^2+15b^4)\log(\sin(c+dx)) - 4(a^4-12a^2b^2+15b^4)\log(a+b\sin(c+dx)) + \frac{2(a^3-ab^2)^2}{(a+b\sin(c+dx))^2} + \frac{4a(a^4-6a^2b^2+5b^4)}{a+b\sin(c+dx)}}{4a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-8*a*b*(3*a^2 - 5*b^2)*Csc[c + d*x] + 4*a^2*(a^2 - 3*b^2)*Csc[c + d*x]^2 +
 4*a^3*b*Csc[c + d*x]^3 - a^4*Csc[c + d*x]^4 + 4*(a^4 - 12*a^2*b^2 + 15*b^4
)*Log[Sin[c + d*x]] - 4*(a^4 - 12*a^2*b^2 + 15*b^4)*Log[a + b*Sin[c + d*x]]
 + (2*(a^3 - a*b^2)^2)/(a + b*Sin[c + d*x])^2 + (4*a*(a^4 - 6*a^2*b^2 + 5*b
^4))/(a + b*Sin[c + d*x]))/(4*a^7*d)
```

Maple [A]

time = 0.57, size = 207, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{1}{4a^3\sin(dx+c)^4} - \frac{-2a^2+6b^2}{2a^5\sin(dx+c)^2} + \frac{\left(a^4-12a^2b^2+15b^4\right)\ln(\sin(dx+c))}{a^7} + \frac{b}{a^4\sin(dx+c)^3} - \frac{2b\left(3a^2-5b^2\right)}{a^6\sin(dx+c)} - \frac{\left(a^4-12a^2b^2+15b^4\right)\ln(}{a^7}}{d}$
default	$\frac{-\frac{1}{4a^3\sin(dx+c)^4} - \frac{-2a^2+6b^2}{2a^5\sin(dx+c)^2} + \frac{\left(a^4-12a^2b^2+15b^4\right)\ln(\sin(dx+c))}{a^7} + \frac{b}{a^4\sin(dx+c)^3} - \frac{2b\left(3a^2-5b^2\right)}{a^6\sin(dx+c)} - \frac{\left(a^4-12a^2b^2+15b^4\right)\ln(}{a^7}}{d}$
risch	$\frac{2i\left(-150b^5e^{5i(dx+c)} + 75b^5e^{3i(dx+c)} + 15b^5e^{11i(dx+c)} - 75b^5e^{9i(dx+c)} + 150b^5e^{7i(dx+c)} - 15b^5e^{i(dx+c)} - 30b\,a^4e^{7i(dx+c)} - 6\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/a^3/sin(d*x+c)^4-1/2*(-2*a^2+6*b^2)/a^5/sin(d*x+c)^2+(a^4-12*a^2*
b^2+15*b^4)/a^7*ln(sin(d*x+c))+1/a^4*b/sin(d*x+c)^3-2*b*(3*a^2-5*b^2)/a^6/s
in(d*x+c)-(a^4-12*a^2*b^2+15*b^4)/a^7*ln(a+b*sin(d*x+c))+(a^4-6*a^2*b^2+5*b
^4)/a^6/(a+b*sin(d*x+c))+1/2*(a^4-2*a^2*b^2+b^4)/a^5/(a+b*sin(d*x+c))^2)
```

Maxima [A]
time = 0.48, size = 236, normalized size = 1.07

$$\frac{\frac{2\,a^4b\sin(dx+c)+4\left(a^4b-12\,a^2b^3+15\,b^5\right)\sin(dx+c)^5-a^5+6\left(a^5-12\,a^3b^2+15\,ab^4\right)\sin(dx+c)^4-4\left(4\,a^4b-5\,a^2b^3\right)\sin(dx+c)^3+\left(4\,a^5-5\,a^3b^2\right)\sin(dx+c)^2}{a^6b^2\sin(dx+c)^6+2\,a^7b\sin(dx+c)^5+a^8\sin(dx+c)^4}-\frac{4\left(a^4-12\,a^2b^2+15\,b^4\right)\log(b\sin(dx+c)+a)}{a^7}+\frac{4\left(a^4-12\,a^2b^2+15\,b^4\right)\log(\sin(dx+c))}{a^7}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((2*a^4*b*sin(d*x + c) + 4*(a^4*b - 12*a^2*b^3 + 15*b^5)*sin(d*x + c)^5
 - a^5 + 6*(a^5 - 12*a^3*b^2 + 15*a*b^4)*sin(d*x + c)^4 - 4*(4*a^4*b - 5*a^
2*b^3)*sin(d*x + c)^3 + (4*a^5 - 5*a^3*b^2)*sin(d*x + c)^2)/(a^6*b^2*sin(d*
x + c)^6 + 2*a^7*b*sin(d*x + c)^5 + a^8*sin(d*x + c)^4) - 4*(a^4 - 12*a^2*b
^2 + 15*b^4)*log(b*sin(d*x + c) + a)/a^7 + 4*(a^4 - 12*a^2*b^2 + 15*b^4)*lo
g(sin(d*x + c))/a^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 754 vs. 2(217) = 434.
time = 0.42, size = 754, normalized size = 3.41

$$-\frac{9\,a^6-77\,a^4b^2+90\,a^2b^4+6\left(a^6-12\,a^4b^2+15\,a^2b^4\right)\cos(dx+c)^4-\left(16\,a^6-149\,a^4b^2+180\,a^2b^4\right)\cos(dx+c)^2+4\left(\left(a^4b^2-12\,a^2b^4+15\,b^6\right)\cos(dx+c)^6-a^6+11\,a^4b^2-3\,a^2b^4-15\,b^6-\left(a^6-9\,a^4b^2-21\,a^2b^4+45\,b^6\right)\cos(dx+c)^4+\left(2\,a^6-21\,a^4b^2-6\,a^2b^4+45\,b^6\right)\cos(dx+c)^2-2\left(a^5b-12\,a^3b^3+15\,ab^5+\left(a^5b-12\,a^3b^3+15\,ab^5\right)\cos(dx+c)^4-2\left(a^5b-12\,a^3b^3+15\,ab^5\right)\cos(dx+c)^2\right)\sin(dx+c)\right)\log(b\sin(dx+c)+a)-4\left(\left(a^4b^2-12\,a^2b^4+15\,b^6\right)\cos(dx+c)^6-a^6+11\,a^4b^2-3\,a^2b^4-15\,b^6-\left(a^6-9\,a^4b^2-21\,a^2b^4+45\,b^6\right)\cos(dx+c)^4+\left(2\,a^6-21\,a^4b^2-6\,a^2b^4+45\,b^6\right)\cos(dx+c)^2-2\left(a^5b-12\,a^3b^3+15\,ab^5+\left(a^5b-12\,a^3b^3+15\,ab^5\right)\cos(dx+c)^4-2\left(a^5b-12\,a^3b^3+15\,ab^5\right)\cos(dx+c)^2\right)\sin(dx+c)\right)\log\left(-\frac{1}{2}\sin(dx+c)\right)-2\left(5\,a^5b+14\,a^3b^3-30\,ab^5-2\left(a^5b-12\,a^3b^3+15\,ab^5\right)\cos(dx+c)^4-2\left(2\,a^5b+19\,a^3b^3-30\,ab^5\right)\cos(dx+c)^2\right)\sin(dx+c)}{4\left(a^7b^2d\cos(dx+c)^6-\left(a^9+3\,a^7b^2\right)d\cos(dx+c)^4+\left(2\,a^9+3\,a^7b^2\right)d\cos(dx+c)^2-\left(a^9+a^7b^2\right)d-2\left(a^8bd\cos(dx+c)^4-2\,a^8bd\cos(dx+c)^2+a^8bd\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(9*a^6 - 77*a^4*b^2 + 90*a^2*b^4 + 6*(a^6 - 12*a^4*b^2 + 15*a^2*b^4)*c
os(d*x + c)^4 - (16*a^6 - 149*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^2 + 4*((a
^4*b^2 - 12*a^2*b^4 + 15*b^6)*cos(d*x + c)^6 - a^6 + 11*a^4*b^2 - 3*a^2*b^4
 - 15*b^6 - (a^6 - 9*a^4*b^2 - 21*a^2*b^4 + 45*b^6)*cos(d*x + c)^4 + (2*a^6
 - 21*a^4*b^2 - 6*a^2*b^4 + 45*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 12*a^3*b^3
+ 15*a*b^5 + (a^5*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 12
*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a)
- 4*((a^4*b^2 - 12*a^2*b^4 + 15*b^6)*cos(d*x + c)^6 - a^6 + 11*a^4*b^2 - 3*
a^2*b^4 - 15*b^6 - (a^6 - 9*a^4*b^2 - 21*a^2*b^4 + 45*b^6)*cos(d*x + c)^4 +
 (2*a^6 - 21*a^4*b^2 - 6*a^2*b^4 + 45*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 12*a
^3*b^3 + 15*a*b^5 + (a^5*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^4 - 2*(a^5
*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x
+ c)) - 2*(5*a^5*b + 14*a^3*b^3 - 30*a*b^5 - 2*(a^5*b - 12*a^3*b^3 + 15*a*b
^5)*cos(d*x + c)^4 - 2*(2*a^5*b + 19*a^3*b^3 - 30*a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))/(a^7*b^2*d*cos(d*x + c)^6 - (a^9 + 3*a^7*b^2)*d*cos(d*x + c)^4
+ (2*a^9 + 3*a^7*b^2)*d*cos(d*x + c)^2 - (a^9 + a^7*b^2)*d - 2*(a^8*b*d*cos
(d*x + c)^4 - 2*a^8*b*d*cos(d*x + c)^2 + a^8*b*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5{(c+dx)}}{(a+b\sin{(c+dx)})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 8.10, size = 327, normalized size = 1.48

$$\frac{\frac{12\,(a^4-12\,a^2b^2+15\,b^4)\log(|\sin(dx+c)|)}{a^7}-\frac{12\,(a^4b-12\,a^2b^3+15\,b^5)\log(|b\sin(dx+c)+a|)}{a^7b}+\frac{6\left(3\,a^4b^2\sin(dx+c)^2-36\,a^2b^4\sin(dx+c)^2+45\,b^6\sin(dx+c)^2+8\,a^5b\sin(dx+c)-84\,a^3b^3\sin(dx+c)+100\,ab^5\sin(dx+c)+6\,a^6-50\,a^4b^2+56\,a^2b^4\right)}{(b\sin(dx+c)+a)^2a^7}-\frac{25\,a^4\sin(dx+c)^4-300\,a^2b^2\sin(dx+c)^4+375\,b^4\sin(dx+c)^4+72\,a^3b\sin(dx+c)^3-120\,ab^3\sin(dx+c)^3-12\,a^4\sin(dx+c)^2+36\,a^2b^2\sin(dx+c)^2-12\,a^3b\sin(dx+c)+3\,a^4}{a^7\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 12*a^2*b^2 + 15*b^4)*log(abs(sin(d*x + c)))/a^7 - 12*(a^4*b
 - 12*a^2*b^3 + 15*b^5)*log(abs(b*sin(d*x + c) + a))/(a^7*b) + 6*(3*a^4*b^2
*sin(d*x + c)^2 - 36*a^2*b^4*sin(d*x + c)^2 + 45*b^6*sin(d*x + c)^2 + 8*a^5
*b*sin(d*x + c) - 84*a^3*b^3*sin(d*x + c) + 100*a*b^5*sin(d*x + c) + 6*a^6
- 50*a^4*b^2 + 56*a^2*b^4)/((b*sin(d*x + c) + a)^2*a^7) - (25*a^4*sin(d*x +
 c)^4 - 300*a^2*b^2*sin(d*x + c)^4 + 375*b^4*sin(d*x + c)^4 + 72*a^3*b*sin(
d*x + c)^3 - 120*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 36*a^2*b^2*
sin(d*x + c)^2 - 12*a^3*b*sin(d*x + c) + 3*a^4)/(a^7*sin(d*x + c)^4))/d
```

Mupad [B]
time = 7.26, size = 563, normalized size = 2.55

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{23\,a^5}{4}-172\,a^3b^2+272\,a\,b^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(27\,a^4b-40\,a^2b^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(-134\,a^4b+200\,a^2b^3+128\,b^5\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(106\,a^4b-336\,a^2b^3+192\,b^5\right)-\frac{a^5}{4}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{5\,a^5}{2}-5\,a^3b^2\right)+a^4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(3\,a^6-276\,a^4b^2+768\,a^2b^4-352\,b^6\right)}{a}}{d\left(16\,a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+16\,a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(32\,a^8+64\,a^6b^2\right)+64\,a^7b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+64\,a^7b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,a^3d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3\,(a^2+4\,b^2)}{32\,a^5}+\frac{3}{32\,a^3}-\frac{9\,b^2}{8\,a^5}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{6\,b\left(\frac{3\,(a^2+4\,b^2)}{16\,a^5}+\frac{3}{16\,a^3}-\frac{9\,b^2}{4\,a^5}\right)}{a}-\frac{192\,a^2b+128\,b^3}{256\,a^6}+\frac{9\,b\,(a^2+4\,b^2)}{8\,a^6}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^4-12\,a^2b^2+15\,b^4\right)}{a^7d}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{8\,a^4d}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\left(a^4-12\,a^2b^2+15\,b^4\right)}{a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(272*a*b^4 + (23*a^5)/4 - 172*a^3*b^2) - tan(c/2 + (d
*x)/2)^3*(27*a^4*b - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^5*(128*b^5 - 134*a^4*
b + 200*a^2*b^3) - tan(c/2 + (d*x)/2)^7*(106*a^4*b + 192*b^5 - 336*a^2*b^3)
 - a^5/4 + tan(c/2 + (d*x)/2)^2*((5*a^5)/2 - 5*a^3*b^2) + a^4*b*tan(c/2 + (
d*x)/2) + (tan(c/2 + (d*x)/2)^6*(3*a^6 - 352*b^6 + 768*a^2*b^4 - 276*a^4*b^
2))/a)/(d*(16*a^8*tan(c/2 + (d*x)/2)^4 + 16*a^8*tan(c/2 + (d*x)/2)^8 + tan(
c/2 + (d*x)/2)^6*(32*a^8 + 64*a^6*b^2) + 64*a^7*b*tan(c/2 + (d*x)/2)^5 + 64
*a^7*b*tan(c/2 + (d*x)/2)^7)) - tan(c/2 + (d*x)/2)^4/(64*a^3*d) + (tan(c/2
+ (d*x)/2)^2*((3*(a^2 + 4*b^2))/(32*a^5) + 3/(32*a^3) - (9*b^2)/(8*a^5)))/d
 - (tan(c/2 + (d*x)/2)*((6*b*((3*(a^2 + 4*b^2))/(16*a^5) + 3/(16*a^3) - (9*
b^2)/(4*a^5)))/a - (192*a^2*b + 128*b^3)/(256*a^6) + (9*b*(a^2 + 4*b^2))/(8
*a^6)))/d + (log(tan(c/2 + (d*x)/2))*(a^4 + 15*b^4 - 12*a^2*b^2))/(a^7*d) +
 (b*tan(c/2 + (d*x)/2)^3)/(8*a^4*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*t
an(c/2 + (d*x)/2)^2)*(a^4 + 15*b^4 - 12*a^2*b^2))/(a^7*d)
```

3.198 $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=474

$$\frac{8a^4b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d}+\frac{12a^2b^2(a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d}+\frac{a^4(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d}$$

```
[Out] 8*a^4*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(9/2)/
d+12*a^2*b^2*(a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^
2-b^2)^(9/2)/d+a^4*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))/(a^2-b^2)^(9/2)/d+1/12*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))^2-3/4*a*cos
(d*x+c)/(a+b)^4/d/(1-sin(d*x+c))+1/12*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1
/12*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))^2+3/4*a*cos(d*x+c)/(a-b)^4/d/(1+sin
(d*x+c))-1/12*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))+1/2*a^4*b*cos(d*x+c)/(a^2
-b^2)^3/d/(a+b*sin(d*x+c))^2+3/2*a^5*b*cos(d*x+c)/(a^2-b^2)^4/d/(a+b*sin(d*
x+c))+4*a^3*b^3*cos(d*x+c)/(a^2-b^2)^4/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.66, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2810, 2729, 2727, 2743, 2833, 12, 2739, 632, 210}

$$\frac{12a^2b^2(a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{9/2}}+\frac{3a^5b\cos(c+dx)}{2d\,(a^2-b^2)^4\,(a+b\sin(c+dx))}+\frac{a^4(2a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{9/2}}+\frac{8a^4b^2\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{9/2}}+\frac{a^4b\cos(c+dx)}{2d\,(a^2-b^2)^3\,(a+b\sin(c+dx))^2}+\frac{4a^3b^3\cos(c+dx)}{d\,(a^2-b^2)^4\,(a+b\sin(c+dx))}-\frac{3a\cos(c+dx)}{4d(a+b)^4(1-\sin(c+dx))}+\frac{3a\cos(c+dx)}{4d(a-b)^4(\sin(c+dx)+1)}+\frac{\cos(c+dx)}{12d(a+b)^3(1-\sin(c+dx))}-\frac{\cos(c+dx)}{12d(a-b)^3(\sin(c+dx)+1)}+\frac{\cos(c+dx)}{12d(a+b)^3(1-\sin(c+dx))^2}-\frac{\cos(c+dx)}{12d(a-b)^3(\sin(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (8*a^4*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(
9/2)*d) + (12*a^2*b^2*(a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/((a^2 - b^2)^(9/2)*d) + (a^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(9/2)*d) + Cos[c + d*x]/(12*(a + b)
^3*d*(1 - Sin[c + d*x])^2) - (3*a*Cos[c + d*x])/(4*(a + b)^4*d*(1 - Sin[c +
 d*x])) + Cos[c + d*x]/(12*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(
12*(a - b)^3*d*(1 + Sin[c + d*x])^2) + (3*a*Cos[c + d*x])/(4*(a - b)^4*d*(1
 + Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4
*b*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) + (3*a^5*b*Cos[
c + d*x])/(2*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (4*a^3*b^3*Cos[c + d*x
])/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(\frac{1}{4(a+b)^3(-1+\sin(c+dx))^2} + \frac{3a}{4(a+b)^4(-1+\sin(c+dx))} + \frac{}{4(a-b)^3(1}\right.$$

$$= -\frac{(3a)\int \frac{1}{1+\sin(c+dx)}\,dx}{4(a-b)^4} + \frac{\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{4(a-b)^3} + \frac{(3a)\int \frac{1}{-1+\sin(c+dx)}\,dx}{4(a+b)^4} + \frac{\int \frac{1}{(-1+\sin(c}}{4(a+}$$

$$= \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2} - \frac{3a\cos(c+dx)}{4(a+b)^4d(1-\sin(c+dx))} - \frac{\cos(c-}{12(a-b)^3d(1+}$$

$$= \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2} - \frac{3a\cos(c+dx)}{4(a+b)^4d(1-\sin(c+dx))} + \frac{\cos(c+}{12(a+b)^3d(1-}$$

$$= \frac{12a^2b^2(a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d} + \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2} - \frac{}{4(a}$$

$$= \frac{12a^2b^2(a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d} + \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2} - \frac{}{4(a}$$

$$= \frac{8a^4b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d} + \frac{12a^2b^2(a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d} + \frac{}{12(}$$

$$= \frac{8a^4b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d} + \frac{12a^2b^2(a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d} + \frac{a^4(2}{}$$

Mathematica [A]
time = 0.67, size = 351, normalized size = 0.74

$$\frac{\frac{96a^2(2a^4+21a^2b^2+12b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}} - \frac{\sec^3(c+dx)\left(-264a^6b-358a^4b^3+8a^2b^5-16b^7-8(44a^6b+55a^4b^3+8a^2b^5-2b^7)\cos(2(c+dx))-2(28a^6b+89a^4b^3-12a^2b^5)\cos(4(c+dx))+22a^5b^2\sin(c+dx)-264a^3b^4\sin(c+dx)+32ab^6\sin(c+dx)+32a^7\sin(3(c+dx))-91a^5b^2\sin(3(c+dx))-244a^3b^4\sin(3(c+dx))-12ab^6\sin(3(c+dx))-17a^5b^2\sin(5(c+dx))-76a^3b^4\sin(5(c+dx))-12ab^6\sin(5(c+dx))\right)}{(a^2-b^2)^4(a+b\sin(c+dx))^2}}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((96*a^2*(2*a^4 + 21*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]])/(a^2 - b^2)^(9/2) - (Sec[c + d*x]^3*(-264*a^6*b - 358*a^4*b^3
 + 8*a^2*b^5 - 16*b^7 - 8*(44*a^6*b + 55*a^4*b^3 + 8*a^2*b^5 - 2*b^7)*Cos[2
*(c + d*x)] - 2*(28*a^6*b + 89*a^4*b^3 - 12*a^2*b^5)*Cos[4*(c + d*x)] + 22*
a^5*b^2*Sin[c + d*x] - 264*a^3*b^4*Sin[c + d*x] + 32*a*b^6*Sin[c + d*x] + 3
2*a^7*Sin[3*(c + d*x)] - 91*a^5*b^2*Sin[3*(c + d*x)] - 244*a^3*b^4*Sin[3*(c
 + d*x)] - 12*a*b^6*Sin[3*(c + d*x)] - 17*a^5*b^2*Sin[5*(c + d*x)] - 76*a^3
*b^4*Sin[5*(c + d*x)] - 12*a*b^6*Sin[5*(c + d*x)]))/((a^2 - b^2)^4*(a + b*S
in[c + d*x])^2))/(96*d)
```

Maple [A]
time = 0.79, size = 354, normalized size = 0.75 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(a-b)^3/(tan(1/2*d*x+1/2*c)+
1)^2-1/2/(a-b)^4*(-2*a-b)/(tan(1/2*d*x+1/2*c)+1)-1/3/(a+b)^3/(tan(1/2*d*x+1
/2*c)-1)^3-1/2/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^4*(-2*a+b)/(tan(1
/2*d*x+1/2*c)-1)+2*a^2/(a-b)^4/(a+b)^4*((1/2*a*b^2*(5*a^2+6*b^2)*tan(1/2*d*
x+1/2*c)^3+1/2*b*(4*a^4+15*a^2*b^2+14*b^4)*tan(1/2*d*x+1/2*c)^2+11/2*b^2*a*
(a^2+2*b^2)*tan(1/2*d*x+1/2*c)+2*b*a^4+7/2*a^2*b^3)/(a*tan(1/2*d*x+1/2*c)^2
+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(2*a^4+21*a^2*b^2+12*b^4)/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.43, size = 1249, normalized size = 2.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^8*b - 16*a^6*b^3 + 24*a^4*b^5 - 16*a^2*b^7 + 4*b^9 - 2*(28*a^8*b
 + 61*a^6*b^3 - 101*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 - 4*(8*a^8*b - 25*
a^6*b^3 + 27*a^4*b^5 - 11*a^2*b^7 + b^9)*cos(d*x + c)^2 - 3*((2*a^6*b^2 + 2
1*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^5 - 2*(2*a^7*b + 21*a^5*b^3 + 12*a^3*b
^5)*cos(d*x + c)^3*sin(d*x + c) - (2*a^8 + 23*a^6*b^2 + 33*a^4*b^4 + 12*a^2
*b^6)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) - 2*(2*a^9 - 8*a^7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 + (17*a^7*b^
2 + 59*a^5*b^4 - 64*a^3*b^6 - 12*a*b^8)*cos(d*x + c)^4 - 2*(4*a^9 - 9*a^7*b
^2 + 3*a^5*b^4 + 5*a^3*b^6 - 3*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^10*
b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d*cos(d*x +
c)^5 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11
)*d*cos(d*x + c)^3*sin(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^
8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^3), 1/6*(2*a^8*b - 8*a^6*b^3 + 12*a^4
*b^5 - 8*a^2*b^7 + 2*b^9 - (28*a^8*b + 61*a^6*b^3 - 101*a^4*b^5 + 12*a^2*b^
7)*cos(d*x + c)^4 - 2*(8*a^8*b - 25*a^6*b^3 + 27*a^4*b^5 - 11*a^2*b^7 + b^9
)*cos(d*x + c)^2 - 3*((2*a^6*b^2 + 21*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^5
- 2*(2*a^7*b + 21*a^5*b^3 + 12*a^3*b^5)*cos(d*x + c)^3*sin(d*x + c) - (2*a^
8 + 23*a^6*b^2 + 33*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^3)*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^9 - 8*a^
7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 + (17*a^7*b^2 + 59*a^5*b^4 - 64*a^
3*b^6 - 12*a*b^8)*cos(d*x + c)^4 - 2*(4*a^9 - 9*a^7*b^2 + 3*a^5*b^4 + 5*a^3
*b^6 - 3*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a
^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d*cos(d*x + c)^5 - 2*(a^11*b - 5*a
^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3*sin
(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)
*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 12.29, size = 632, normalized size = 1.33

$$\frac{\frac{3\,(2a^6+21a^4b^2+12a^2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)\sqrt{a^2-b^2}}+\frac{3\left(5a^5b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^6b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15a^4b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+14a^2b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+11a^5b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+22a^3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4a^6b+7a^4b^3\right)}{(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)^2}+\frac{2\left(3a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+24a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+9ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-24a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-3b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-56a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 \text{[illegible]}\right)}{(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)\,\text{[illegible]}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/3*(3*(2*a^6 + 21*a^4*b^2 + 12*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*(5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 + 14*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^5*b^2*tan(1/2*d*x + 1/2*c) + 22*a^3*b^4*tan(1/2*d*x + 1/2*c) + 4*a^6*b + 7*a^4*b^3)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 2*(3*a^5*tan(1/2*d*x + 1/2*c)^5 + 24*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 9*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 9*a^4*b*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 3*b^5*tan(1/2*d*x + 1/2*c)^4 - 10*a^5*tan(1/2*d*x + 1/2*c)^3 - 56*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 36*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 36*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1/2*d*x + 1/2*c) + 24*a^3*b^2*tan(1/2*d*x + 1/2*c) + 9*a*b^4*tan(1/2*d*x + 1/2*c) - 15*a^4*b - 20*a^2*b^3 - b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d

Mupad [B]
time = 11.22, size = 1099, normalized size = 2.32

$$\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(2\,a^6\,b+21\,a^4\,b^3+12\,a^2\,b^5\right)}{a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(14\,a^7+23\,a^5\,b^2+242\,a^3\,b^4+36\,a\,b^6\right)}{3\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(2\,a^5+21\,a^3\,b^2+12\,a\,b^4\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(30\,a^4\,b+71\,a^2\,b^3+4\,b^5\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{a^2\,\left(42\,a^4\,b+61\,a^2\,b^3+2\,b^5\right)}{3\,\left(a^2-b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(4\,a^6\,b+16\,a^4\,b^3+43\,a^2\,b^5\right)}{3\,\left(a^2-b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(-2\,a^7+65\,a^5\,b^2+131\,a^3\,b^4+16\,a\,b^6\right)}{3\,\left(a^2-b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(22\,a^6\,b+233\,a^4\,b^3+153\,a^2\,b^5+12\,b^7\right)}{3\,\left(a^2-b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(2\,a^4+21\,a^2\,b^2+12\,b^4\right)}{a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-6\,a^6+105\,a^4\,b^2+208\,a^2\,b^4+8\,b^6\right)}{3\,\left(a^2-b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^2+12\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(2\,a^2+12\,b^2\right)+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-a^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2-4\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(a^2-4\,b^2\right)+8\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^2\,\mathrm{atan}\left(\frac{\frac{a^2\,\left(2\,a^4+21\,a^2\,b^2+12\,b^4\right)\,\left(2\,a^8\,b-8\,a^6\,b^3+12\,a^4\,b^5-8\,a^2\,b^7+2\,b^9\right)}{2\,{\left(a+b\right)}^{9/2}\,{\left(a-b\right)}^{9/2}}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^4+21\,a^2\,b^2+12\,b^4\right)\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}{{\left(a+b\right)}^{9/2}\,{\left(a-b\right)}^{9/2}}}{2\,a^6+21\,a^4\,b^2+12\,a^2\,b^4}\right)\,\left(2\,a^4+21\,a^2\,b^2+12\,b^4\right)}{d\,{\left(a+b\right)}^{9/2}\,{\left(a-b\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^4/(a + b*sin(c + d*x))^3,x)

[Out] ((3*tan(c/2 + (d*x)/2)^8*(2*a^6*b + 12*a^2*b^5 + 21*a^4*b^3))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) - (2*tan(c/2 + (d*x)/2)^5*(36*a*b^6 + 14*a^7 + 242*a^3*b^4 + 23*a^5*b^2))/(3*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (4*tan(c/2 + (d*x)/2)^7*(12*a*b^4 + 2*a^5 + 21*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(30*a^4*b + 4*b^5 + 71*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (a^2*(42*a^4*b + 2*b^5 + 61*a^2*b^3))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (10*tan(c/2 + (d*x)/2)^4*(4*a^6*b + 43*a^2*b^5 + 16*a^4*b^3))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^3*(16*a*b^6 - 2*a^7 + 131*a^3*b^4 + 65*a^5*b^2))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^6*(22*a^6*b + 12*b^7 + 153*a^2*b^5 + 233*a^4*b^3))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a^3*tan(c/2 + (d*x)/2)^9*(2*a^4 + 12*b^4 + 21*a^2*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) - (a*tan(c/2 + (d*x)/2)*(8*b^6 - 6*a^6 + 208*a^2*b^4 + 105*a^4*b^2))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)))/(d*(tan(c/2 + (d*x)/2)^4*(2*a^2 + 12*b^2) - tan(c/2 + (d*x)/2)^6*(2*a^2 + 12*b^2) + a^2*tan(c/2 + (d*x)/2)^10 - a^2 + tan(c/2 + (d*x)/2)^2*(a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^8*(a^2 - 4*b^2) + 8*a*b*tan(c/2 + (d*x)/2)^3 - 8*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 - 4*a*b*tan(c/2 + (d*x)/2

```
))) + (a^2*atan(((a^2*(2*a^4 + 12*b^4 + 21*a^2*b^2)*(2*a^8*b + 2*b^9 - 8*a^
2*b^7 + 12*a^4*b^5 - 8*a^6*b^3))/(2*(a + b)^(9/2)*(a - b)^(9/2)) + (a^3*tan
(c/2 + (d*x)/2)*(2*a^4 + 12*b^4 + 21*a^2*b^2)*(a^8 + b^8 - 4*a^2*b^6 + 6*a^
4*b^4 - 4*a^6*b^2))/((a + b)^(9/2)*(a - b)^(9/2)))/(2*a^6 + 12*a^2*b^4 + 21
*a^4*b^2))*(2*a^4 + 12*b^4 + 21*a^2*b^2))/(d*(a + b)^(9/2)*(a - b)^(9/2))
```

3.199 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=350

$$-\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}-\frac{a^2(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}-\frac{2b^2(3a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a^2-b^2}}\right.}{\left(a^2-b^2\right)^{7/2}d}$$

```
[Out] -4*a^2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)
/d-a^2*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^
2)^(7/2)/d-2*b^2*(3*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2
))/(a^2-b^2)^(7/2)/d+1/2*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1/2*cos(d*x+c)
/(a-b)^3/d/(1+sin(d*x+c))-1/2*a^2*b*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c
))^2-3/2*a^3*b*cos(d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-2*a*b^3*cos(d*x+c)
/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.40, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2810, 2727, 2743, 2833, 12, 2739, 632, 210}

$$-\frac{a^2(2a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{4a^2b^2\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{2b^2(3a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{a^2b\cos(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}-\frac{2ab^3\cos(c+dx)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{3a^3b\cos(c+dx)}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^3(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2d(a-b)^3(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-4*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^
(7/2)*d) - (a^2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/((a^2 - b^2)^(7/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]/(2*(a + b)^3*
d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) - (
a^2*b*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) - (3*a^3*b*C
os[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (2*a*b^3*Cos[c + d*
x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))} + \frac{1}{2(a-b)^3(1+\sin(c+dx))} - \frac{\cdots}{(a^2-b^2)\,(\cdots}\right.$$

$$= \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} - \frac{(2ab^2)\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{(a^2-b^2)^2} - \frac{a^2\int \frac{1}{(a+b\sin(\cdots}}{a^2-\cdots}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a^2b\cos(\cdots}{2\,(a^2-b^2)^2\,d(a\cdots}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a^2b\cos(\cdots}{2\,(a^2-b^2)^2\,d(a\cdots}$$

$$= -\frac{2b^2(3a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cdots}{2(a\cdots}$$

$$= -\frac{2b^2(3a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cdots}{2(a\cdots}$$

$$= -\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} - \frac{2b^2(3a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} + \frac{\cdots}{2(\cdots}$$

$$= -\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} - \frac{a^2(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} - \frac{2b^2\cdots}{\cdots}$$

Mathematica [A]
time = 2.09, size = 212, normalized size = 0.61

$$\frac{-\frac{2(2a^4+11a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right) - \frac{ab\cos(c+dx)(4a^3+3ab^2+b(3a^2+4b^2)\sin(c+dx))}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 + 11*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/(a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
))) - (a*b*Cos[c + d*x]*(4*a^3 + 3*a*b^2 + b*(3*a^2 + 4*b^2)*Sin[c + d*x]))
/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

Maple [A]
time = 0.54, size = 258, normalized size = 0.74

method	result
derivativedivides	$\frac{-\frac{1}{(a-b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2\left(\frac{\left(\frac{5}{2}a^3b^2+a\,b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b\left(4a^4+11a^2b^2+6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\right.\right.}\right.}{}}{d}$
default	$\frac{-\frac{1}{(a-b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2\left(\frac{\left(\frac{5}{2}a^3b^2+a\,b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b\left(4a^4+11a^2b^2+6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\right.\right.}\right.}{}}{d}$
risch	$\frac{i\left(-11ia^2b^3e^{5i(dx+c)}+29ib^3a^2e^{i(dx+c)}-2ib\,a^4e^{5i(dx+c)}+2ia^2b^3e^{3i(dx+c)}-2ib^5e^{i(dx+c)}+4ib^5e^{3i(dx+c)}+6a^5e^{4i(dx+c)}+33\right.}{\left(1+e^{2i(dx+c)}\right)\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)-1/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-2/(
a-b)^3/(a+b)^3*(((5/2*a^3*b^2+a*b^4)*tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4+11*a
^2*b^2+6*b^4)*tan(1/2*d*x+1/2*c)^2+1/2*a*b^2*(11*a^2+10*b^2)*tan(1/2*d*x+1/
2*c)+1/2*a^2*b*(4*a^2+3*b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c
)+a)^2+1/2*(2*a^4+11*a^2*b^2+2*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.44, size = 934, normalized size = 2.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(8*a^6*b + a^4*b^3 - 11
*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos(d*
x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) - (
2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log
(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(
d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 -
2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a
^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^
9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c
) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x
 + c)), 1/2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + (8*a^6*b + a^4*b^3 -
 11*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos
(d*x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c)
- (2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^7 - 6*a^
5*b^2 + 6*a^3*b^4 - 2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^
2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*co
s(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(
d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b
^8 + b^10)*d*cos(d*x + c))]
```

<u>Sympy</u> [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

<u>Giac</u> [A]
time = 6.09, size = 384, normalized size = 1.10

$$-\frac{\frac{(2a^4+11a^2b^2+2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{2\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^2b-b^3\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+11a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+6b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+11a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+10ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4a^4b+3a^2b^3}{(a^6-3a^4b^2+3a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((2*a^4 + 11*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + a
rctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan
```

```
(1/2*d*x + 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(ta
n(1/2*d*x + 1/2*c)^2 - 1)) + (5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^4*ta
n(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 11*a^2*b^3*tan(1/2*
d*x + 1/2*c)^2 + 6*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 10*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b + 3*a^2*b^3)/((a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c)
+ a)^2))/d
```

Mupad [B]
time = 10.36, size = 627, normalized size = 1.79

$$\frac{\frac{5\,\left(2\,a^4\,b+a^2\,b^3\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^5+a^3\,b^2+12\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^4\,b+6\,a^2\,b^3+7\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^4\,b+11\,a^2\,b^3+2\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-2\,a^4+29\,a^2\,b^2+18\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^4+11\,a^2\,b^2+2\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{d\,\left(a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2+4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^2+4\,b^2\right)+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\mathrm{atan}\left(\frac{\frac{\left(2\,a^4+11\,a^2\,b^2+2\,b^4\right)\,\left(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7\right)}{2\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^4+11\,a^2\,b^2+2\,b^4\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}}{2\,a^4+11\,a^2\,b^2+2\,b^4}\right)\,\left(2\,a^4+11\,a^2\,b^2+2\,b^4\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((5*(2*a^4*b + a^2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2 +
 (d*x)/2)^3*(12*a*b^4 + 2*a^5 + a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2) + (2*tan(c/2 + (d*x)/2)^2*(2*a^4*b + 7*b^5 + 6*a^2*b^3))/(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2) - (3*tan(c/2 + (d*x)/2)^4*(2*a^4*b + 2*b^5 + 11*a^2*b^
3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (a*tan(c/2 + (d*x)/2)*(18*b^4 - 2
*a^4 + 29*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (a*tan(c/2 + (d*x
)/2)^5*(2*a^4 + 2*b^4 + 11*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(
d*(a^2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c/2 + (d*x)/2)^2*(a^2 + 4*b^2) + ta
n(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c
/2 + (d*x)/2))) - (atan((((2*a^4 + 2*b^4 + 11*a^2*b^2)*(2*a^6*b - 2*b^7 + 6
*a^2*b^5 - 6*a^4*b^3))/(2*(a + b)^(7/2)*(a - b)^(7/2)) + (a*tan(c/2 + (d*x)
/2)*(2*a^4 + 2*b^4 + 11*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a +
 b)^(7/2)*(a - b)^(7/2)))/(2*a^4 + 2*b^4 + 11*a^2*b^2))*(2*a^4 + 2*b^4 + 11
*a^2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.200 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$-\frac{(2a^4-9a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{(5a^2-6b^2)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d}+\frac{\cot}{2ad(a+}$$

```
[Out] -(2*a^4-9*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a
^4/(a^2-b^2)^(3/2)/d+3*b*arctanh(cos(d*x+c))/a^4/d-1/2*(5*a^2-6*b^2)*cot(d*
x+c)/a^3/(a^2-b^2)/d+1/2*cot(d*x+c)/a/d/(a+b*sin(d*x+c))^2+1/2*(2*a^2-3*b^2
)*cot(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.53, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2802, 3135, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{(2a^2-3b^2)\cot(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{(2a^4-9a^2b^2+6b^4)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\left(a^2-b^2\right)^{3/2}}-\frac{(5a^2-6b^2)\cot(c+dx)}{2a^3d\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^4*(a^2 - b^2)^(3/2)*d)) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - ((
5*a^2 - 6*b^2)*Cot[c + d*x])/(2*a^3*(a^2 - b^2)*d) + Cot[c + d*x]/(2*a*d*(a
 + b*Sin[c + d*x])^2) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*(a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
```

```
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx \\
&= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^2(c+dx)\left(3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)} \\
&= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \\
&= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \\
&= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \\
&= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \\
&= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \\
&= -\frac{\left(2a^4-9a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} -
\end{aligned}
$$

Mathematica [A]
time = 3.38, size = 195, normalized size = 0.97

$$
\frac{-\frac{2\left(2a^4-9a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} - a\cot\left(\frac{1}{2}(c+dx)\right) + 6b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 6b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \frac{a^2b\cos(c+dx)}{(a+b\sin(c+dx))^2} + \frac{ab\left(-3a^2+4b^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))} + a\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
[Out] ((-2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^2 - b^2)^(3/2) - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]]
```

```
- 6*b*Log[Sin[(c + d*x)/2]] - (a^2*b*Cos[c + d*x])/(a + b*Sin[c + d*x])^2 +
 (a*b*(-3*a^2 + 4*b^2)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x]))
 + a*Tan[(c + d*x)/2])/(2*a^4*d)
```

Maple [A]
time = 0.58, size = 299, normalized size = 1.48

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{2\left(\frac{\frac{a\,b^2\left(5a^2-6b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{b\left(4a^4+3a^2b^2-10b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{a\,b^2\left(11a^2-14b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-2b^2}+\frac{a^2b\left(4a\right.}{2a^2-}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\right.}{a^4}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{2\left(\frac{\frac{a\,b^2\left(5a^2-6b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{b\left(4a^4+3a^2b^2-10b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{a\,b^2\left(11a^2-14b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-2b^2}+\frac{a^2b\left(4a\right.}{2a^2-}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\right.}{a^4}}{d}$
risch	$\frac{i\left(-2ia^3b\,\mathrm{e}^{5i(dx+c)}+3ia\,b^3\mathrm{e}^{5i(dx+c)}+20ib\,a^3\mathrm{e}^{3i(dx+c)}-24ib^3a\,\mathrm{e}^{3i(dx+c)}+6a^4\mathrm{e}^{4i(dx+c)}-3b^2\mathrm{e}^{4i(dx+c)}a^2-6b^4\mathrm{e}^{4i(dx+c)}-1\right.}{\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(-ib\,\mathrm{e}^{2i(dx+c)}+ib+2a\,\mathrm{e}^{i(dx+c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a^3*tan(1/2*d*x+1/2*c)-2/a^4*((1/2*a*b^2*(5*a^2-6*b^2)/(a^2-b^2)*t
an(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4+3*a^2*b^2-10*b^4)/(a^2-b^2)*tan(1/2*d*x+1/
2*c)^2+1/2*a*b^2*(11*a^2-14*b^2)/(a^2-b^2)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*(4*
a^2-5*b^2)/(a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1
/2*(2*a^4-9*a^2*b^2+6*b^4)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2)))-1/2/a^3/tan(1/2*d*x+1/2*c)-3/a^4*b*ln(tan(1/2*d*x
+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(191) = 382$.
time = 0.68, size = 1394, normalized size = 6.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - 2*(8*a^6*b - 1
7*a^4*b^3 + 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 +
12*a*b^5 - 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^
4*b^2 - 3*a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)
*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*s
in(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*
sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) -
2*(2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 6*(2*a^5*b^2 - 4*a
^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b
- a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a^5*b^2 - 4*a^3*b^4 + 2*a*b^6
 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3 - a^2*
b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-
1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^2
- 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d*co
s(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)), -1/2*
((5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - (8*a^6*b - 17*a^4*b^3
+ 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 + 12*a*b^5 -
 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^4*b^2 - 3*
a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)*sin(d*x +
 c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
+ c))) - (2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 3*(2*a^5*b^
2 - 4*a^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)
^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(2*a^5*b^2 - 4*a^3*b^4 +
2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3
 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c)
)*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x
+ c)^2 - 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^
6)*d*cos(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c))
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 13.76, size = 339, normalized size = 1.68

$$-\frac{\frac{2\,(2\,a^4-9\,a^2b^2+6\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-a^4b^2)\sqrt{a^2-b^2}}+\frac{2\left(5\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-10\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+11\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-14\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a^4b-5\,a^2b^3\right)}{(a^6-a^4b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2}+\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*
sqrt(a^2 - b^2)) + 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*
x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b^3*tan(1/2*d*x + 1/2
*c)^2 - 10*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 1
4*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b - 5*a^2*b^3)/((a^6 - a^4*b^2)*(a*tan
(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 6*b*log(abs(tan(1/
2*d*x + 1/2*c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3 - (6*b*tan(1/2*d*x + 1/2*c)
 - a)/(a^4*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 7.85, size = 1762, normalized size = 8.72

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{a^2-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(7\,a\,b^3-6\,a^3\,b\right)}{a^2-b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(a^4+9\,a^2\,b^2-12\,b^4\right)}{a^2-b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(a^4+12\,a^2\,b^2-16\,b^4\right)}{a^2-b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(6\,a^4\,b+a^2\,b^3-10\,b^5\right)}{a\,\left(a^2-b^2\right)}}{d\,\left(2\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(4\,a^5+8\,a^3\,b^2\right)+2\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,a^4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\,a^4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{3\,b\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{\mathrm{atan}\left(\text{[illegible]}\right)\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}\,\left(a^4-\frac{9\,a^2\,b^2}{2}+3\,b^4\right)\,2{}\mathrm{i}}{d\,\left(a^{10}-3\,a^8\,b^2+3\,a^6\,b^4-a^4\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*sin(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (a^2 - (2*tan(c/2 + (d*x)/2)*(7*a*b^3 - 6*a^
3*b))/(a^2 - b^2) + (tan(c/2 + (d*x)/2)^4*(a^4 - 12*b^4 + 9*a^2*b^2))/(a^2
- b^2) + (2*tan(c/2 + (d*x)/2)^2*(a^4 - 16*b^4 + 12*a^2*b^2))/(a^2 - b^2) +
 (2*tan(c/2 + (d*x)/2)^3*(6*a^4*b - 10*b^5 + a^2*b^3))/(a*(a^2 - b^2)))/(d*
(2*a^5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^3*(4*a^5 + 8*a^3*b^2) + 2*
a^5*tan(c/2 + (d*x)/2) + 8*a^4*b*tan(c/2 + (d*x)/2)^2 + 8*a^4*b*tan(c/2 + (
d*x)/2)^4)) - (3*b*log(tan(c/2 + (d*x)/2)))/(a^4*d) - (atan((((-(a + b)^3*(
a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6
*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7 + 60*a^4
*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) + (((2*a^10*b - 2*a^8*b^3)/
(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20
*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4
+ 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^
```

10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + ((-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7 + 60*a^4*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) - (((2*a^10*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))/((2*(6*a^4*b + 18*b^5 - 27*a^2*b^3))/(a^8 - a^6*b^2) + (2*tan(c/2 + (d*x)/2)*(4*a^6 - 18*b^6 + 39*a^2*b^4 - 24*a^4*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2) - ((-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7 + 60*a^4*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) + (((2*a^10*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + ((-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7 + 60*a^4*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) - (((2*a^10*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*2i)/(d*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))

3.201 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=289

$$\frac{(2a^4-19a^2b^2+20b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d}-\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}+\frac{(17a^2-60b^2)\cot(c+d}{6a^5d}$$

```
[Out] -1/2*b*(9*a^2-20*b^2)*arctanh(cos(d*x+c))/a^6/d+1/6*(17*a^2-60*b^2)*cot(d*x
+c)/a^5/d-(a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/b/d+1/6*(3*a^2-5*b^2)*cot(d
*x+c)*csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^2-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d
/(a+b*sin(d*x+c))^2+1/6*(3*a^2-20*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d/(a+b*s
in(d*x+c))+(2*a^4-19*a^2*b^2+20*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b
^2)^(1/2))/a^6/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2803, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}+\frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d}-\frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd}+\frac{(3a^2-20b^2)\cot(c+dx)\csc(c+dx)}{6a^3bd(a+b\sin(c+dx))}+\frac{(2a^4-19a^2b^2+20b^4)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (b*(9*a^2 - 20*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^6*d) + ((17*a^2 - 60*b^2)*Cot[c + d*x])/(6*a^5*d) - ((a^2 - 5*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(a^4*b*d) + ((3*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*
a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(6*a^3*b*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2803

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m
 + 1)/Sin[e + f*x]^3)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e
+ f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[(3*a^2 +
 b^2*(m - 2))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(3*a^2*b*f*(m + 1)
*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{\int \csc^3(c+dx)(2(3 \ldots}{\ldots}$$

$$= \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{(3a^2-20b^2)\,\mathrm{c}\ldots}{6a^3bd(a \ldots}$$

$$= -\frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\mathrm{co}\ldots}{3a\ldots}$$

$$= \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\,\mathrm{co}\ldots}{6a^2bd(a \ldots}$$

$$= \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\,\mathrm{co}\ldots}{6a^2bd(a \ldots}$$

$$= -\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\,\mathrm{c}\ldots}{\ldots}$$

$$= -\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\,\mathrm{c}\ldots}{\ldots}$$

$$= \frac{(2a^4-19a^2b^2+20b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} - \frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+\ldots}{2a^6d}$$

Mathematica [A]
time = 6.17, size = 459, normalized size = 1.59

$$\frac{(2a^4-19a^2b^2+20b^4)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} + \frac{\left(2a^2\cos\left(\frac{1}{2}(c+dx)\right)-9b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{3a^5d} + \frac{3b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^3d} + \frac{(-9a^2b+20b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} + \frac{(9a^2b-20b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} - \frac{3b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-2a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5d} + \frac{a^2b\cos(c+dx)-b^3\cos(c+dx)}{2a^4d(a+b\sin(c+dx))^2} + \frac{3a^2b\cos(c+dx)-8b^3\cos(c+dx)}{2a^5d(a+b\sin(c+dx))} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2]
 + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) + ((2*a^2
*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5*d) + (
3*b*Csc[(c + d*x)/2]^2)/(8*a^4*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(
24*a^3*d) + ((-9*a^2*b + 20*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^6*d) + ((9*a^2
*b - 20*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^6*d) - (3*b*Sec[(c + d*x)/2]^2)/(8
*a^4*d) + (Sec[(c + d*x)/2]*(-2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/
2]))/(3*a^5*d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(2*a^4*d*(a + b*Si
n[c + d*x])^2) + (3*a^2*b*Cos[c + d*x] - 8*b^3*Cos[c + d*x])/(2*a^5*d*(a +
b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^3*d)
```

Maple [A]
time = 0.67, size = 360, normalized size = 1.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/a^5*(1/3*tan(1/2*d*x+1/2*c)^3*a^2-3*a*b*tan(1/2*d*x+1/2*c)^2-5*a^2
*tan(1/2*d*x+1/2*c)+24*b^2*tan(1/2*d*x+1/2*c))+2/a^6*(((5/2*a^3*b^2-5*a*b^4
)*tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4-a^2*b^2-18*b^4)*tan(1/2*d*x+1/2*c)^2+1/
2*a*b^2*(11*a^2-26*b^2)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*(4*a^2-9*b^2))/(a*tan(
1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(2*a^4-19*a^2*b^2+20*b^4)/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1
/24/a^3/tan(1/2*d*x+1/2*c)^3-1/8*(-5*a^2+24*b^2)/a^5/tan(1/2*d*x+1/2*c)+3/8
/a^4*b/tan(1/2*d*x+1/2*c)^2+1/2/a^6*b*(9*a^2-20*b^2)*ln(tan(1/2*d*x+1/2*c))
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 972 vs. 2(274) = 548.
time = 0.65, size = 2027, normalized size = 7.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3
*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 38*a^3*b^3
+ 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*
b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 +
 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^6)*cos(d*x + c)^4 - (2*a^6 - 15*a^
4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)
*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*
cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
```

```
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b
^4 + 20*a*b^6)*cos(d*x + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*
a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4
 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 +
(9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 -
38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) + 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 2
0*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c
)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5
+ 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b -
59*a^4*b^3 + 45*a^2*b^5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2
*b^5)*cos(d*x + c))*sin(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4
*(a^9*b - a^7*b^3)*d*cos(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a
^6*b^4)*d*cos(d*x + c)^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 +
(a^10 - a^6*b^4)*d)*sin(d*x + c)), 1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*
b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x
 + c)^3 - 6*(4*a^5*b - 38*a^3*b^3 + 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20
*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2
 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 + 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^
6)*cos(d*x + c)^4 - (2*a^6 - 15*a^4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)
^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 -
b^2)*cos(d*x + c))) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b^4 + 20*a*b^6)*cos(d*x
 + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 +
 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x +
 c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^
5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*c
os(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(18*a^5*b^2 -
58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^
4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^
4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)
^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b - 59*a^4*b^3 + 45*a^2*b^
5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2*b^5)*cos(d*x + c))*si
n(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4*(a^9*b - a^7*b^3)*d*c
os(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a^6*b^4)*d*cos(d*x + c)
^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 + (a^10 - a^6*b^4)*d)*si
n(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 7.21, size = 451, normalized size = 1.56

$$\frac{\frac{12\,(9\,a^2b-20\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^6}+\frac{24\,(2\,a^4-19\,a^2b^2+20\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^6}+\frac{24\left(5\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-10\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-18\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+11\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-26\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a^4b-9\,a^2b^3\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2a^6}+\frac{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+72\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^9}-\frac{198\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-440\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+72\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(12*(9*a^2*b - 20*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 + 24*(2*a^4
- 19*a^2*b^2 + 20*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) + 24*(5*
a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*
tan(1/2*d*x + 1/2*c)^2 - a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 18*b^5*tan(1/2*d*
x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 26*a*b^4*tan(1/2*d*x + 1/2
*c) + 4*a^4*b - 9*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1
/2*c) + a)^2*a^6) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d*x + 1/2
*c)^2 - 15*a^6*tan(1/2*d*x + 1/2*c) + 72*a^4*b^2*tan(1/2*d*x + 1/2*c))/a^9
- (198*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 440*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a
^3*tan(1/2*d*x + 1/2*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b*tan(1
/2*d*x + 1/2*c) + a^3)/(a^6*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 7.31, size = 1261, normalized size = 4.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (tan(c/2 + (d*x)/2)*((3*(a^2 + 4*b^2))/(8
*a^5) + 1/(4*a^3) - (9*b^2)/(2*a^5)))/d + (tan(c/2 + (d*x)/2)^6*(5*a^4 - 80
*b^4 + 16*a^2*b^2) + tan(c/2 + (d*x)/2)^4*((29*a^4)/3 - 304*b^4 + 72*a^2*b^
2) - a^4/3 + tan(c/2 + (d*x)/2)^2*((13*a^4)/3 - (40*a^2*b^2)/3) - tan(c/2 +
 (d*x)/2)^3*(156*a*b^3 - (170*a^3*b)/3) - (tan(c/2 + (d*x)/2)^5*(144*b^5 -
55*a^4*b + 104*a^2*b^3))/a + (5*a^3*b*tan(c/2 + (d*x)/2))/3)/(d*(8*a^7*tan(
c/2 + (d*x)/2)^3 + 8*a^7*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^5*(16*a^
7 + 32*a^5*b^2) + 32*a^6*b*tan(c/2 + (d*x)/2)^4 + 32*a^6*b*tan(c/2 + (d*x)/
2)^6)) + (log(tan(c/2 + (d*x)/2))*(9*a^2*b - 20*b^3))/(2*a^6*d) - (3*b*tan(
c/2 + (d*x)/2)^2)/(8*a^4*d) + (atan((((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^
4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (
d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 + ((-(a + b)*(a - b))^(1/
2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4
```

- (19*a^2*b^2)/2))/(a^8 - a^6*b^2))*1i)/(a^8 - a^6*b^2) + ((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 - ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2))*1i)/(a^8 - a^6*b^2))/((18*a^6*b - 400*b^7 + 560*a^2*b^5 - 211*a^4*b^3)/a^10 + (2*tan(c/2 + (d*x)/2)*(4*a^6 - 200*b^6 + 230*a^2*b^4 - 58*a^4*b^2))/a^9 - ((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 + ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2)))/(a^8 - a^6*b^2) + ((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 - ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2)))/(a^8 - a^6*b^2)))*(-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*2i)/(d*(a^8 - a^6*b^2))

3.202 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=492

$$-\frac{\sqrt{a^2-b^2}\,(2a^4-29a^2b^2+42b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d}$$

```
[Out] 1/8*b*(45*a^4-200*a^2*b^2+168*b^4)*arctanh(cos(d*x+c))/a^8/d-1/30*(91*a^4-6
45*a^2*b^2+630*b^4)*cot(d*x+c)/a^7/d+1/8*(8*a^4-79*a^2*b^2+84*b^4)*cot(d*x+
c)*csc(d*x+c)/a^6/b/d-1/30*(15*a^4-187*a^2*b^2+210*b^4)*cot(d*x+c)*csc(d*x+
c)^2/a^5/b^2/d-1/3*cot(d*x+c)*csc(d*x+c)/b/d/(a+b*sin(d*x+c))^2+1/12*a*cot(
d*x+c)*csc(d*x+c)^2/b^2/d/(a+b*sin(d*x+c))^2+1/60*(5*a^4-60*a^2*b^2+63*b^4)
*cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d/(a+b*sin(d*x+c))^2+7/20*b*cot(d*x+c)*csc
(d*x+c)^3/a^2/d/(a+b*sin(d*x+c))^2-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d/(a+b*sin
(d*x+c))^2+1/12*(4*a^4-54*a^2*b^2+63*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^4/b^2/d
/(a+b*sin(d*x+c))-(2*a^4-29*a^2*b^2+42*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))
/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^8/d
```

Rubi [A]
time = 1.39, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2805, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{(4a^4-54a^2b^2+63b^4)\cot(c+dx)\csc^2(c+dx)}{12a^4b^2d(a+b\sin(c+dx))}-\frac{\sqrt{a^2-b^2}\,(2a^4-29a^2b^2+42b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d}-\frac{(91a^4-645a^2b^2+630b^4)\cot(c+dx)}{30a^7d}+\frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc(c+dx)}{8a^6bd}-\frac{(15a^4-187a^2b^2+210b^4)\cot(c+dx)\csc^2(c+dx)}{30a^5b^2d}+\frac{(5a^4-60a^2b^2+63b^4)\cot(c+dx)\csc^2(c+dx)}{60a^3b^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((Sqrt[a^2 - b^2]*(2*a^4 - 29*a^2*b^2 + 42*b^4)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(a^8*d)) + (b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*ArcT
anh[Cos[c + d*x]])/(8*a^8*d) - ((91*a^4 - 645*a^2*b^2 + 630*b^4)*Cot[c + d*
x])/(30*a^7*d) + ((8*a^4 - 79*a^2*b^2 + 84*b^4)*Cot[c + d*x]*Csc[c + d*x])/
(8*a^6*b*d) - ((15*a^4 - 187*a^2*b^2 + 210*b^4)*Cot[c + d*x]*Csc[c + d*x]^2
)/(30*a^5*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^
2) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(12*b^2*d*(a + b*Sin[c + d*x])^2) + ((
5*a^4 - 60*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a^3*b^2*d*(a
+ b*Sin[c + d*x])^2) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3)/(20*a^2*d*(a + b*S
in[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x])
^2) + ((4*a^4 - 54*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^4*b
^2*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2805

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(5*a*f*Sin[
e + f*x]^5)), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x]
)^m/Sin[e + f*x]^4)*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m -
 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m
*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x]
, x], x] + Simp[Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Sin[e + f
*x]^2)), x] + Simp[a*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*m*(m
 - 1)*Sin[e + f*x]^3)), x] - Simp[b*(m - 4)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^(m + 1)/(20*a^2*f*Sin[e + f*x]^4)), x]) /; FreeQ[{a, b, e, f, m}, x] &&
 NeQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{7b\cot(c+dx)\csc^3(c}{20a^2d(a+b\sin(c+}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{(5a^4-60a^2b^2+63b^4}{60a^3b^2d(a-}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{(5a^4-60a^2b^2+63b^4}{60a^3b^2d(a-}$$

$$= -\frac{(15a^4-187a^2b^2+210b^4)\cot(c+dx)\csc^2(c+dx)}{30a^5b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$$= \frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc(c+dx)}{8a^6bd} - \frac{(15a^4-187a^2b^2+210b^4)\cot}{30a^5b^2d}$$

$$= -\frac{(91a^4-645a^2b^2+630b^4)\cot(c+dx)}{30a^7d} + \frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc}{8a^6bd}$$

$$= -\frac{(91a^4-645a^2b^2+630b^4)\cot(c+dx)}{30a^7d} + \frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc}{8a^6bd}$$

$$= \frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{(91a^4-645a^2b^2+630b^4)\cot}{30a^7d}$$

$$= \frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{(91a^4-645a^2b^2+630b^4)\cot}{30a^7d}$$

$$= -\frac{\sqrt{a^2-b^2}\,(2a^4-29a^2b^2+42b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} + \frac{b(45a^4-200a^2b^2}{}$$

Mathematica [A]
time = 1.13, size = 448, normalized size = 0.91

$$\frac{-\frac{3840(2a^6-31a^4b^2+71a^2b^4-42b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+480b(45a^4-200a^2b^2+168b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-480b(45a^4-200a^2b^2+168b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2a\cot(c+dx)\csc^6(c+dx)\left(-784a^6+3256a^4b^2+7860a^2b^4-12600b^6+2(384a^6-2131a^4b^2-6315a^2b^4+9450b^6)\cos(2(c+dx))+(-368a^6+824a^4b^2+6060a^2b^4-7560b^6)\cos(4(c+dx))+182a^4b^2\cos(6(c+dx))-1290a^2b^4\cos(6(c+dx))+1260b^6\cos(6(c+dx))-8156a^5b\sin(c+dx)+42270a^3b^3\sin(c+dx)-37800ab^5\sin(c+dx)+3956a^5b\sin(3(c+dx))-20715a^3b^3\sin(3(c+dx))+18900ab^5\sin(3(c+dx))-608a^5b\sin(5(c+dx))+3975a^3b^3\sin(5(c+dx))-3780ab^5\sin(5(c+dx))\right)}{(b+a\csc(c+dx))^2}}{3840a^8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-3840*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 480*b*(45*a^4 - 200*a^2*b^2 + 1
68*b^4)*Log[Cos[(c + d*x)/2]] - 480*b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*Log[
Sin[(c + d*x)/2]] + (2*a*Cot[c + d*x]*Csc[c + d*x]^6*(-784*a^6 + 3256*a^4*b
^2 + 7860*a^2*b^4 - 12600*b^6 + 2*(384*a^6 - 2131*a^4*b^2 - 6315*a^2*b^4 +
9450*b^6)*Cos[2*(c + d*x)] + (-368*a^6 + 824*a^4*b^2 + 6060*a^2*b^4 - 7560*
b^6)*Cos[4*(c + d*x)] + 182*a^4*b^2*Cos[6*(c + d*x)] - 1290*a^2*b^4*Cos[6*(
c + d*x)] + 1260*b^6*Cos[6*(c + d*x)] - 8156*a^5*b*Sin[c + d*x] + 42270*a^3
*b^3*Sin[c + d*x] - 37800*a*b^5*Sin[c + d*x] + 3956*a^5*b*Sin[3*(c + d*x)]
- 20715*a^3*b^3*Sin[3*(c + d*x)] + 18900*a*b^5*Sin[3*(c + d*x)] - 608*a^5*b
*Sin[5*(c + d*x)] + 3975*a^3*b^3*Sin[5*(c + d*x)] - 3780*a*b^5*Sin[5*(c + d
*x)]))/(b + a*Csc[c + d*x])^2)/(3840*a^8*d)
```

Maple [A]
time = 0.84, size = 559, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{3}+8a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24b\,a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40a\,b^3\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{32a^7}}{}$
default	$\frac{\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{3}+8a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24b\,a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40a\,b^3\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{32a^7}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32/a^7*(1/5*a^4*tan(1/2*d*x+1/2*c)^5-3/2*b*tan(1/2*d*x+1/2*c)^4*a^3-
7/3*tan(1/2*d*x+1/2*c)^3*a^4+8*a^2*b^2*tan(1/2*d*x+1/2*c)^3+24*b*a^3*tan(1/
2*d*x+1/2*c)^2-40*a*b^3*tan(1/2*d*x+1/2*c)^2+22*a^4*tan(1/2*d*x+1/2*c)-216*
a^2*b^2*tan(1/2*d*x+1/2*c)+240*b^4*tan(1/2*d*x+1/2*c))-1/160/a^3/tan(1/2*d*
```

```
x+1/2*c)^5-1/96*(-7*a^2+24*b^2)/a^5/tan(1/2*d*x+1/2*c)^3-1/32*(22*a^4-216*a
^2*b^2+240*b^4)/a^7/tan(1/2*d*x+1/2*c)+3/64*b/a^4/tan(1/2*d*x+1/2*c)^4-1/4/
a^6*b*(3*a^2-5*b^2)/tan(1/2*d*x+1/2*c)^2-1/8/a^8*b*(45*a^4-200*a^2*b^2+168*
b^4)*ln(tan(1/2*d*x+1/2*c))-2/a^8*(((5/2*a^5*b^2-19/2*a^3*b^4+7*a*b^6)*tan(
1/2*d*x+1/2*c)^3+1/2*b*(4*a^6-9*a^4*b^2-21*a^2*b^4+26*b^6)*tan(1/2*d*x+1/2*
c)^2+1/2*a*b^2*(11*a^4-49*a^2*b^2+38*b^4)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*(4*a
^4-17*a^2*b^2+13*b^4))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+
1/2*(2*a^6-31*a^4*b^2+71*a^2*b^4-42*b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(467) = 934$.
time = 0.72, size = 2571, normalized size = 5.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^
7 + 67*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 3
7*a^5*b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 60*(2*(2*a^5*b - 29*a
^3*b^3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*
a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 4
2*a*b^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6
 - 2*a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*
b^4 + 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*
cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b
*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2)) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6)*cos(d*x + c)
 + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 +
 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d
```

x + c)^4 - 6(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a
^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168
*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(
d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^
2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2 - 400*a^3*b^4
 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*
(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200
*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5
 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*
b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^
4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d
x + c) + 1/2) - 2((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)*cos(d*x + c)^
5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^3 + 15*(53*a^6
*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(2*a^9*b*d*cos(
d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c)^2 - 2*a^9*b*
d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x + c)^4 + (2*a^
10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*x + c)), -1/24
0*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^7 + 67
*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 37*a^5*
b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 120*(2*(2*a^5*b - 29*a^3*b^
3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*a^5*b
 - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b
^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6 - 2*
a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*b^4 +
 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*cos(d
*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt
(a^2 - b^2)*cos(d*x + c))) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6
)*cos(d*x + c) + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 -
 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 16
8*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x
 + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200
*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 +
 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)
*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2
 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d
x + c)^6 + 6(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45
*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^
3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c
)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*
a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*
log(-1/2*cos(d*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)
*cos(d*x + c)^5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^
3 + 15*(53*a^6*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(
2*a^9*b*d*cos(d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c
)^2 - 2*a^9*b*d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x

```
+ c)^4 + (2*a^10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*
x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**6/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 11.78, size = 731, normalized size = 1.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/960*(120*(45*a^4*b - 200*a^2*b^3 + 168*b^5)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^8 + 960*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a^8) + 960*(5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 19*a^3*b
^4*tan(1/2*d*x + 1/2*c)^3 + 14*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*tan(1
/2*d*x + 1/2*c)^2 - 9*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*b^5*tan(1/2*d
*x + 1/2*c)^2 + 26*b^7*tan(1/2*d*x + 1/2*c)^2 + 11*a^5*b^2*tan(1/2*d*x + 1/
2*c) - 49*a^3*b^4*tan(1/2*d*x + 1/2*c) + 38*a*b^6*tan(1/2*d*x + 1/2*c) + 4*
a^6*b - 17*a^4*b^3 + 13*a^2*b^5)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d
*x + 1/2*c) + a)^2*a^8) - (12330*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 54800*a^2*b
^3*tan(1/2*d*x + 1/2*c)^5 + 46032*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(
1/2*d*x + 1/2*c)^4 + 6480*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 7200*a*b^4*tan(1
/2*d*x + 1/2*c)^4 - 720*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 1200*a^2*b^3*tan(1/2
*d*x + 1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 240*a^3*b^2*tan(1/2*d*x +
 1/2*c)^2 + 45*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^8*tan(1/2*d*x + 1/2*c
)^5) - (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^11*b*tan(1/2*d*x + 1/2*c)^4 -
70*a^12*tan(1/2*d*x + 1/2*c)^3 + 240*a^10*b^2*tan(1/2*d*x + 1/2*c)^3 + 720*
a^11*b*tan(1/2*d*x + 1/2*c)^2 - 1200*a^9*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a
^12*tan(1/2*d*x + 1/2*c) - 6480*a^10*b^2*tan(1/2*d*x + 1/2*c) + 7200*a^8*b^
4*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 7.30, size = 1614, normalized size = 3.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + b*sin(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a^3*d) - (tan(c/2 + (d*x)/2)^3*((a^2 + 4*b^2)/(32
*a^5) + 1/(24*a^3) - (3*b^2)/(8*a^5)))/d + (tan(c/2 + (d*x)/2)*(1/(8*a^3) -
 (3*(a^2 + 4*b^2))/(32*a^5) - (6*b*((6*b*((3*(a^2 + 4*b^2))/(32*a^5) + 1/(8
*a^3) - (9*b^2)/(8*a^5)))/a - (384*a^2*b + 256*b^3)/(1024*a^6) + (9*b*(a^2
+ 4*b^2))/(16*a^6)))/a + (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(32*a^5) + 1/(
8*a^3) - (9*b^2)/(8*a^5)))/a^2 + (3*b*(384*a^2*b + 256*b^3))/(512*a^7)))/d
- (tan(c/2 + (d*x)/2)^3*((187*a^5*b)/15 - 14*a^3*b^3) + a^6/5 + tan(c/2 + (
d*x)/2)^4*((263*a^6)/15 + 112*a^2*b^4 - (358*a^4*b^2)/3) + tan(c/2 + (d*x)/
2)^5*(1216*a*b^5 + (1519*a^5*b)/6 - 1360*a^3*b^3) - tan(c/2 + (d*x)/2)^2*((
29*a^6)/15 - (14*a^4*b^2)/5) + tan(c/2 + (d*x)/2)^8*(22*a^6 + 448*b^6 - 368
*a^2*b^4 - 56*a^4*b^2) + tan(c/2 + (d*x)/2)^6*((125*a^6)/3 + 2176*b^6 - 211
2*a^2*b^4 + 112*a^4*b^2) + (8*tan(c/2 + (d*x)/2)^7*(30*a^6*b + 104*b^7 + 36
*a^2*b^5 - 149*a^4*b^3))/a - (7*a^5*b*tan(c/2 + (d*x)/2))/10)/(d*(32*a^9*ta
n(c/2 + (d*x)/2)^5 + 32*a^9*tan(c/2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^7*(64
*a^9 + 128*a^7*b^2) + 128*a^8*b*tan(c/2 + (d*x)/2)^6 + 128*a^8*b*tan(c/2 +
(d*x)/2)^8)) + (tan(c/2 + (d*x)/2)^2*((3*b*((3*(a^2 + 4*b^2))/(32*a^5) + 1/
(8*a^3) - (9*b^2)/(8*a^5)))/a - (384*a^2*b + 256*b^3)/(2048*a^6) + (9*b*(a^
2 + 4*b^2))/(32*a^6)))/d - (log(tan(c/2 + (d*x)/2))*(45*a^4*b + 168*b^5 - 2
00*a^2*b^3))/(8*a^8*d) - (3*b*tan(c/2 + (d*x)/2)^4)/(64*a^4*d) - (atan((((-
(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^
6 + 121*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b
- 672*a^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13) + ((-(a + b)*(a - b)
)^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(
a^4 + 21*b^4 - (29*a^2*b^2)/2))/a^8)*1i)/a^8 + ((-(a + b)*(a - b))^(1/2)*(a
^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^6 + 121*a^10*b^4 - (169*a
^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b - 672*a^6*b^7 + 1136*a^8*
b^5 - 538*a^10*b^3))/(4*a^13) - ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c
/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(a^4 + 21*b^4 - (29*a^2*b^
2)/2))/a^8)*1i)/a^8)/(((45*a^10*b)/2 - 1764*b^11 + 5082*a^2*b^9 - (10649*a^
4*b^7)/2 + (9731*a^6*b^5)/4 - (1795*a^8*b^3)/4)/a^14 + (tan(c/2 + (d*x)/2)*
(16*a^10 - 3528*b^10 + 9282*a^2*b^8 - 8549*a^4*b^6 + 3185*a^6*b^4 - 406*a^8
*b^2))/(2*a^13) - ((-(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)
*((2*a^14 - 84*a^8*b^6 + 121*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 +
 (d*x)/2)*(61*a^12*b - 672*a^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13)
 + ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a
^14*b^2))/(4*a^13))*(a^4 + 21*b^4 - (29*a^2*b^2)/2))/a^8))/a^8 + ((-(a + b)
*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^6 + 121
*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b - 672*a
^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13) - ((-(a + b)*(a - b))^(1/2)
*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(a^4 + 2
1*b^4 - (29*a^2*b^2)/2))/a^8))/a^8))*(-(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4
```

```
- (29*a^2*b^2)/2)*2i)/(a^8*d)
```

3.203 $\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=271

$$\frac{a^3\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)}+\frac{3a^2b\cos^2(e+fx)^{\frac{1+p}{2}}\,{}_2F_1\left(\frac{1+p}{2},\frac{2+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)}{fg(2+p)}$$

```
[Out] a^3*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p
)/f/g/(1+p)+3*a^2*b*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*
p],[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)+b^3*(c
os(f*x+e)^2)^(1/2+1/2*p)*hypergeom([2+1/2*p, 1/2+1/2*p],[3+1/2*p],sin(f*x+e
)^2)*sin(f*x+e)^3*(g*tan(f*x+e))^(1+p)/f/g/(4+p)+3*a*b^2*hypergeom([2, 3/2+
1/2*p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/g^3/(3+p)
```

Rubi [A]

time = 0.27, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2801, 3557, 371, 2682, 2657, 2671}

$$\frac{a^3(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{3a^2b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{fg(p+2)}+\frac{3ab^2(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{b^3\sin^3(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+4}{2};\frac{p+6}{2};\sin^2(e+fx)\right)}{fg(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^2*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hyper
geometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*
(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^3*(Cos[e + f*x]^2)^((1 + p)/2)
*Hypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e +
 f*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a*b^2*Hypergeometric2F
1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^
3*(3 + p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
```

```
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx &= \int \left(a^3(g\tan(e+fx))^p+3a^2b\sin(e+fx)(g\tan(e+fx))^p+3a\right.\\ &= a^3\int (g\tan(e+fx))^p\,dx+\left(3a^2b\right)\int \sin(e+fx)(g\tan(e+fx\\ &= \frac{\left(a^3g\right)\operatorname{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx,x,g\tan(e+fx)\right)}{f}+\frac{\left(3ab^2g\right)\operatorname{Subst}\left(\int\right.}{}\\ &= \frac{a^3\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)}+\frac{3a^2b}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 14.62, size = 4791, normalized size = 17.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^3*(2 + p)*Appell
F1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2
*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + p)*(3*a^2*AppellF1[1 + p/2,
p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(AppellF1[1
 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[
1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e +
 f*x)/2]))*(g*Tan[e + f*x])^p*(-1/8*(b^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p) -
 a^3*Sin[e + f*x]^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p + ((3*I)/8)*b^3*Sin[2*(
e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (3*b^3*Sin[2*(e + f*x)]^2*Sin[3
*(e + f*x)]*Tan[e + f*x]^p)/8 - (I/8)*b^3*Sin[2*(e + f*x)]^3*Sin[3*(e + f*x
)]*Tan[e + f*x]^p + Cos[e + f*x]^3*(a^3*Cos[3*(e + f*x)]*Tan[e + f*x]^p - I
*a^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Cos[2*(e + f*x)]^3*((I/8)*b^3*Cos[3
*(e + f*x)]*Tan[e + f*x]^p + (b^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8) + Sin
[e + f*x]^2*((-3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 + ((3*I)/2)*a^2*b
*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Sin[e + f*x]*((-3*a*b^
2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/4 + ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Sin
[3*(e + f*x)]*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]
*Tan[e + f*x]^p)/4) + Cos[2*(e + f*x)]^2*((-3*b^3*Sin[3*(e + f*x)]*Tan[e +
f*x]^p)/8 - (3*a*b^2*Sin[e + f*x]*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/4 + ((3*
I)/8)*b^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + Cos[3*(e + f*x
)]*(((-3*I)/8)*b^3*Tan[e + f*x]^p - ((3*I)/4)*a*b^2*Sin[e + f*x]*Tan[e + f*
x]^p - (3*b^3*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/8)) + Cos[3*(e + f*x)]*((-1/
8*I)*b^3*Tan[e + f*x]^p - I*a^3*Sin[e + f*x]^3*Tan[e + f*x]^p - (3*b^3*Sin[
2*(e + f*x)]*Tan[e + f*x]^p)/8 + ((3*I)/8)*b^3*Sin[2*(e + f*x)]^2*Tan[e + f
*x]^p + (b^3*Sin[2*(e + f*x)]^3*Tan[e + f*x]^p)/8 + Sin[e + f*x]^2*(((-3*I)
/2)*a^2*b*Tan[e + f*x]^p - (3*a^2*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/2) + S
in[e + f*x]*(((-3*I)/4)*a*b^2*Tan[e + f*x]^p - (3*a*b^2*Sin[2*(e + f*x)]*Ta
n[e + f*x]^p)/2 + ((3*I)/4)*a*b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)) + Cos
[e + f*x]^2*((3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 + 3*a^3*Sin[e + f*
x]*Sin[3*(e + f*x)]*Tan[e + f*x]^p - ((3*I)/2)*a^2*b*Sin[2*(e + f*x)]*Sin[3
*(e + f*x)]*Tan[e + f*x]^p + Cos[3*(e + f*x)]*(((3*I)/2)*a^2*b*Tan[e + f*x]
^p + (3*I)*a^3*Sin[e + f*x]*Tan[e + f*x]^p + (3*a^2*b*Sin[2*(e + f*x)]*Tan[
e + f*x]^p)/2) + Cos[2*(e + f*x)]*(((-3*I)/2)*a^2*b*Cos[3*(e + f*x)]*Tan[e
+ f*x]^p - (3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2)) + Cos[e + f*x]*(((
3*I)/4)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (3*I)*a^3*Sin[e + f*x]^2*Si
n[3*(e + f*x)]*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*
```

```
Tan[e + f*x]^p)/2 - ((3*I)/4)*a*b^2*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]*Tan
[e + f*x]^p + Cos[2*(e + f*x)]^2*((-3*a*b^2*Cos[3*(e + f*x)]*Tan[e + f*x]^p
)/4 + ((3*I)/4)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Sin[e + f*x]*((3*I
)*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p + 3*a^2*b*Sin[2*(e + f*x)]*Sin[3*(e
 + f*x)]*Tan[e + f*x]^p) + Cos[3*(e + f*x)]*((-3*a*b^2*Tan[e + f*x]^p)/4 -
3*a^3*Sin[e + f*x]^2*Tan[e + f*x]^p + ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Tan[
e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/4 + Sin[e + f*x]*(
-3*a^2*b*Tan[e + f*x]^p + (3*I)*a^2*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p)) + C
os[2*(e + f*x)]*(((-3*I)/2)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*I)*a
^2*b*Sin[e + f*x]*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*a*b^2*Sin[2*(e + f*x
)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 + Cos[3*(e + f*x)]*((3*a*b^2*Tan[e +
f*x]^p)/2 + 3*a^2*b*Sin[e + f*x]*Tan[e + f*x]^p - ((3*I)/2)*a*b^2*Sin[2*(e
+ f*x)]*Tan[e + f*x]^p))) + Cos[2*(e + f*x)]*((3*b^3*Sin[3*(e + f*x)]*Tan[e
 + f*x]^p)/8 + (3*a^2*b*Sin[e + f*x]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 -
 ((3*I)/4)*b^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*b^3*Si
n[2*(e + f*x)]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8 + Sin[e + f*x]*((3*a*b^
2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 - ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Sin
[3*(e + f*x)]*Tan[e + f*x]^p) + Cos[3*(e + f*x)]*(((3*I)/8)*b^3*Tan[e + f*x
]^p + ((3*I)/2)*a^2*b*Sin[e + f*x]^2*Tan[e + f*x]^p + (3*b^3*Sin[2*(e + f*x
)]*Tan[e + f*x]^p)/4 - ((3*I)/8)*b^3*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p + Si
n[e + f*x]*(((3*I)/2)*a*b^2*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]*Tan[
e + f*x]^p)/2)))))/(f*(1 + p)*(2 + p)*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^
2)^p*Sec[e + f*x]^2*Tan[(e + f*x)/2]*(a^3*(2 + p)*AppellF1[(1 + p)/2, p, 1,
 (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(6*a*b*(2 + p)*A
ppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] - 6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (1 + p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ...
```

Maple [F]

time = 1.12, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(fx+e\right)\right)^{3}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="fricas")
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*(g*tan(f*x + e))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g\tan(e+fx))^p (a+b\sin(e+fx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(g*tan(f*x+e))**p,x)
[Out] Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\tan(e+f\,x)\right)}^p\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^3,x)
[Out] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^3, x)
```

3.204 $\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=186

$$\frac{a^2\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)}+\frac{2ab\cos^2(e+fx)^{\frac{1+p}{2}}\,{}_2F_1\left(\frac{1+p}{2},\frac{2+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right.}{fg(2+p)}$$

```
[Out] a^2*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p
)/f/g/(1+p)+2*a*b*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p]
,[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)+b^2*hype
rgeom([2, 3/2+1/2*p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/g^3/
(3+p)
```

Rubi [A]

time = 0.16, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2801, 3557, 371, 2682, 2657, 2671}

$$\frac{a^2(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{fg(p+2)}+\frac{b^2(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (2*a*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^2*Hypergeometric2F1[2, (3 + p)/2
, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx &= \int \left(a^2(g\tan(e+fx))^p+2ab\sin(e+fx)(g\tan(e+fx))^p+b^2\operatorname{si}\right.\\ &= a^2\int (g\tan(e+fx))^p\,dx+(2ab)\int \sin(e+fx)(g\tan(e+fx))^p\\ &= \frac{(a^2g)\operatorname{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx,x,g\tan(e+fx)\right)}{f}+\frac{(b^2g)\operatorname{Subst}\left(\int \frac{x^{2-}}{(g^2+}\right.}{}\\ &= \frac{a^2\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)}+\frac{2ab\cos}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 13.43, size = 2464, normalized size = 13.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

```
[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^2*(2 + p)*Appell
F1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4
*b*(b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] - b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*(g*Tan[e + f*
x])^p*(-1/4*(b^2*Cos[2*(e + f*x)]^3*Tan[e + f*x]^p) + (I/4)*b^2*Sin[2*(e +
f*x)]*Tan[e + f*x]^p + I*a^2*Sin[e + f*x]^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p
 + (b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/2 - (I/4)*b^2*Sin[2*(e + f*x)]^3
*Tan[e + f*x]^p + Cos[e + f*x]^2*(a^2*Cos[2*(e + f*x)]*Tan[e + f*x]^p - I*a
^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p) + Cos[2*(e + f*x)]^2*((b^2*Tan[e + f*x]
^p)/2 + a*b*Sin[e + f*x]*Tan[e + f*x]^p - (I/4)*b^2*Sin[2*(e + f*x)]*Tan[e
+ f*x]^p) + Sin[e + f*x]*(I*a*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p + a*b*Sin[2
*(e + f*x)]^2*Tan[e + f*x]^p) + Cos[2*(e + f*x)]*(-1/4*(b^2*Tan[e + f*x]^p)
 - a*b*Sin[e + f*x]*Tan[e + f*x]^p - a^2*Sin[e + f*x]^2*Tan[e + f*x]^p - (b
^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/4) + Cos[e + f*x]*((-I)*a*b*Cos[2*(e
+ f*x)]^2*Tan[e + f*x]^p + a*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p + 2*a^2*Sin[
e + f*x]*Sin[2*(e + f*x)]*Tan[e + f*x]^p - I*a*b*Sin[2*(e + f*x)]^2*Tan[e +
 f*x]^p + Cos[2*(e + f*x)]*(I*a*b*Tan[e + f*x]^p + (2*I)*a^2*Sin[e + f*x]*T
an[e + f*x]^p))))/(f*(1 + p)*(2 + p)*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2
)^p*Sec[e + f*x]^2*Tan[(e + f*x)/2]*(a^2*(2 + p)*AppellF1[(1 + p)/2, p, 1,
(3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b*(b*(2 + p)*Appell
F1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b
*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan[e + f*x]^(-1 + p))/((1 + p)*(
2 + p)) + (Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(a^2*(2 +
 p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + 4*b*(b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] - b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p,
2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan
[e + f*x]^p)/((1 + p)*(2 + p)) + (2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1
 + p)*Tan[(e + f*x)/2]*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])*(a^2*(2 + p)*AppellF1[(1 + p)/2, p, 1,
(3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b*(b*(2 + p)*Appell
F1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b
*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan[e + f*x]^p)/((1 + p)*(2 + p))
 + (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^2*(2 + p)*(-(
```

```
((1 + p)*AppellF1[1 + (1 + p)/2, p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (p*(1 +
 p)*AppellF1[1 + (1 + p)/2, 1 + p, 1, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + 4*b*((a*
(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2)/2 + a*(1 + p)*Tan[(e + f*x)/2]*((-2*(1 + p/2)*Ap
pellF1[2 + p/2, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + ((1 + p/2)*p*AppellF1[2 + p/2
, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/(2 + p/2)) + b*(2 + p)*((-2*(1 + p)*AppellF1[1 + (1
 + p)/2, p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[
(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1 + p)/
2, 1 + p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) - b*(2 + p)*((-3*(1 + p)*AppellF1[1
 + (1 + p)/2, p, 4, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1
+ p)/2, 1 + p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p))))*Tan[e + f*x]^p)/((1 + p)*(2
+ p))))
```

Maple [F]
time = 1.04, size = 0, normalized size = 0.00

$$\int (a+b\sin(fx+e))^2 (g\tan(fx+e))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*(g*tan(f*x
+ e))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g\tan(e+fx))^p (a+b\sin(e+fx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(g*tan(f*x+e))**p,x)
```

```
[Out] Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\tan(e+f\,x))^p\,(a+b\,\sin(e+f\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^2,x)
```

```
[Out] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^2, x)
```

3.205 $\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=129

$$\frac{a\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)}+\frac{b\cos^2(e+fx)^{\frac{1+p}{2}}\,{}_2F_1\left(\frac{1+p}{2},\frac{2+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sin(}{fg(2+p)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(1+p)/
f/g/(1+p)+b*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p],[2+1/
2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(1+p)/f/g/(2+p)
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2801, 3557, 371, 2682, 2657}

$$\frac{a(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{fg(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f
*x])^(1 + p))/(f*g*(1 + p)) + (b*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometri
c2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e
 + f*x])^(1 + p))/(f*g*(2 + p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
```

```
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx = \int \left(a(g\tan(e+fx))^p + b\sin(e+fx)(g\tan(e+fx))^p\right)\,dx$$

$$= a\int (g\tan(e+fx))^p\,dx + b\int \sin(e+fx)(g\tan(e+fx))^p\,dx$$

$$= \frac{(ag)\text{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{(b\cos^{1+p}(e+fx)\sin}{}$$

$$= \frac{a\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{b\cos^2(e}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 4.95, size = 849, normalized size = 6.58

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

```
[Out] (2*(a + b*Sin[e + f*x])*Tan[(e + f*x)/2]*(a*(2 + p)*AppellF1[(1 + p)/2, p,
1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*Appell
F1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e
+ f*x)/2])*(g*Tan[e + f*x])^p)/(f*(Sec[(e + f*x)/2]^2*(a*(2 + p)*AppellF1[(
1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(
1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
```

```
2]^2]*Tan[(e + f*x)/2]) - 16*p*Cos[(e + f*x)/2]*Csc[e + f*x]^3*Sec[e + f*x]
*Sin[(e + f*x)/2]^5*(a*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*AppellF1[1 + p/2, p, 2, 2 +
 p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]) + 2*p*Csc[
e + f*x]*Sec[e + f*x]*Tan[(e + f*x)/2]*(a*(2 + p)*AppellF1[(1 + p)/2, p, 1,
 (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*AppellF1
[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e +
f*x)/2]) + 2*(1 + p)*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(b*AppellF1[1 + p/
2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (a*(2 + p)*(-A
ppellF1[(3 + p)/2, p, 2, (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + p*AppellF1[(3 + p)/2, 1 + p, 1, (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2])/(3 + p) + (2*b*(2 + p)*(-2*AppellF1[2 + p/2
, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[2 +
p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
f*x)/2]^2)/(4 + p))))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int (a+b\sin(fx+e))(g\tan(fx+e))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x)
[Out] int((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g\tan(e+fx))^p (a+b\sin(e+fx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))**p,x)
```

```
[Out] Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\tan(e+f\,x))^p\, (a+b\,\sin(e+f\,x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x)),x)
```

```
[Out] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x)), x)
```

3.206 $\int \frac{(g \tan(e+fx))^p}{a+b\sin(e+fx)}\, dx$

Optimal. Leaf size=284

$$\frac{ag\left(1-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)^{\frac{1}{2}(-1+p)} {}_2F_1\left(\frac{1-p}{2},\frac{1-p}{2};\frac{3-p}{2};\frac{\cos^2(e+fx)-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}}{1-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}}\right)\sin^2(e+fx)^{\frac{1-p}{2}}(g\tan(e+fx))^{-1+p}}{(a^2-b^2)\, f(-1+p)} + \cdots$$

```
[Out] a*g*(1-b^2*cos(f*x+e)^2/(-a^2+b^2))^(-1/2+1/2*p)*Hypergeometric2F1(1/2-1/2*
p,1/2-1/2*p,3/2-1/2*p,(cos(f*x+e)^2-b^2*cos(f*x+e)^2/(-a^2+b^2))/(1-b^2*cos
(f*x+e)^2/(-a^2+b^2)))*(sin(f*x+e)^2)^(1/2-1/2*p)*(g*tan(f*x+e))^(-1+p)/(a^
2-b^2)/f/(-1+p)+b*AppellF1(1/2-1/2*p,-1/2*p,1,3/2-1/2*p,cos(f*x+e)^2,b^2*co
s(f*x+e)^2/(-a^2+b^2))*cos(f*x+e)*(g*tan(f*x+e))^p/(-a^2+b^2)/f/(-1+p)/((si
n(f*x+e)^2)^(1/2*p))
```

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\, dx$$

Verification is not applicable to the result.

```
[In] Int[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]),x]
```

```
[Out] Defer[Int][(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\, dx = \int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\, dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 858 vs. 2(284) = 568.
time = 13.04, size = 858, normalized size = 3.02

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Tan[e + f*x]^(1 + p)*(g*Tan[e + f*x])^p*((a^2 - b^2)*(1 + p)*AppellF1[(2 +
 p)/2, -1/2, 1, (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*
Tan[e + f*x] + a*(b*(2 + p)*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, ((-a
^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + p)*Hypergeometric2F1[1/2, 1 + p/2,
2 + p/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*b*f*(1 + p)*(2 + p)*(a + b*S
in[e + f*x])*((Sec[e + f*x]^2*Tan[e + f*x]^p*((a^2 - b^2)*(1 + p)*AppellF1[
(2 + p)/2, -1/2, 1, (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]
^2]*Tan[e + f*x] + a*(b*(2 + p)*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2,
((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + p)*Hypergeometric2F1[1/2, 1 + p
/2, 2 + p/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*b*(2 + p)) + (Tan[e + f*
x]^(1 + p)*((a^2 - b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 1, (4 + p)/2, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 + (a^2 - b^2)*(
1 + p)*Tan[e + f*x]*((2*(-1 + b^2/a^2)*(2 + p)*AppellF1[1 + (2 + p)/2, -1/2
, 2, 1 + (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e +
 f*x]^2*Tan[e + f*x])/(4 + p) + ((2 + p)*AppellF1[1 + (2 + p)/2, 1/2, 1, 1
+ (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2
*Tan[e + f*x])/(4 + p)) + a*(-(a*(1 + p)*Hypergeometric2F1[1/2, 1 + p/2, 2
+ p/2, -Tan[e + f*x]^2]*Sec[e + f*x]^2) - 2*a*(1 + p/2)*(1 + p)*Sec[e + f*x
]^2*(-Hypergeometric2F1[1/2, 1 + p/2, 2 + p/2, -Tan[e + f*x]^2] + 1/Sqrt[1
+ Tan[e + f*x]^2]) + b*(1 + p)*(2 + p)*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeo
metric2F1[1, (1 + p)/2, (3 + p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1
- ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2)^(-1)))))/(a^2*b*(1 + p)*(2 + p))))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(g \tan (fx+e))^p}{a+b \sin (fx+e)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x)
[Out] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+b*sin(f*x+e)),x)
[Out] Integral((g*tan(e + f*x))**p/(a + b*sin(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\tan(e+f\,x))^p}{a+b\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p/(a + b*sin(e + f*x)),x)
[Out] int((g*tan(e + f*x))^p/(a + b*sin(e + f*x)), x)
```

3.207 $\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=737

$$\frac{a^2\cos(e+fx)\left(1-\cos^2(e+fx)\right)^{\frac{1}{2}(-1+q)}\left(1-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)^{-2+\frac{3-q}{2}+\frac{1}{2}(-1+q)}\left(\left(2(a^2-b^2)+b^2(1+q)\cos^2(e+\right.\right.}{2}$$

```
[Out] 1/2*a^2*cos(f*x+e)*(1-cos(f*x+e)^2)^(-1/2+1/2*q)/(1-b^2*cos(f*x+e)^2/(-a^2+
b^2))*((2*a^2-2*b^2+b^2*(1+q)*cos(f*x+e)^2)*HurwitzLerchPhi(a^2*cos(f*x+e)^
2/(a^2-b^2)/(-1+cos(f*x+e)^2),1,1/2-1/2*q)-b^2*(-1+q)*cos(f*x+e)^2*HurwitzL
erchPhi(a^2*cos(f*x+e)^2/(a^2-b^2)/(-1+cos(f*x+e)^2),1,3/2-1/2*q))*sin(f*x+
e)*(sin(f*x+e)^2)^(-1/2-1/2*q)*(g*tan(f*x+e))^q/(a^2-b^2)^2/(-a^2+b^2)/f-a^
2*cos(f*x+e)*(1-b^2*cos(f*x+e)^2/(-a^2+b^2))^(-1/2+1/2*q)*Hypergeometric2F1
(1/2-1/2*q,1/2-1/2*q,3/2-1/2*q,(cos(f*x+e)^2-b^2*cos(f*x+e)^2/(-a^2+b^2))/(
1-b^2*cos(f*x+e)^2/(-a^2+b^2)))*sin(f*x+e)*(sin(f*x+e)^2)^(-1/2-1/2*q)*(g*t
an(f*x+e))^q/(a^2-b^2)^2/f/(-1+q)+b^2*cos(f*x+e)*(1-b^2*cos(f*x+e)^2/(-a^2+
b^2))^(-1/2+1/2*q)*Hypergeometric2F1(1/2-1/2*q,1/2-1/2*q,3/2-1/2*q,(cos(f*x
+e)^2-b^2*cos(f*x+e)^2/(-a^2+b^2))/(1-b^2*cos(f*x+e)^2/(-a^2+b^2)))*sin(f*x
+e)*(sin(f*x+e)^2)^(-1/2-1/2*q)*(g*tan(f*x+e))^q/(a^2-b^2)^2/f/(-1+q)-2*a*b
*AppellF1(1/2-1/2*q,-1/2*q,2,3/2-1/2*q,cos(f*x+e)^2,b^2*cos(f*x+e)^2/(-a^2+
b^2))*cos(f*x+e)*(g*tan(f*x+e))^q/(a^2-b^2)^2/f/(-1+q)/((sin(f*x+e)^2)^(1/2
*q))
```

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$$

Mathematica [A]
time = 5.29, size = 695, normalized size = 0.94

$$\frac{\cos(e+fx)\sin(e+fx)(g\tan(e+fx))^p\left(a(2+p)\left((a^2+b^2)\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)-2b^2\,{}_2F_1\left(2,\frac{1+p}{2};\frac{3+p}{2};\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)\right)+2b(-a^2+b^2)(1+p)F_1\left(\frac{2+p}{2};-\frac{1}{2},2;\frac{4+p}{2};-\tan^2(e+fx),\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)\tan(e+fx)\right)}{f(1+p)(a+b\sin(e+fx))^2\left(a(2+p)\left((a^2+b^2)\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)-2b^2\,{}_2F_1\left(2,\frac{1+p}{2};\frac{3+p}{2};\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)\right)+2b(-a^2+b^2)F_1\left(\frac{2+p}{2};-\frac{1}{2},2;\frac{4+p}{2};-\tan^2(e+fx),\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)\tan(e+fx)+2b(-a^2+b^2)(1+p)F_1\left(\frac{2+p}{2};-\frac{1}{2},2;\frac{4+p}{2};-\tan^2(e+fx),\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)\tan(e+fx)+\frac{2b(-a^2+b^2)(2+p)\left(\left(-4+\frac{4b^2}{a^2}\right)F_1\left(\frac{4+p}{2};-\frac{1}{2},3;\frac{6+p}{2};-\tan^2(e+fx),\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)+F_1\left(\frac{4+p}{2};\frac{1}{2},2;\frac{6+p}{2};-\tan^2(e+fx),\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)\right)\tan^3(e+fx)}{4+p}+a(2+p)\left(-2b^2\left(-{}_2F_1\left(2,\frac{1+p}{2};\frac{3+p}{2};\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)+\frac{1}{\left(1+\left(1-\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)^2}\right)+(a^2+b^2)\left(-{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+fx)\right)+\frac{1}{1+\left(1-\frac{b^2}{a^2}\right)\tan^2(e+fx)}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(g*Tan[e + f*x])^p*(a*(2 + p)*((a^2 + b^2)*Hyper
geometric2F1[1, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 2*b^
2*Hypergeometric2F1[2, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2]
) + 2*b*(-a^2 + b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 2, (4 + p)/2, -Tan[e
 + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]))/(f*(1 + p)*(a + b*
Sin[e + f*x])^2*(a*(2 + p)*((a^2 + b^2)*Hypergeometric2F1[1, (1 + p)/2, (3
+ p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 2*b^2*Hypergeometric2F1[2, (1 + p)
/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(-a^2 + b^2)*AppellF1[
(2 + p)/2, -1/2, 2, (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]
^2]*Tan[e + f*x] + 2*b*(-a^2 + b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 2, (4
 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x] + (2*
b*(-a^2 + b^2)*(2 + p)*((-4 + (4*b^2)/a^2)*AppellF1[(4 + p)/2, -1/2, 3, (6
+ p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + AppellF1[(4 + p)/
2, 1/2, 2, (6 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Tan[
e + f*x]^3)/(4 + p) + a*(2 + p)*(-2*b^2*(-Hypergeometric2F1[2, (1 + p)/2, (
3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (1 + (1 - b^2/a^2)*Tan[e + f*x]^
2)^(-2)) + (a^2 + b^2)*(-Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, (-1 + b
^2/a^2)*Tan[e + f*x]^2] + (1 + (1 - b^2/a^2)*Tan[e + f*x]^2)^(-1)))))
```

Maple [F]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{(g\tan(fx+e))^p}{(a+b\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
[Out] integral(-(g*tan(f*x + e))^p/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2
 - b^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\tan{(e+fx)})^p}{(a+b\sin{(e+fx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+b*sin(f*x+e))**2,x)
[Out] Integral((g*tan(e + f*x))**p/(a + b*sin(e + f*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="giac")
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\tan(e+f\,x))^p}{(a+b\,\sin{(e+f\,x)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p/(a + b*sin(e + f*x))^2,x)
[Out] int((g*tan(e + f*x))^p/(a + b*sin(e + f*x))^2, x)
```

3.208 $\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=26

$$\text{Int}((a+b\sin(e+fx))^m(g\tan(e+fx))^p,x)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx = \int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$$

Mathematica [A]
time = 1.66, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
[Out] Integrate[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g\tan{(e+fx)})^p\,(a+b\sin{(e+fx)})^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(g*tan(f*x+e))**p,x)
```

```
[Out] Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x))**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(g\,\mathrm{tan}\left(e+f\,x\right)\right)}^p\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^m,x)
[Out] int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^m, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section 986
4.2 Listing of Grading functions 986

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```